Abstract—We consider a group of problems related to development, progress, and change which are solved through formal models and methods. Among such models are conceptual models, mathematical and computational models, and methods for optimization and operations research problem-solving. Such models and methods shall enable the evaluation of the predicted values of the operational (pragmatic) properties of systems depending on the IT used for system progress and changes in general. The corresponding IT usage, dynamic capability, organizational capability, human and social capability, innovation capability, and potential system indicators can be measured as a result. The analytical estimation of such indicators depending on the variables and options in the mathematical problems to be solved becomes possible. This could lead to a solution to contemporary research problems using predictive analytical mathematical models and methods.

I. INTRODUCTION

Group of development, progress and change problems have three considerations.

First, the distinguishing feature of these problems is the necessity of progress in modelling and organization. This is crucial to overcome most problems in developing nations. We consider innovation, digitalization, and information technology use as a kind of system reaction to the need for change and a reaction to the necessity of progress, generally caused by the environment. Unfortunately, there is a gap between the need for change and its organization (including innovation, digitalization, information technology use, the rise in human capabilities, etc.) and the available means to research, plan, and organize change. Further, we will discuss these issues as progress problems. One cannot solve progress problems and organize progress if mechanisms are not yet researched in detail.

Second, to organize progress, especially with mathematical models and methods, one shall measure it. Such measurement is preferably done using analytical, predictive dependencies of progress quality measures depending on the variables and parameters of the problems. Unfortunately, a gap exists between the need to estimate progress properties (for example, innovation, digitalization, information technology performance measurement, human capability assessment, dynamic capabilities estimation, etc.) and the available models, methods, and frameworks (especially analytical and predictive ones) for such estimation. It is problematic to organize something one cannot estimate and predict.

Third, effective and efficient methods to solve problems shall welcome models, methods, knowledge, and framework sharing, preferably among most nations. That is why it is better to use models and methods which enable the formal description and automation of problem-solving (for example, based on mathematical programming solvers), problem description sharing (for example, based on conceptual models), and model construction (for example, based on model-driven architecture). Unfortunately, there is a discrepancy between reality and the need for such models, methods, and frameworks to improve problem-solving. We suggest concepts, models, and methods that make it possible to overcome the above-mentioned gaps. Progress problems are considered in unified, threefold way.

First, the conceptual models of different types of progress problems are taken into account. These types include problems of innovation, digitalization estimation and organization, information technology performance estimation, human capability estimation, and dynamic capability estimation. This concept includes a systematic, unified consideration of the system and its environment function because of the interplay between the two. System environment changes are seen as causing system reactions. Causes and reactions are taken into account in the functioning domain—both for the system and its environment. Reactions (changes in function) are possible in some moments and form a common sequence of possible alternative reactions, again, for both the system and its environment. Thus, such sequence of moments is used to integrate a system’s models and its environmental function. A system may react to environmental changes because information operates on the border with its environment. Information operations may cause further (possibly an alternative) functioning of the system to provide the necessary changes, which may further cause changes in functioning effects and in goals achieved. As a result, a conceptual model of possible functioning chains for the system in different environmental conditions for given technologies (including information technology and innovation technology) is created. Depending on the technologies used, different chains of functioning are possible with different characteristics. The concept determines which cause–effect
relations shall be considered when modeling different changes of functions and their effects. Such a conceptual model is considered as a basis to build further (graph-theoretical, functional, program) models and to explain the formation of effects obtained because of innovation, digitalization, information technology use, human capability use, and dynamic capability use. The appropriate properties of various kinds of indicator estimation (such as potential, capability, effectiveness, and performance) are suggested as part of a unified process of "pragmatic correspondence measure" estimation. This measures how (random) effects are predicted according to the model of progress of possible functioning in a (random) changing environment corresponding to effect values, demanded by the environment in different circumstances at different moments of changing function. This unified estimation suggests unifying estimation routines and building progress problem models as mathematical problems (but based on conceptual models).

Second, estimation for various kinds of properties is suggested according to a unified routine. Unified estimation is possible because of a unified (pragmatic, i.e., based on effects) correspondence measure, which is based on the characteristics of predicted functioning effects (measured results of practical activity) and the predicted environmental demands in various conditions. These correspondence measures depend on particular changes during functioning in a changing environment; that is, they depend on the kind of change, progress, and cause–effect characteristics of particular progress mechanism(s). For example, correspondence measures depend on information operation results involving dynamic, organizational, and human capabilities. These cause–effect relationships are encoded by functional dependencies of transitions between states as a result of changing function. Such states and their transitions form a functional model of appropriate property manifestation. As a result, based on the suggested functional model, it is possible to build a model of progress indicator dependencies from parameters and variables in mathematical progress problems (for example, as objective function(s) and constraints) and to solve them based on known mathematical methods.

Third, decisions in various kinds of (mathematical) progress problems are based on the unification of models built. This is possible because all models used such decisions are based on the initial conceptual model, which describes all possible chains of states and transitions as a result of cause–effect relationships, including those between the environment and its system. Thus, unified models of different kinds (graph-theoretical, parametric, functional, program) can be created based on the initial concept model. These unified models correspond to various aspects of states and transition chains, and various meta-models represent such transitions, but they have a common concept model and can be integrated based on such model or its representation (in the form of some ontology, for example). This makes it possible to create a common framework and instruments of modeling and exchange conceptual and other models for different kinds of progress problem-solving.

