Air Navigation: Adaptive Filtration of Parameters of Motion of Manoeuvrable UAVs

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Abstract—A novel approach to recurrent filtering of trajectory parameters of unmanned aerial vehicles (UAVs), including areas of intense trajectory maneuver, has been proposed in the article. A procedure for the synthesis of an adaptive recurrent filter based on the separation principle has been developed. A technique for simplifying the adaptive filter in order to reduce the computational complexity of the filtering algorithm has been proposed. The results of modeling synthesized adaptive filters have been presented and a comparative analysis of the proposed filters with traditional ones, which showed an increase in the accuracy of the estimated trajectory parameters of the UAV by 50-70 percent in the area of an intensive trajectory maneuver, has been carried out.

I. Introduction

Inclusion of unmanned aerial vehicles (UAV) in the air traffic common with the pilot-controlled aviation created the considerable problems of safety in the air space for the pilot-controlled aviation in the last decade [1]. UAVs are a quite new element of the air traffic control system unlike pilot-controlled aircrafts with a pilot aboard, flying the airplane from the airplane, and also passengers or load [2]. The indicated problem will be only intensified in forthcoming years. The air traffic control system is to provide a reliable discovery and tracking of pilotless vehicles with the purpose of prevention of collisions and other incidents in air space [2], [3] for the decision of tasks of integration of pilotless aviation. There is an important scientific and technical task related to the necessity of their integration into national air spaces of the developed countries and into the air space of the whole world [4].

It is known that UAVs, unlike airplanes, carry out plenty of maneuvers of different intensity in the process of the motion in the air space, which considerably complicates their discovery in the air space, and also tracking (supervision) by means of informative facilities (radio-locators) of the air traffic control system [5], [6]. Administrative decisions on prevention of dangerous rapprochements and other near-accidents in air space [7], [8] are made in the air traffic control system on the basis of trajectory information about the tracked aircrafts.

It is known that the measuring informative systems that apply the radio-location stations or laser radars for measuring current positions are mostly used for determination and clarification of parameters of UAVs trajectories. Thus, it is expedient to use recurrent filters for treatment of acting information as an amount of the trajectory measuring is usually large enough (hundreds are thousands of measuring) and it is inefficient to keep them in the computer memory. Most known and widely used is Kalman filter [9], [10]. A number of modifications of this filter that allow steadily tracking a UAV in the determined areas of trajectories is known. However, similar filters lead to appearance impermissible significant dynamic errors of the tracking that result in a loss of tracking by such objects.

The method of regulation of recurrent filter, consisting of adding of measuring of additional member to the matrix of errors, is offered in the work [11], which allows to carry out tracking of a UAV in the area of weak maneuver. However, it is impossible to carry out the estimation of veritable parameters of intensity of maneuver here, and a quite serious dynamic error of tracking, that can result in a loss of tracking by the object, is present.

To solve this problem, filters with maneuver detection, estimation of its intensity and switching to a recurrent filter with an appropriate motion model are used. However, such an approach often requires large expenditures of computational resources and does not significantly reduce the dynamic error of the tracking in the transition areas.

Another approach is to use adaptive recurrent filters. The adaptive filter suggested in [12] makes it possible to effectively solve the problem of tracking objects that maneuver relative to a linear trajectory, while the dynamic error of tracking of such objects is reduced by 2 - 3 times compared to filters with maneuver detection. However, the use of such an adaptive filter for solving the problems of processing the data of trajectory measurements of UAVs is not possible, since in the filter under consideration, the basic model is a linear motion model, and the model of UAV movement at sufficiently long tracking intervals is not such. As a result, the filter constantly operates in the mode of tracking a maneuvering object, which leads to the appearance of an additional dynamic error.

The aim of this article is to synthesize an adaptive filter, which takes into account the peculiarity of the UAV motion model, to ensure the possibility of an effective assessment of the parameters of the trajectory maneuver and stable tracking by measuring information systems in such trajectory segments.

For the development of software, which was used by the authors to simulate UAV motion and information processing, the authors used the C# programming system as well as the MathCad mathematical computing package.

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II. Formalization of the Problem

For successful tracking of UAVs by measuring information systems in the trajectory maneuver section, it is necessary to use a motion model where there are additional terms describing such a maneuver.

The article uses the designations of variables that were introduced in [12] and other works by S. Kuzmin, since he examined the practical issues of recurrent filtration in measuring systems in the most detailed way.

The model of the UAV movement in the trajectory maneuver section can be represented in the form

$$\mathbf{υ}_n = \mathbf{Φ}_n \mathbf{υ}_{n-1} + \mathbf{Γ}_n \mathbf{g}_{mt} + \mathbf{G}_n \mathbf{η}_n \quad (1)$$

where $\mathbf{Φ}_n \mathbf{υ}_{n-1}$ is the equation of the basic unperturbed trajectory (it is advisable to use a polynomial of the second degree for an UAV), $\mathbf{g}_{mt}$ - l is the dimensional vector of disturbances in the section of the UAV's trajectory maneuver, $\mathbf{η}_n$ - p is the dimensional vector of disturbances caused by the influence of the external environment, $\mathbf{Γ}_n$ и $\mathbf{G}_n$ are known matrices that allow transforming the elements of vectors $\mathbf{g}_{mt}$ и $\mathbf{η}_n$ into a single coordinate system used in the motion model.

It should be noted that the measurement of the parameters of UAV trajectories by measuring information systems (for example, radar systems) is usually carried out in spherical coordinate systems [13], but even an unperturbed trajectory of motion is described there by equations containing derivatives of order higher than two, which extremely complicates the recurrent filter, increases its dimension makes it unrealizable in real time.

To simplify the filter, in [12], it has been proposed to carry out filtering in the topocentric coordinate system [13], and to maintain target tracking, transform the extrapolated coordinates into the spherical system of the radar station.

Thus, the synthesized filter should provide recurrent filtering of incoming measurements of the UAV motion parameters, the equations of motion of which are described by relation (1), and also provide effective tracking of the object in the trajectory maneuver section, when the values of the vector $\mathbf{g}_{mt}$ elements are a priori unknown.

III. Synthesis of the Adaptive Recurrent Filter

At the stage of the trajectory maneuver, the disturbance of the trajectory (1) of the UAV can be represented by a random process [12], the average value of which can take values from a fixed set of states in the range $\left[-g_{m\max},\ldots,0,\ldots,+g_{m\max}\right]$. Transitions from state i to state j occur with probability π_{ij}, the time spent in a state i is a random variable with a probability density $\omega\left(t_i\right)$. Such a process is a semi-Markov random process [12].

In work [14], it was concluded that the separation principle is effective for constructing adaptive systems. It is advisable to use it to build an adaptive filter.

When estimating the vector of UAV motion parameters $\hat{\mathbf{υ}}_n$ we will assume that the posterior probability density $\omega\left(\mathbf{υ}_n \mid \left\{\mathbf{Y}\right\}_n\right)$ of vector $\mathbf{υ}_n$ is known according to the data n - a measured sequence of measurements $\left\{\mathbf{Y}\right\}_n$. Then, with a quadratic loss function, the posterior mathematical expectation (estimate) can be calculated using the Bayes formula.

$$m_{\mathbf{υ}_n} = \hat{\mathbf{υ}}_n = \int\limits_{(\Theta)} \mathbf{υ}_n \omega\left(\mathbf{υ}_n \mid \left\{\mathbf{Y}\right\}_n\right) d\mathbf{υ}_n . \quad (2)$$

Here, $\left(\Theta\right)$ is the space of possible values of the estimated parameter.

In the case of the presence of a disturbing effect, the posterior probability density can be represented in the form [15]

$$\omega\left(\mathbf{υ}_n \mid \left\{\mathbf{Y}\right\}_n\right) = \int\limits_{(\mathbf{g}_m)} \omega\left(\mathbf{υ}_n \mathbf{g}_{mt}, \left\{\mathbf{Y}\right\}_n\right) \times \\ \times \omega\left(\mathbf{g}_{mt} \mid \left\{\mathbf{Y}\right\}_n\right) d\mathbf{g}_{mt} d\mathbf{υ}_n . \quad (3)$$

Here, $\left(\mathbf{g}_m\right)$ is the space of possible values of the trajectory maneuver intensities.

$$\hat{\mathbf{υ}}_n = \int\limits_{(\Theta)} \mathbf{υ}_n \int\limits_{(\mathbf{g}_m)} \omega\left(\mathbf{υ}_n \mid \mathbf{g}_{mt}, \left\{\mathbf{Y}\right\}_n\right) \times \\ \times \omega\left(\mathbf{g}_{mt} \mid \left\{\mathbf{Y}\right\}_n\right) d\mathbf{g}_{mt} d\mathbf{υ}_n . \quad (4)$$

Substituting (3) into (2), we obtain the relation

After transforming the expression (4) we get

$$\hat{\mathbf{υ}}_n = \int\limits_{(\mathbf{g}_m)} \hat{\mathbf{υ}}_n\left(\mathbf{g}_{mt}\right) \omega\left(\mathbf{g}_{mt} \mid \left\{\mathbf{Y}\right\}_n\right) d\mathbf{g}_{mt} . \quad (5)$$

In [12,15], an original approach to calculating the values of the probability density $\omega\left(\mathbf{g}_{mt} \mid \left\{\mathbf{Y}\right\}_n\right)$, which are necessary to obtain the value $\hat{\mathbf{υ}}_n$, that is, to solve the problem of adaptive filtering.