II. THE RESEARCH ON THE SYSTEMS CHANGE AND THE USE OF INFORMATION TECHNOLOGIES

The solutions of many modern practical problems that arise for practitioners require the study of systems’ functions, characterized by the necessity to consider changes in these functions because of environmental changes. These changes are initiated by informational actions, which, in turn, are caused by environmental influences. Such actions are required to implement measurements, change state checks, transfer information, and prescribe further actions, in some cases including actions to change the system itself. Then the actual change is implemented. Under the changed conditions, the delivery is better adapted to real functioning effects, different from those that would occur with-out the use of informational actions and so without reactions to environmental influences. The abovementioned practical problems are of different types from different domains. Such problems include those in the improvement of enterprises and organizations [1], the modernization of critical technologies’ provision systems or infrastructures [2] [3], the digitalization of enterprises and economies [4], society informatization [5] [6], as well as other problems that are of crucial importance for the socioeconomic development of the countries [7]. Many such problems formalized as those of improving kinds of social, societal, technological systems [8] [9]. Technical systems are understood as interconnected complexes of parts that include technical devices. Social systems include humans and organisations. Further we consider complex technological systems (CTSs) example. Such systems may include parts of various types, in particular groups of people and instructions for performing actions related to various types of relationships among people, in addition to technical devices, including devices intended to operate information. Such CTSs are classified as technological, organizational, societal or technical systems, depending on the research objectives [10] [11]. Need to improve systems due to changed requirements and influences of system environment lead, first, to the need for the CTS staff to perform transitional actions to achieve new goals or to resolve inconsistencies and then to the implementation of these transitional actions and, as a result, to new and improved CTS functions to achieve, perhaps, new goals.

The research on the use of information technologies (IT) traditionally implemented based on the operational properties of such use [12] [13]. The operational properties of the objects under study [14] are an extensive class of properties of various objects, such that these properties characterize the results of the activity with these objects. Therefore, the operational properties form the basis of the quality of objects under research. These properties are manifested at the boundary of the object in which the activity is implemented and in the environment. Operational properties are characterized by the effects (main results) of activity at the boundary, and then by these effects compliance with the requirements of the environment. Activity is always implemented using certain information operations (at least using the senses or speech). In-
formation operations are elements of activity whose objectives are to obtain information, not to exchange matter and energy. Information operations are implemented in accordance with certain information technology whose objective is to describe the use of information operations. Unfortunately, the mechanisms of the formation of activity effects and the subsequent formation of operational properties, taking into account the use of information operations, including modern (digital) IT, have not been studied in sufficient detail in order to predict the effects of activity with mathematical models, depending on the selected characteristics of the information operations used, as mathematical problems of evaluation, analysis, and synthesis. This is primarily because there are no suitable models and methods for analytically describing the effects of information operations and the operational properties. This, in turn, is related to the absence of a universally accepted concept of the manifestation of the effects of information operations, particularly for non-information (material) effects that are obtained by non-information (material) operations, which depend on the information operations under study. The non-information effects of such operations vary with the implementation of the dependencies between environmental changes, further information, and subsequently non-information operations.

Since information operations lead to changes in non-information operations but do not directly lead to non-information effects, it is necessary to develop the concept of information and non-information action dependencies and the concept of effect manifestation as a result of such dependencies. It is the development of the concept of such dependencies that causes conceptual difficulties. Therefore, further research is directed to the description of the dependencies between information and non-information actions and between information and non-information effects, but first of all an analytical description using mathematical models is required. The mathematical models developed of the formation of usage effects of information operations, including non-information (material) effects that are changed as a result of the information effects, are designed to analytically evaluate the operational properties of the use of information operations. The models are also used to evaluate other operational properties, especially the complex operational property of the system potential, which are measured taking into account the necessary use of information operations. The results presented in this study are aimed at bridging the gap between the need to solve research problems of operational properties based on mathematical models and methods [8] [9] [15] [16] and the lack of the necessary concepts and methodology for solving usage problems of information operations in the sense of formalizing them as mathematical problems of estimation, analysis, and planning by operational properties indicators [17]. The system potential with regard of digitalization [18] [19] is investigated to consider the aforementioned complex of CTS functioning features and to solve the problems specified. To solve practical tasks of the research of the system’s potential, developing the concept for the solution of the research problem was necessary. Such a concept and methodology were developed based on theory of CTS functioning efficiency works and others.