Since the parameter $\mathbf{g}_{mt}$ takes only fixed values, and taking into account the properties of a definite integral [16], expression (5) is transformed to the form

$$\hat{\mathbf{υ}}_n = \sum_{j=-k}^{k} \hat{\mathbf{υ}}_n\left(\mathbf{g}_{Mjn}\right) P\left(\mathbf{g}_{mjn} \mid \left\{\mathbf{Y}\right\}_n\right), \quad (6)$$

where $\mathbf{g}_{mjn}$ is one of j $\left(j=-k,-\left(k-1\right),\ldots,0,1,\ldots,k-1,k\right)$ the fixed values of the parameter $\mathbf{g}_m$; $P\left(\mathbf{g}_{mjn} \mid \{\mathbf{Y}\}_n\right)$ is the posterior probability of the event $\mathbf{g}_{mjn} = \mathbf{g}_{mj}$ obtained from n measurement $\{\mathbf{Y}\}_n$ results.

We apply Bayes' rule [17] to calculate $P\left(\mathbf{g}_{mjn} \mid \{\mathbf{Y}\}_n\right)$, then

$$P\left(\mathbf{g}_{mjn} \mid \{\mathbf{Y}\}_n\right) = \frac{P\left(\mathbf{g}_{mjn} \mid \{\mathbf{Y}\}_{n-1}\right)\omega\left(\mathbf{Y}_n \mid \mathbf{g}_{mj(n-1)}\right)}{\sum_{j=-k}^{k} P\left(\mathbf{g}_{mjn} \mid \{\mathbf{Y}\}_{n-1}\right)\omega\left(\mathbf{Y}_n \mid \mathbf{g}_{mj(n-1)}\right)}. \quad (7)$$

Here, $P\left(\mathbf{g}_{mjn} \mid \{\mathbf{Y}\}_{n-1}\right)$ is the a priori probability of the maneuver parameter $\mathbf{g}_{mj}$ calculated from $n-1$ measurement results at the nth filtering step. To calculate this probability, taking into account the semi-mark nature of the maneuver

$$P\left(\mathbf{g}_{mjn} \mid \{\mathbf{Y}\}_{n-1}\right) = \sum_{i=-k}^{k} \pi_{ij} P\left(\mathbf{g}_{mi(n-1)} \mid \{\mathbf{Y}\}_{n-1}\right) \quad (8)$$

process, we apply the relation [12]

where $\pi_{ij} = P\left(\mathbf{g}_{mn} = \mathbf{g}_{mj} \mid \mathbf{g}_{m(n-1)} = \mathbf{g}_{mi}\right)$ is the probability of transition of the maneuver process from the state i at the $n-1$ the step into j state at the n -th step.

Conditional probability density $\omega\left(\mathbf{Y}_n \mid \mathbf{g}_{mj(n-1)}\right)$ of the measured values of the parameters vector $\mathbf{Y}_n$, if the trajectory maneuver at the $n-1$-th step had value $\mathbf{g}_{mj}$, is usually approximated by a normal distribution [12, 15] with mathematical expectation $\hat{\mathbf{Y}}_{enj}$ and variance $\boldsymbol{\sigma}_n^2$, and is calculated by the formulas

$$\hat{\mathbf{Y}}_{enj} = \mathbf{H}_n\left[\mathbf{\Phi}_{n-1}\boldsymbol{\upsilon}_{n-1} + \mathbf{\Gamma}_{n-1}\mathbf{g}_{mj}\right] \quad (9)$$

$$\boldsymbol{\sigma}_n^2 = \mathbf{H}_n \mathbf{\Psi}_{en} \mathbf{H}_n^{\mathrm{T}} + \boldsymbol{\sigma}_{Y_n}^2 \quad (10)$$

Here, $\mathbf{H}_n$, $\mathbf{\Phi}_{n-1}$ are well-known observation matrices and extrapolation of Kalman filter. Then, taking into account relations (7) - (10) and denoting $P\left(\mathbf{g}_{mjn} \mid \{\mathbf{Y}\}_n\right) = P_{nj}$, we get an expression for the posterior probability density

$$P_{nj} = \frac{\sum_{i=-k}^{k} \pi_{ij} P\left(\mathbf{g}_{mi(n-1)} \mid \{\mathbf{Y}\}_{n-1}\right) \exp\left[-\frac{\left(\hat{\mathbf{Y}}_n - \hat{\mathbf{Y}}_{enj}\right)^2}{2\boldsymbol{\sigma}_n^2}\right]}{\sum_{j=-k}^{k}\sum_{i=-k}^{k} \pi_{ij} P\left(\mathbf{g}_{mi(n-1)} \mid \{\mathbf{Y}\}_{n-1}\right) \exp\left[-\frac{\left(\hat{\mathbf{Y}}_n - \hat{\mathbf{Y}}_{enj}\right)^2}{2\boldsymbol{\sigma}_n^2}\right]} \quad (11)$$