III. Conceptual Foundations of Techniques Used to Estimate IT Results

Informational actions check compliance and develop transitional actions, and their results are prescriptions to implement subsequent actions. Therefore, CTS study should consider the CTS as it improves because of systematic changes in requirements and other environmental influences (that is, in changing conditions). Such an improvement of the CTS in changing conditions is that one which requires the use of informational actions, performed in accordance with corresponding IT [20]. Shortcomings that occur because of changes in the environment can be eliminated in different ways, and the methods used to address these shortcomings depend on those to perform informational actions. When improving a CTS, methods to perform informational operations and eliminate deficiencies are chosen based on models that describe the dependencies of the characteristics of operational properties of such a CTS from possible actions of different types and from their characteristics. Such ways and characteristics form a variety of choices. According to the practice, because of regular impacts from the environment, regular verification and, perhaps, transitional actions are necessary. This leads further to the regular implementation of informational actions related to regular verification, the assessment of the system and the environment, and, if necessary, the development of methods of transitional actions aimed at the changes described above. Then after the informational actions are implemented, the developed transitional actions are performed. Such transitional actions, on the one hand, should regularly lead to the improved results of operation in the conditions of ongoing changes in the environment but, on the other hand, also lead to additional costs to provide such results [21]. The need to perform informational and then transitional operations is caused by changing environmental influences and is typical not only for the complex technical and complex technological systems, considered as an example but also for other systems. This is especially true when studying digitalization in various industries, described by such popular terms as digital production, digital medicine, digital economy, and digital state [4] [22] [12]. As evidenced by the analysis of digitalization [23] [24] [25], its research is based on the dynamic capabilities, organizational capabilities of the system use and the ability of the system and its operating personnel to change functions so that it better meets changing conditions, improves, and achieves changing operation goals [26] [15] [27] [28]. In particular, the results of the relevant research are described in publications in the fields of improvement, strategic planning and development [29] and the digitalization of the economy [13] as well as industrial [30] and military systems [31]. Such studies are conducted with the use of dynamic, organizational, and strategic capabilities [28]. However, to examine the results of digitalization analytically, these results must be connected with the projected results of the system’s functions in which these
dynamic capabilities and skills use in practice the necessary conceptual and methodological apparatuses that create analytical models and solve practical problems as mathematical problems. This is primarily due to the novelty of the studied properties of systems that function in the conditions of changes in environmental influences and change their functions because of informational and transient (possibly non-informational) operations. The new property of the CTS potential proposed by author is an operational property that characterizes the CTS’s ability to achieve changing (i.e., actual and possible) goals during operation (in a changing environment). This depends on the characteristics of the “target” and “transition” functions of the CTS, including the informational actions performed to check the state of the CTS and the environment, develop prescriptions for performing technological operations, and bring the prescriptions to the executors. The indicator of this property is evaluated depending on the composition and characteristics of possible actions of different types that form a set of choices in the problems solved. The CTS’s potential property is a complex operational (pragmatic, praxeological) property of the system—that is, a property that describes the results of the system’s functions and the results’ compliance with requirements in different conditions of the environment [32]. The complexity of the system’s potential is caused by the following: the complexity of the description of action in achieving the activities with the system; the complexity of dependencies at the boundary of the system and environment in different conditions; the complexity of the description of the goals of the system; and the complexity of the description of activities involved in improving the system. The results obtained in the study of the potential of systems are used in the study of other operational properties of systems that operate under changing environmental influences and under changing functions because of informational and transient non-informational actions. They are proposed to solve practical problems of improved functions (including digitalization problems, problems of improving activities by industry) as corresponding mathematical problems. The property of the system’s potential is studied analytically based on the conceptualization and subsequent methodologization of the research problems of the system’s potential using features of the functions, which are improved under changing conditions of the environment. These features include the simultaneity of the disclosing states of the functions and sufficiently describe the possible cause-and-effect relationships of the states considered as the results of actions. The simultaneous disclosure of the system states as workplace states helps evaluate possible CTS states and then compare them with the requirements of the environment at specified times. The possible cause-and-effect relationships among states help specify possible sequences of state changes as well as possible causes of such sequences and their results. These possible sequences depend on the states of the environment and on informational and, subsequently, non-informational operations. Conceptually and corresponding to the mathematical modeling of such possible sequences, depending on the states of the environment, ways to perform informational and non-informational operations are needed to build such models, which allows the description of functioning results when using the system’s capabilities and informational operations. The system’s potential is represented by the characteristics (e.g., expectations, distribution boundaries, modes) of a corresponding random variable describing measures of compliance of the system states with requirements or characteristics that describe such measures of compliance (e.g., random vectors, graphs).

IV. SYSTEM FUNCTIONING AND CHANGE SET-THEORETIC MODELING

To built models of IT use semantic method of conceptualization suggested. It is illustrated in Figure 1. According

Fig. 1. The four triangles of the activity meaning

method proposed, semantic triangle of Gottlob Frege [33] and Ogden&Richards [34] used in four ways, with four triangles. First ($T_1$) used to model concept of reality. Second ($T_2$) used to model concept of mathematical model of reality. This model obtained due to schematizing of the initial conceptual model of reality. Third ($T_3$) used to model concept of mathematical model of planned activity. This model obtained through planning schematizing as mathematical objects. Fourth ($T_4$) used to model concept of activity realization. The four triangles depicts how signs and models used for human’s activity. Activity changes through four triangles repetition once system and its environment change. Thus, information operations used at each stage of activity change and leads to change in functioning through sequence of change started by objects of reality (system and its environment) changes. Information about activity realization includes vector of objects of reality characteristics as well as information for person (or actuator device) about activity fulfillment. Let us sequentially define objects of interest as complex set theoretic and graphs structures sequences. In such sequences each consecutive object defined based on previous ones. Such definitions can be made based on relations or predicates sequentially defined based
on basic objects. Basic elements are objects which reflects elements of system and its environment.

\( O^s \) - set of system’ elements (which, probably, includes subsets structure);

\( O^e \) - set of system environment elements (which, probably, includes subsets structure);

\( O := O^s \cup O^e \) - set of system and its environment elements;

\( M'^e \subseteq O^e \times O^s \) - set of possible relations (relational structures or graphs) between environment elements (which, probably, includes subsets structure);

\( M'^s \subseteq O^s \times O^s \) - set of possible relations (relational structures or graphs) between system’s elements (which, probably, includes subsets structures);

\( C(O^s) \) - set of environment elements characteristics (which probably, includes subset structure);

\( C(O^e) \) - set of system’ elements characteristics (which probably, includes subset structure), \( C := C(O^s) \cup C(O^e) \);

\( S^e \in B(C(O^e)) \) - set of possible states of the environment (which, possibly, includes subsets);

\( S'^e \in B(C(O^e)) \) - set of possible states of the system (which, possibly, includes subsets);

\( S := S^e \cup S'^e \) - universal set of system and its environment states;

\( M \subseteq S \times S \) - universal set of system and its environment transitions;

\( A_u \subseteq M \) - set of actions of system or its environment;

\( I_u \) - description (prescription) of action, i.e. information how action performed and which results of action could be obtained due to its realization, \( I_u \) allows to link elements of models together in order to predict results of functioning and to change functioning course;

\( i_u \in I_u \) - alternative information which can be used to plan and perform action \( A_u \), \( i_u \) may describe objects, states, ways of action: \( i_u \subseteq p^s_u, i_u^a, i^e_u, i^e_u^s, i^e_u^p, p^u_i \), where \( i^u \) - information about objects (workplaces parts) which planned to perform action \( A_u \), \( i^u_a \) - information about starting states planned to perform action \( A_u \), \( i^e_u \) - information about possible planned ending states of action \( A_u \), \( I_u^a \) - information about planned instructions, prescriptions to perform action \( A_u \), \( I_u^e \) - information about possible planned alternative ways of action, i.e. which ending states planned as possible after planned instructions fulfilled from planned beginning states, why and how possibly;

\( I_u \subseteq I^a_u, I^b_u, I^e_u, I^p_u, I_u^a \), where \( I_u^a \) - information about objects (workplaces parts) possible for alternative use to perform action \( A_u \), \( I_u^b \) - information about alternative starting states possible for performing action \( A_u \), \( I_u^e \) - information about alternative possible ending states of action \( A_u \), \( I_u^p \) - information about alternative instructions, prescriptions to perform action \( A_u \), \( I_u^a \) - information about possible alternative ways of action, i.e. which ending states can result from which beginning states and why;

\( u \subseteq U \), where \( U \) - universal index, variable multidimensional array of natural numbers which corresponds to variable designated sets elements numbers;

\( a_u \subseteq A_u \) - way of action \( A_u \), particular transition which results due to action \( A_u := a_u \);

\( M'^s \subseteq S^s \times S^s \) - set of possible state transitions between states of the system (which, possibly, includes subsets), associated with sets of actions of system elements;

\( M'^e \subseteq S^e \times S^e \) - set of possible state transitions between states of the environment (which, possibly, includes subsets), associated with sets of actions of system’s environment;

\( M'^e \subseteq S^e \times S^s \) - set of possible state transitions between states of the environment and states of the system (which, possibly, includes subsets), associated with sets of change actions of system environment directed to system’s elements;

\( M'^s \subseteq S^s \times S^s \) - set of possible state transitions between states of the system and states of the environment (which, possibly, includes subsets), associated with sets of change actions of system’s elements directed to system’s environment elements;

\( M = \{ a_u \}, M \supseteq M'^s \cup M'^e \cup M'^s ; \)

\( C(a_u) \) - characteristics of way of actions \( a_u \);

\( C(M) := \{ C(a_u) \} \) - characteristics of state transitions;

\( C := C(M) \cup C(O) \); \( A := \{ A_u \} \) - set of possible actions;

\( I_a := \{ i_u \} \) - information about action \( A_u \) alternatives \( i_u \), which can be planned for realization;

\( I := \{ I_u \} \) - information about set of possible actions alternatives, \( I \supseteq C \);

\( T := (I, I(A \times A)) \) - technology of actions, i.e. information about possible actions alternatives and about their possible alternative sequences \( I(A \times A) \).