From the analysis of the obtained relation (6), we can conclude that in practice, the problem of estimating the desired vector $\hat{\boldsymbol{\upsilon}}_n$ is reduced to weighted averaging of estimates $\hat{\boldsymbol{\upsilon}}_n\left(\mathbf{g}_{mjn}\right)$ that can be obtained by any known method for fixed values $\mathbf{g}_{mjn}$.

To obtain $\hat{\boldsymbol{\upsilon}}_n\left(\mathbf{g}_{mjn}\right)$ estimates, it is advisable to apply Kalman filters, since they are traditionally used in measuring information systems and have proven themselves well. Then the synthesized adaptive filter is a set of Kalman filters for obtaining estimates $\hat{\boldsymbol{\upsilon}}_n\left(\mathbf{g}_{mjn}\right)$, which are summed at the output with the weight coefficients P_{nj} to find the desired estimate. The block diagram of the resulting adaptive filter is shown in Fig. 1.

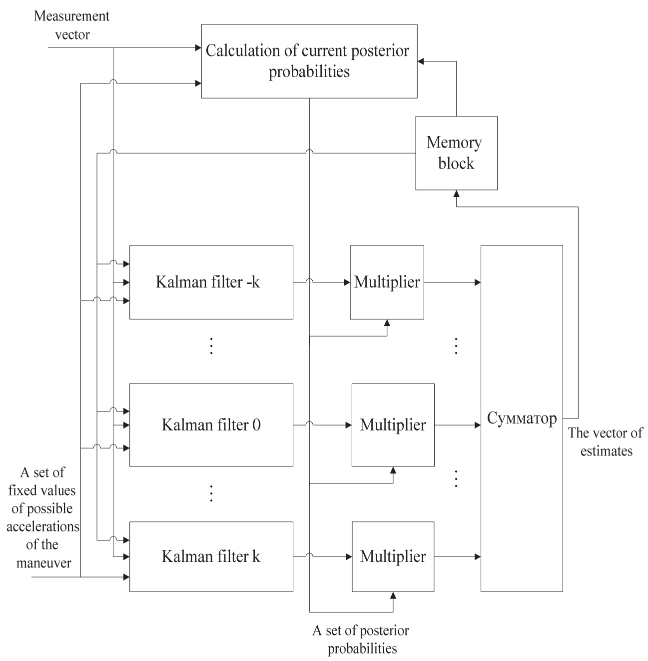

Fig. 1. General structure of a recurrent adaptive filter

The set of operations that are performed in the Kalman filter is known [9, 10, 12] and can be represented as the following expressions

$$\begin{aligned}
\hat{\boldsymbol{\upsilon}}_{en} &= \boldsymbol{\Phi}_n \hat{\boldsymbol{\upsilon}}_{n-1};\\
\boldsymbol{\Psi}_{en} &= \boldsymbol{\Phi}_n \boldsymbol{\Psi}_{n-1} \boldsymbol{\Phi}_n^{\mathrm{T}};\\
\mathbf{K}_n &= \boldsymbol{\Psi}_{en} \mathbf{H}_n^{\mathrm{T}} \left(\mathbf{H}_n \boldsymbol{\Psi}_{en} \mathbf{H}_n^{\mathrm{T}} + \mathbf{R}_n \right);\\
\hat{\boldsymbol{\upsilon}}_n &= \hat{\boldsymbol{\upsilon}}_{en} + \mathbf{K}_n \left(\mathbf{Y}_n - \mathbf{H}_n \hat{\boldsymbol{\upsilon}}_{en} \right);\\
\boldsymbol{\Psi}_n &= \boldsymbol{\Psi}_{en} - \mathbf{K}_n \mathbf{H}_n \boldsymbol{\Psi}_{en}.
\end{aligned} \qquad (12)$$

From the analysis of relations (12), it can be concluded that the values of the filter gains $\mathbf{K}_n$, as well as the covariance matrices of the parameter estimates errors $\boldsymbol{\Psi}_n$, do not depend on the specific values of the elements of the vector of possible values of the trajectory maneuver $\mathbf{g}_{mjn}$ intensities. Thus, these parameters can be calculated in a separate block and used in all $2k+1$ Kalman filters at once. Given this circumstance, we can conclude that the computational costs required to implement the considered adaptive filter are significantly reduced, which is an important factor for real-time information processing systems.

Then the block diagram of the adaptive filter can be presented in the following form (Fig. 2).

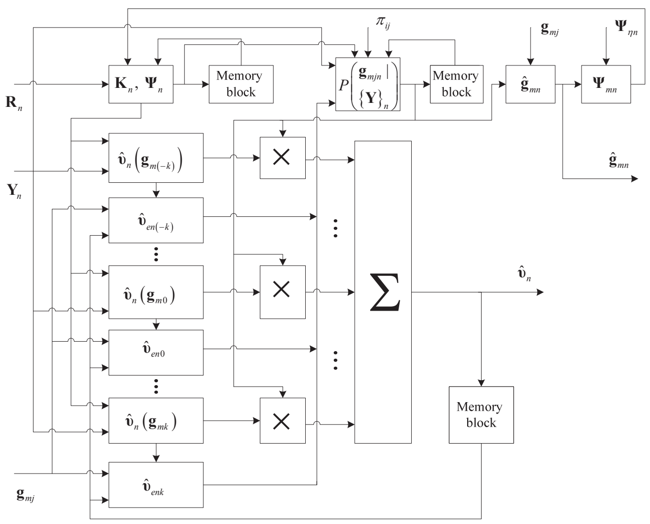

Fig. 2. A block diagram of the adaptive filter

Let us consider the content of the major operations that are implemented in the adaptive filter, considering the previously mentioned features of processing information about the parameters of the UAV in the trajectory maneuver section.

IV. An example of implementation of the adaptive filter

It should be noted that measurements of UAV coordinates by radar stations are usually performed in the second spherical coordinate system [13] $R,\ \varepsilon,\ \gamma$. However, due to the peculiarities of such a system, acceleration can reach significantly larger values than in rectangular coordinate systems.

For example, at a flight speed of V = 18 m / s at a distance R of 500 m at R = 0, the radial acceleration value will be $\ddot{R} \approx 28$ m/s^2, while in rectangular inertial coordinate systems the maximum acceleration value does not exceed the gravitational acceleration value of $g_з \approx$ of 10 m/s^2. This will lead to the fact that the filter will work constantly in the maneuver mode, in addition, more blocks are required that implement the Kalman filters, which are tuned to a certain intensity of the maneuver. Therefore, it is advisable to process information in an adaptive filter in a rectangular coordinate system.

Let us apply the topocentric coordinate system [13] for this, as it is directly related to the location of the measuring information system. Its beginning is on the surface of the Earth. The axis y is directed to the zenith along the normal to the surface, while the axes x and z lie in the horizontal plane. The direction of the axis z is given by the azimuth $A_з$ ($A_з$ is the angle counted from the direction northward clockwise to the axis z). The axis x complements the system to the right.

Topocentric rectangular coordinates are expressed in terms

$$\begin{aligned}
x &= R\sin\varepsilon\cos\gamma;\\
y &= R\sin\varepsilon\sin\gamma;\\
z &= R\cos\varepsilon.
\end{aligned} \qquad (13)$$

of spherical coordinates $R,\ \varepsilon,\ \gamma$, using the relations

Here, ε $\left(0 \le \varepsilon \le \pi\right)$ is the axis between the axis z and the line from the reference point to the point $a\left(R,\ \varepsilon,\ \gamma\right)$. The angle γ $\left(-\pi \le \gamma \le \pi\right)$ is the linear angle of the dihedral angle formed by the horizontal plane and the plane passing through the axis z and point a.

We will assume that at the $\left(n-1\right)$-th step, the vector of estimates of the parameters of the UAV trajectory is obtained

$$\hat{\boldsymbol{\upsilon}}_{n-1} = \left(\hat{x}_{n-1}, \hat{\dot{x}}_{n-1}, \hat{\ddot{x}}_{n-1}, \hat{y}_{n-1}, \hat{\dot{y}}_{n-1}, \hat{\ddot{y}}_{n-1}, \hat{z}_{n-1}, \hat{\dot{z}}_{n-1}, \hat{\ddot{z}}_{n-1} \right)^{\mathrm{T}},$$

posterior probabilities of possible values of maneuver intensities

$$P\left(\mathbf{g}_{mj} \mid \left\{\mathbf{Y}\right\}_n\right) \left(j = -k, -\left(k-1\right), \ldots, 0, 1, \ldots, k-1, k\right),$$