Action \( A_u \) defined as the result of its alternative description \( i_u \) fulfillment (mapping to the reality) \( i_u : i_u \rightarrow A_u \) (action performing mapping);

\( i_u^a \in I_u^a \) - information about action, selected for functioning;

\( I_u^p \) - complex of information about actions, selected for functioning;

\( A^s \subseteq I_u^p \) - set of ways of actions, planned for functioning;

\( i_u(A^s \times A^s) \) - information about actions sequences, possible for functioning if ways of actions planned;

\( I_u^p(A^s \times A^s) \) - information about actions sequences, selected for functioning once ways of actions planned;

\( I_u(A^s \times A^s) \) - information about set of possible alternative actions sequences which can be planned for functioning if ways of actions \( A^s \) planned;

\( I_u(A^s \times A^s) \) - information about set of possible alternative actions sequences selected for functioning once ways of actions \( A^s \) planned;

\( \Pi_u := I_u(A^s \times A^s) \) - possible alternative plans of environment actions, including needed information about planned results of such plans realization \( I^e \); \( \Pi_u^* := \{ \pi_u^* \} \); \( \pi_u^* \) - given plan of environment actions, including needed information about planned results of such plan realization \( I^{e\prime} \);

\( \bar{E}_u \) - event which is \( \pi_u^* \) will be performed and \( I^{e\prime} \) obtained as a result;

\( I_u/\bar{E}_u \) - possible sequences of \( i_u \) under condition of \( \bar{E}_u \);

\( \Pi_u^* := I_u(A^s \times A^s) \) - possible alternative plans of system’ actions, including needed information about planned results of such plans realization \( I^{e\prime} \); \( \Pi_u^* := \{ \pi_u^* \} \);
\[ \pi^*_s = I^*_s (A^* \times A^*), \]  
\[ I^*_s \] - given plan of system' actions, including needed information about planned results of plan realization \( I^{res} \); 
\[ \hat{A}_u \] - event, which is \( \pi^*_s / \hat{E} \) will be performed and \( I^{res} \) obtained as a result, under condition \( \hat{E} \) happened; 
\[ \pi^*_i (\hat{E}_u) := I_i (A^* \times A^*), i_i - \text{ plan of system' actions in a given unchanged conditions of the environment which resulted in } I^{res}, \text{ i.e. under condition event } \hat{E} \text{ happened, } \pi^*_i \in P_i^s / \hat{E}; \]  
\{ \hat{T}_1; \ldots; \hat{T}_n \} - set of n moments, where alternations of the system' functioning is possible;  
\[ i = \{i_1; \ldots; i_n\} \] - set of \( n \) information operations alternatives, each of \( i_i \in I_i \) leads to possible alternation of the system' functioning:  
\[ \hat{A}_i / (\hat{A}_i-1; \ldots; \hat{A}_i), i = 1, [I_i] \] - sequence of conditional events that at moment \( T_i \) event \( \hat{A}_i \) happens after previous ones at \( T_{i-1} \); 
\[ \text{Then, } \hat{A}_u - \text{ event which is } \pi^*_s / \hat{E} \text{ will be performed and } I^{res} \text{ obtained as a result, under condition } \hat{E} \text{ happened and } \hat{A}_1; \hat{A}_2; \ldots; \hat{A}_u \text{ happened at } T = \{T_1; \ldots; T_n\} \text{ respectively, } \]  
\[ P(\hat{A}_u) = P(A_1) \times P(A_2) \times P(\hat{A}_u), \text{ sum}_i \{\hat{A}_i\} = 1; \]  
\[ S_i^*(i_i, \pi^*_i, \pi^*_i) - \text{ system' state at moment } T_i \text{ as a result of } i_i \in I_i \text{ alternative realization}; \]  
\[ S_i^*(i_i, \pi^*_i, \pi^*_i) / \hat{A}_i - \text{ realization of system' state at moment } T_i \text{ as a result of } i_i \in I_i \text{ alternative fulfillment (result of event } \hat{A}_i); \]  
\[ S_i^*(i_i, \pi^*_i, \pi^*_i) / \hat{E}_i - \text{ realization of system state at moment } T_i \text{ as a result of } i_i \in I_i \text{ alternative fulfillment (result of event } \hat{E}_i); \]  
\[ S_i^*(i_i, \pi^*_i, \pi^*_i) / \hat{A}_i \] and \( \hat{E}_i \) are conditionally independent:  
\[ w(u) = P(\hat{B}_u / (\hat{A}_u \cap \hat{E}_u)) = \prod_{i=1}^{I_i} P(\hat{E}_u)P(\hat{A}_u)P(\hat{B}_u); \]  
\[ 0 \leq w(u) \leq 1 \] - where multi-index \( u \) running through only one dimension, which corresponds to the branch of the tree of possible states. Value \( w(u) \) is probabilistic measure and may take values from \([0, 1]\). It is conditional probability of efficient functioning under conditions of event such functioning will be realized.