the covariance matrix of the random component and control noise $\boldsymbol{\Psi}_{\eta(n-1)}$, as well as the covariance matrix of the estimates errors

$$\mathbf{\Psi}_{n-1} = \begin{pmatrix} \psi_{11(n-1)} & \psi_{12(n-1)} & \cdots & \psi_{18(n-1)} & \psi_{19(n-1)} \\ \psi_{21(n-1)} & \psi_{22(n-1)} & \cdots & \psi_{28(n-1)} & \psi_{29(n-1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \psi_{81(n-1)} & \psi_{82(n-1)} & \cdots & \psi_{88(n-1)} & \psi_{89(n-1)} \\ \psi_{91(n-1)} & \psi_{92(n-1)} & \cdots & \psi_{98(n-1)} & \psi_{99(n-1)} \end{pmatrix} \quad (14)$$

It should be considered that the elements $\psi_{ii(n-1)}$ $(i = 1,\dots,9)$ are the variances of the errors of the corresponding parameters of the vector $\hat{\mathbf{\upsilon}}_{n-1}$ estimates, and the remaining elements are the corresponding covariance.

Extrapolation of the trajectory parameters for each of the possible ones $\mathbf{g}_{mj}$ $\left(j = -k, -(k-1), \dots, 0, 1, \dots, k-1, k \right)$ is carried out according to the formulas

$$\hat{\mathbf{\upsilon}}_{en}\left(\mathbf{g}_{mj}\right) = \mathbf{\Phi}_n \hat{\mathbf{\upsilon}}_{n-1} + \mathbf{\Phi}_{mn}\mathbf{g}^{p}_{mj} \quad (15)$$

Where the matrices $\mathbf{\Phi}_n$, $\mathbf{\Phi}_{mn}$ are the matrices of extrapolation of the state and maneuver vectors, respectively. Wherein

$$\mathbf{\Phi}_n = \begin{pmatrix} \mathbf{\Phi}_e & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_e & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Phi}_e \end{pmatrix} \quad (16)$$

where $\mathbf{\Phi}_e = \begin{pmatrix} 1 & \tau_e & \tau_e^2/2 \\ 0 & 1 & \tau_e \\ 0 & 0 & 1 \end{pmatrix}$ is the matrix of extrapolation of a part of the state vector corresponding to a specific coordinate and its two derivatives.

The matrix $\mathbf{\Phi}_{mn}$ has the form

$$\mathbf{\Phi}_{mn} = \begin{pmatrix} \mathbf{\Phi}_{emn} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_{emn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Phi}_{emn} \end{pmatrix}, \quad \mathbf{\Phi}_{emn} = \begin{pmatrix} 0 & 0 & \tau_e^2/2 \\ 0 & 0 & \tau_e \\ 0 & 0 & 1 \end{pmatrix}, \quad (17)$$

where τ_e is an interval of extrapolation, and an extended vector of possible intensities of the maneuver is $\mathbf{g}^{p}_{mj} = \left(0,\ 0,\ \ddot{x}_{mj},\ 0,\ 0,\ \ddot{y}_{mj},\ 0,\ 0,\ \ddot{z}_{mj}\right)$.

The calculation of the covariance matrix of extrapolation errors is carried out according to the formula

$$\mathbf{\Psi}_{en} = \mathbf{\Phi}_n \mathbf{\Psi}_{n-1} \mathbf{\Phi}_n^{\mathrm{T}}. \quad (18)$$

The vector of filter gains in the corresponding block is calculated in accordance with the expression

$$\mathbf{K}_n = \mathbf{\Psi}_{en}\mathbf{H}_n^{\mathrm{T}}\left(\mathbf{H}_n\mathbf{\Psi}_{en}\mathbf{H}_n^{\mathrm{T}} + \mathbf{R}_n\right), \quad (19)$$

where the observation matrix is

$\mathbf{H}_n = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$, the covariance matrix of measurement errors $\mathbf{R}_n = \begin{pmatrix} \sigma^2_{x_n} & 0 & 0 \\ 0 & \sigma^2_{y_n} & 0 \\ 0 & 0 & \sigma^2_{z_n} \end{pmatrix}$ has a diagonal form, since, in accordance with the recommendations presented in [12], it is advisable to switch to a simplified filtering method and not take into account the covariance that occurs when the matrix of measurement errors is recalculated from a spherical coordinate system to a rectangular one. In this case, the loss of accuracy does not exceed 5 - 15 percent, but the computational costs are significantly reduced.

The covariance matrix of the parameter estimates errors is determined by the formula

$$\mathbf{\Psi}_n = \mathbf{\Psi}_{en} - \mathbf{K}_n\mathbf{H}_n\mathbf{\Psi}_{en} \quad (20)$$

The estimates of the filtered parameters for each of the possible values of the maneuver $\mathbf{g}_{mj}$ intensities are calculated by the formula

$$\hat{\mathbf{\upsilon}}_{nj} = \hat{\mathbf{\upsilon}}_{en}\left(\mathbf{g}_{mj}\right) + \mathbf{K}_n\left(\mathbf{Y}_n - \mathbf{H}_n\hat{\mathbf{\upsilon}}_{en}\left(\mathbf{g}_{mj}\right)\right), \quad j = -k,\dots,0,\dots,k. \quad (21)$$

Posterior probabilities (weights of possible values of the intensities of the maneuver) are determined based on the expression

$$P_{nj} = \frac{\sum_{i=-k}^{k} \pi_{ij} P\left(\mathbf{g}_{mi(n-1)} \,|\, \{\mathbf{Y}\}_{n-1}\right) \exp\left[-\frac{\left(\hat{\mathbf{Y}}_n - \hat{\mathbf{Y}}_{enj}\right)^2}{2\left(\mathbf{\Psi}_n + \mathbf{\Psi}_{en}\right)^2}\right]}{\sum_{j=-k}^{k}\sum_{i=-k}^{k} \pi_{ij} P\left(\mathbf{g}_{mi(n-1)} \,|\, \{\mathbf{Y}\}_{n-1}\right) \exp\left[-\frac{\left(\hat{\mathbf{Y}}_n - \hat{\mathbf{Y}}_{enj}\right)^2}{2\left(\mathbf{\Psi}_n + \mathbf{\Psi}_{en}\right)^2}\right]}, \quad (22)$$

$j = -k, \ldots, 0, \ldots, k.$

The estimation of the UAV motion parameters is calculated in accordance with the formula

$$\hat{\boldsymbol{\upsilon}}_n = \sum_{j=-k}^{k} P_{nj} \hat{\boldsymbol{\upsilon}}_{nj}. \quad (23)$$

The elements of the vector of maneuver intensities are estimated by the formula

$$\hat{\mathbf{g}}_{mn} = \sum_{j=-k}^{k} P_{nj} \mathbf{g}_{mj}. \quad (24)$$

From the analysis of expressions (15) - (24) it follows that in the case when the measurements along the coordinates are considered independent [12], then the filter under consideration can be divided into three independent filters, which will lead to a decrease in the dimension of the processed matrices to three instead of nine, and this will significantly reduce the computational costs required to implement the filter.

V. Filter simplification

The considered recurrent adaptive filter requires for its implementation a sufficiently large number of matrix operations due to the need $(k+1)$ times to calculate $\hat{\boldsymbol{\upsilon}}_{en}\left(\mathbf{g}_{mj}\right)$, $\hat{\boldsymbol{\upsilon}}_{nj}$ and P_{nj} at each step of processing information about the UAV. In [12], an approach was proposed that makes it possible to simplify the adaptive filter. Its essence lies in the fact that a weighted estimate of the extrapolated values of the parameters is found, which is then used in one ordinary Kalman filter (12). In this case, the specified estimate is calculated by the formula

$$\hat{\boldsymbol{\upsilon}}_{en} = \sum_{j=-k}^{k} P_{nj} \hat{\boldsymbol{\upsilon}}_{en}\left(\mathbf{g}_{mj}\right) \quad (25)$$

The block diagram of such a simplified filter is shown in Figure 3.

VI. Simulation results

To study the effectiveness of the proposed filter, statistical modeling of its work was carried out. To obtain the results, 78 realizations were made, which ensured a confidence level of 97.5 percent.

Simulation of the UAV motion [18, 19, 20] was carried out by integrating the equations of motion in a topocentric coordinate system using the fourth-order Runge-Kutt method [16]

$$\begin{aligned} \ddot{x} &= -\mu x r^{-3} - 2\left(\dot{z}\omega_y - \dot{y}\omega_z\right) + x\omega_з^2 - m\omega_x; \\ \ddot{y} &= -\mu\left(y + r_з\right) r^{-3} - 2\left(\dot{x}\omega_z - \dot{z}\omega_x\right) + \left(y + r_з\right)\omega_з^2 - m\omega_y; \\ \ddot{z} &= -\mu z r^{-3} - 2\left(\dot{y}\omega_x - \dot{x}\omega_y\right) + z\omega_з^2 - m\omega_z, \end{aligned} \quad (26)$$

where the geocentric gravity parameter is $\mu = 3{,}98603 \cdot 10^{14}\ \mathrm{m^3/s^2}$; components of the row vector of the angular velocity of the Earth's rotation $\boldsymbol{\omega}_з = \left(\omega_x, \omega_y, \omega_z\right)$ are determined according to the expressions $\omega_x = \omega_з \cos\varphi_з \sin A_з$, $\omega_y = \omega_з \sin\varphi_з$, $\omega_z = \omega_з \cos\varphi_з \cos A_з$; $\varphi_з$ is the latitude of the position of the measuring information system; the parameter is $m = x\omega_x + \left(y + r_з\right)\omega_y + z\omega_z$; $r_з$ is the radius of the Earth; $A_з$ is the azimuth of the topocentric coordinate system; angular velocity of rotation of the Earth is $\omega_з = 0{,}727211 \cdot 10^{-4}\ rad/\mathrm{s}$.