Set-theoretic model of system functioning according plans \( \Pi^*_u / \hat{E}_u \) in the given conditions of the environment due to its functioning according \( \pi^*_u \) is tuple \( M^{sf} \):

\[ M^{sf} (\Pi^*_u) = < T, U, i_u, I^{res}, \pi^*_u, \hat{E}; \Pi^*_u, O(\pi^*_u), C(\pi^*_u), S(\pi^*_u), M(\pi^*_u); \pi^*_u \in \Pi^*_u \> > \]

This model allows to estimate probability distribution of \( \hat{\omega} \):

\[ \hat{\omega}(\hat{E}_u) = \lim_{i=1}^{I_u} \frac{P(\hat{E}_u)P(\hat{B}_u), I^{res} = 1, [\hat{u}]}{ \Pi_u } \]

which is probabilities of complex event \( P(\hat{B}_u / ((\hat{E}_u) \cup (\hat{A}_u)) \text{ in the different alternative conditions (under condition of given events } \hat{E}_u, \text{ and taking into account probabilities of different alternatives } P(\hat{A}_u) (\text{discrete random variable of probabilistic measure). This variable corresponds to the } u \text{ indexes running through multiple dimensions for alternative } \hat{A}_u \text{ but same } \hat{E}_u. \)

Set-theoretic model of system functioning according set of plans \( \Pi^*_u \) used in various alternative \( \hat{E}_u \) conditions of the environment due to its functioning according to the different alternatives \( \pi^*_u \in P_i^u \) is the tuple \( M^{sf} \):

\[ M^{sf} (\Pi^*_u) = < T, U, i_u, I^{res}, \pi^*_u, \hat{E}; \Pi^*_u, O(\pi^*_u), C(\pi^*_u), S(\pi^*_u), M(\pi^*_u); \pi^*_u \in \Pi^*_u \> > \]

This model allows to estimate multidimensional probabilistic measure \( \hat{\Omega} \):

\[ \hat{\Omega} := \{(P(\hat{A}_u), P(\hat{E}_u), \Pi_u, I^{res} = 1, [\hat{U}], i\} \]

where indexes are running through all possible \( \pi^*_u \in \Pi^*_u \) and all possible \( u \) in \( \hat{A}_u \) - i.e., through multiple dimensions of the complex index \( U \).

Set-theoretic model of system functioning at variable moments \( T_u \) in \( T \) according set of plans \( \Pi^*_u \) used in various alternative \( \hat{E}_u \) conditions of the environment due to its functioning according to the different alternatives \( \pi^*_u \in P_i^u \) is the tuple \( M^{sf} (T_u) \):

\[ M^{sf} (T_u) = < T, U, i_u, I^{res}, \pi^*_u, \hat{E}; \Pi^*_u, O(\pi^*_u), C(\pi^*_u), S(\pi^*_u), M(\pi^*_u); \pi^*_u \in \Pi^*_u \> > \]

This model allows to estimate multidimensional probabilistic measure \( \hat{\Omega}(T_u) \):

\[ \hat{\Omega}(T_u) := \{(P(\hat{A}_u), P(\hat{E}_u), \Pi_u, I^{res} = 1, [\hat{U}], T_u = 1, [\hat{U}] \}; \]
where indexes are running through all possible $T_u$, $p_{i_u}^\psi \in \Pi_u^c$ and all possible $u$ in $A_{ui}$ - i.e., through all possible dimensions of the complex index $U$ which corresponds to probabilistic measures changes.

V. SYSTEM CAPABILITY RESEARCH MATHEMATICAL PROBLEMS STATEMENTS

A. Estimation Problem Statement

Given: $O, C, S, M, T, U, \Pi^c, \Pi^*(\pi^c)$. Estimate:

\[ \Omega(T, \Pi^*, \Pi^*; O, C, S, M), \]
\[ C(\Omega(T, \Pi^*, \Pi^*; O, C, S, M)), \]

where $\Omega$ – multidimensional probabilistic measure. This measure defined on dimensions $u \in U$ for $O, C, S, M$ dimensions (as parameters) and on $T, \Pi^c, \Pi^*$ dimensions (as variables). Its values are twice vague probabilistic predicates values according equation 1 at coordinates given by $u$. Predicates form probabilistic random measures structures according equations 2,3,4,5. $C(\Omega(T; O, C, S, M))$ – multidimensional probabilistic measure defined on dimensions $y \geq u \in U$ for $O_y, C_y, S_y, M_y$ dimensions for additional coordinates at $y \in Y$ as appropriate (additional to $u$) variables dimensions and $T, \Pi^c, \Pi^*$ dimensions (as variables) as twice vague probabilistic predicates values according equation 1 at coordinates given by $u$. Predicates form probabilistic random measures structures according equations 2,3,4,5.

B. Analysis Problem Statement

Given: $O \in O, C \in C, S \in S, M \in M, T, U, \Pi^c, \Pi^*(\pi^c)$. Calculate:

\[ \delta(O, C, S, M, T) = \]
\[ \{\Omega(T; O, C, S, M) - \Omega(T; O_y, C_y, S_y, M_y)\}, \]

where $\Omega(T; O_y, C_y, S_y, M_y)$ – multidimensional probabilistic measure defined on dimensions $y \geq u \in U$ for $O_y, C_y, S_y, M_y$ dimensions for additional coordinates at $y \in Y$ as appropriate (additional to $u$) variables dimensions and $T, \Pi^c, \Pi^*$ dimensions (as variables) as twice vague probabilistic predicates values according equation 1 at coordinates given by $u$. Predicates form probabilistic random measures structures according equations 2,3,4,5.

\[ \delta(O, C, S, M, T) \] provides finite differences of probabilistic measures over additional coordinates of $y$ compared to $u$. These coordinates corresponds to possible variables values $O, C, S, M, T$.

\[ F_y(\Omega(T; O_y, C_y, S_y, M_y)) \] – approximation function.