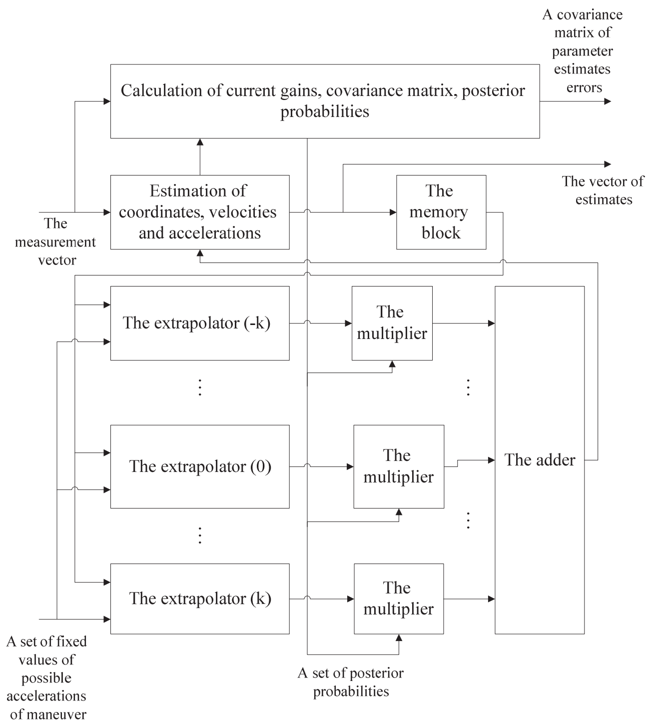

Fig. 3. A block diagram of a simplified adaptive filter.

To simulate the motion (aerodynamic forces of the UAV airframe, thrust forces of the engines) and the UAV maneuver, the corresponding components of the flight $\left(\ddot{x}_p, \ddot{y}_p, \ddot{z}_p\right)$ and maneuver $(\ddot{x}_м, \ddot{y}_м, \ddot{z}_м)$ vectors were added to the equations of motion at the necessary times.

A UAV maneuver was simulated moving along a circular trajectory with a radius of 600 m. The trajectory maneuver was carried out under the influence of continuous thrust during ten periods of the radar station's review. The components of the maneuver vector were

$$\left(10\ \mathrm{m/s^2},\ 10\ \mathrm{m/s^2},\ 10\ \mathrm{m/s^2}\right).$$

The division into five possible values of the intensities of the maneuver $(-16,-16,-16)$, $(-8,-8,-8)$, $(0,0,0)$, $(8,8,8)$, $(16,16,16)$ was carried out in the adaptive filter. The relative ball dynamic error $\sigma_{drel} = \sigma_{\mathrm{shd}} / \sigma_{shmeas}$ was used as a characteristic of the filter efficiency, while $\sigma_{\mathrm{sh}} = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$.

The processing of the incoming information was carried out using the usual Kalman filter [9], an adaptive filter with a linear motion model [12], the proposed adaptive filter.

The simulation results showed that σ_{drel} did not exceed 0.5 for the proposed filter in the maneuver section, 1.2 for the filter [12], 1.9 for the Kalman filter [9]. At the same time, the filter [12] was constantly working in the maneuver mode. At the stationary section of measurement processing, the value σ_{drel} for the filter under consideration exceeded the analogous value for the Kalman filter [9] by 12 percent, and for the filter [12] by 21 percent.

VII. Conclusion

A recurrent adaptive filter which makes it possible to efficiently process information about the UAV both during normal flight and in the trajectory maneuver section has been developed.

A feature of the filter is the use of a quadratic model of motion in the constituent filters or in the extrapolators of the simplified filter, which is more consistent with the actual movement of the UAV at the intervals of their tracking with measuring information systems.

The synthesis is based on the principle of separation.

The developed filter showed its superiority over the analogue and the conventional Kalman filter when working in the UAV trajectory section, providing a dynamic error of 50 - 70 percent less than the filters with which the comparison was made.

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