C. System and Technologies Synthesis Problem Statement

Given: $O, C, S, M, T, Y \supset U, \Pi^c, \Pi^*(\pi^c), O, C, S, M$.

Find \( \arg \max_{y \in Y} \{\Omega(T; O_y, C_y, S_y, M_y)\} \),

where \( \{O^*, C^*, S^*, M^*\} \) – vector of optimal values, taken from $O, C, S, M, y \in Y$ – variable dimensions for $O, C, S, M$; $\Omega(T; O_y, C_y, S_y, M_y)$ – multidimensional probabilistic measure defined on dimensions $y \geq u \in U$ for $O_y, C_y, S_y, M_y$ dimensions for additional coordinates at $y \in Y$ as appropriate (additional to $u$) variables dimensions and $T, \Pi^c, \Pi^*$ dimensions (as variables) as twice vague probabilistic predicates values according equation 1 at coordinates given by $u$. Predicates form probabilistic random measures structures according equations 2,3,4,5.

D. Functional Models Examples

Suggested models and mathematical problems statements was formulated based on set theoretic constructs. Set theoretic objects used shall be constructed (calculated). Such calculation performed with use of functional dependencies. Examples of such dependencies was shown in equations 1,2,3,4,5. In these equations probabilistic measures are computed according algebra of probabilities. It is based on elementary probabilities $B_{ui}$ computed according predicate expression 1 and expressions to compute elementary probabilities $A_{ui}$, $E_{ui}$. To compute these probabilities states characteristics required. In general, such characteristics expressed as random variables of functioning effects. Functioning effects are given as information $i_u$ elementary effects of the possible ways $a_u$ of actions $A_u$. These elementary effects associated with beginning (start) $s_u^b$ or end (finish) $s_u^e$ states for each $a_u$. Let these effects are $i_u = r_u$ – resource quantity, $g_u$ – product quantity. Among other characteristics of $s_u^b$ and $s_u^e$ states are special resource - time $i_u$ to perform action $A_u$ according way of action $a_u$, which results in moments $T_u^b$ of beginning state activation and moment $T_u^e$ of end state activation. General elementary functional model takes form of $c_u$:

\[ c_u := \{c_u^b, c_u^e, t_u, p_u\}; \]

Among other characteristics of $a_u$ states are elementary probabilities $p_u$ of states activation. Information $i_u$ about $a_u$ realization includes vector of $a_u$ characteristics $c_u$ as well as information $i_u^p$ for person or actuator device about $a_u$ fulfillment as well as other information about $a_u$ (relations with other $a_u$ and other $A_u$, rules about $a_u$ fulfillment). Let us explain possible structure of these vectors according used example of effects and assumption two parameters of probability distribution (namely, the left $i_u$ and the right $r_u$ margins of the distribution) are enough for the probability
distributions explications [35] and the number of possible ending states of $a_u$ is $n$:

$$e^b_u := \langle r^b_u, r^b_{u+1}, g^b_u, g^b_{u+1} \rangle;$$

$$c^b_u := \langle s^b_u, s^b_{u+1}, x^b_u \rangle;$$

$$r^b_{u+1} \cdots r^b_{u+n}, g^b_{u+1} \cdots g^b_{u+n} >;$$

$$t^b_u := \langle T^b_u, T^b_{u+1}, T^b_{u+2}, \cdots, T^b_{u+n} \rangle;$$

$$p^b_u := \langle p^b_{u+1}, p^b_{u+2}, \cdots, p^b_{u+n} \rangle.$$  \hfill (11)

Beginning states of next $a_u$ are equal to ending states of previous $a_u$. Beginning and ending states counted according function $\mathbf{f}^{\mathbf{e}}(\{c_u\})$:

$$\mathbf{r}^{\mathbf{e}}(\{c_u\}) := \langle r^{\mathbf{e}}(\{c_u\}), g^{\mathbf{e}}(\{c_u\}), T^{\mathbf{b}}(\{c_u\}), p^{\mathbf{b}}(\{c_u\}) \rangle;$$

$$r^{\mathbf{e}}_{u+1} := r^{\mathbf{e}}_{u+1} + r^{\mathbf{e}}_{u+2};$$

$$\cdots$$

$$r^{\mathbf{e}}_{u+n} := r^{\mathbf{e}}_{u+n} + r^{\mathbf{e}}_{u+n+1};$$

$$g^{\mathbf{e}}_{u+1} := g^{\mathbf{e}}_{u+1} + g^{\mathbf{e}}_{u+2};$$

$$\cdots$$

$$g^{\mathbf{e}}_{u+n} := g^{\mathbf{e}}_{u+n} + g^{\mathbf{e}}_{u+n+1};$$

$$T^{\mathbf{e}}_{u+1} := T^{\mathbf{e}}_{u+1} + T^{\mathbf{e}}_{u+2};$$

$$\cdots$$

$$T^{\mathbf{e}}_{u+n} := T^{\mathbf{e}}_{u+n} + T^{\mathbf{e}}_{u+n+1};$$

$$p^{\mathbf{e}}_{u+1} := p^{\mathbf{b}}_{u+1} p^{\mathbf{e}}_{u+2};$$

$$\cdots$$

$$p^{\mathbf{e}}_{u+n} := p^{\mathbf{b}}_{u+n} p^{\mathbf{e}}_{u+n+1}.$$  \hfill (12)

Let us denote $\mathbf{C}_u$— matrix of all possible $a_u$ characteristics; $\mathbf{G}_u$— graph of all possible $a_u$ sequences; $\mathbf{F}_u(\mathbf{G})$— traverse of graph $\mathbf{G}_u$— with use of $\mathbf{f}^{\mathbf{e}}(\{c_u\})$ function performed to each sequential $a_u$ at $\mathbf{G}_u$; $\mathbf{C}_u$— matrix of all $a_u$ characteristics for root of $\mathbf{G}_u$— graph; Then

$$\mathbf{C}_u = \mathbf{F}_u(\mathbf{C}_u) \cdots (\mathbf{F}_u(\mathbf{C}_u)).$$  \hfill (13)

allows to compute all $\mathbf{C}_u$ elements in the right sequence given by graph $\mathbf{G}$. $\mathbf{C}_u$— multidimensional matrix of demanded values $r^d_u, g^d_u, T^d_u$ of resources spent, products produced, moments to finish $a_u$ for different $a^d_u$.

Based on $\mathbf{C}_u$ and $\mathbf{C}_u$ predicates $p(S^i_u, S^i_{u+1}; \mathbb{R})$ can be computed for all elements, so matrix $\mathbf{p}$ of predicates values (probabilities matrix) can be computed:

$$\mathbf{p}(\mathbf{C}_u, \mathbf{C}_u^d) := \langle p_0(\mathbf{C}_u, \mathbf{C}_u^d, \mathbb{R}, \mathbf{G}) = \right.\cdot$$

$$\left. F_{\mathbf{r}_u}(r^d_u; \mathbf{G}, \mathbb{R}) F_{\mathbf{g}_u}(g^d_u; \mathbf{G}, \mathbb{R}) F_{T^d_u}(T^d_u; \mathbf{G}, \mathbb{R}). \right.$$  \hfill (14)

Here, $F_{\mathbf{r}_u}(r^d_u; \mathbf{G}, \mathbb{R})$— cumulative probability distribution function of resources $r_u$, projected to spent for $a_u$ constructed according $\mathbb{R} (i_{\mathbb{R}}= \text{relation}), \mathbf{G}$ (according sequences and sets of states formed by use of this graph) for argument $r^d_u$ of demanded resource value;

$$F_{\mathbf{g}_u}(g^d_u; \mathbf{G}, \mathbb{R})$$— cumulative probability distribution function of product produced $g_u$, projected as a result of $a_u$ constructed according $\mathbb{R} (i_{\mathbb{R}}= \text{relation}), \mathbf{G}$ (according sequences and sets of states formed by use of this graph) for argument $g^d_u$ of demanded product value produced;

$$F_{T^d_u}(T^d_u; \mathbf{G}, \mathbb{R})$$— cumulative probability distribution function of moment $T^d_u$ of time to end $a_u$, constructed according $\leq \text{relation}, \mathbf{G}$ (according sequences and sets of states formed by use of this graph) for argument $T^d_u$ of demanded moment to end $a_u$.

As a result, task of $\mathbf{p}(\mathbf{C}_u, \mathbf{C}_u^d)$ computation can be considered as tensor-based computation of multidimensional probabilistic measure $\Omega(T_u)$ according equations (10,11,12,13,14) and then - to solve mathematical problems, for example - synthesis (equation 9) problem.

VI. CONCLUSION

The results obtained shall enable the evaluation of the predicted values of the operational (pragmatic) properties of systems depending on the characteristics of IT used for system progress and characteristics of change in general. This could lead to a solution to contemporary research problems using predictive analytical mathematical models and methods. Examples of such research problems are those related to sustainable development planning, digitalization planning, IT efficiency estimation, analysis, and synthesis of the organizational, innovation, and dynamic capabilities of systems. Possible aspects include choosing the best development plans, the best digitalization scenarios, and the best innovation contexts. Suggested indicators can be used, similarly, to estimate indicators of human and social, organizational, and dynamic capabilities for systems functioning. This addresses the existing gap between the need to solve research problems in pragmatic properties regarding digitalization based on mathematical models and methods and the lack of necessary concepts and methodology for solving such problems. Examples of such a problem include innovation, development, digitalization analysis and synthesis, societal systems research and development, and public program planning. Further research should enable the estimation of organizational and dynamic capability indicators used for social systems functioning in changing environments pertaining to IT use; this depends on environmental characteristics—for example, climate change, pandemics, or collaborative characteristics. A software prototype of system change and progress modeling shall allow for the creation of a variety of interrelated conceptual, diagrammatic, probabilistic, functional, and programming models of changing systems functioning in dynamic environments with respect to information operations. Models created with such software shall enable change in and the use of information operations in dynamic environment visualization and in the use of Internet and semantic technology for model construction. As a result, such models shall help in estimating the indicators of various systems capabilities, indicators of progress, digitalization, and
innovation and, further, in solving problems of analysis and synthesis of change based on such a measure.

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