Analysis of Capacity of Picocell with Dominating Video Streaming Traffic

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***Abstract*—This paper focuses on the research of centralized wireless networks where main load is formed with on-line video streaming services. In case of congestion channel resources are not enough for satisfying all the users requirements. This brings to degradation of video playback (appearance of rebufferings, forced transcoding to lower bit rate etc). Congestion has probabilistic nature, since it depends not only on number of users but also on there location relatively to a base station and chosen bit rates of videos. For considered networks we propose a model and two techniques of congestion probability estimation. Basing on these estimations network capacity is analyzed.**

I. Introduction

In recent years on-line multimedia technologies have become increasingly popular. The most widely these technologies are distributed in video transmission over centralized wireless networks. Such networks may contain a sufficient number of users. When resources are not enough for requiring to quality of service (QoS) for each user, then network is considered to be congested. This is accompanied with users video playback degradation. The most common problems are: rebuffering (state of streaming invoked when the play back buffer is emptied) [1], jitter (variation of playback speed) [2], playback smoothness (frequency of video bit rate switching) [3], etc.

For enhancement of network coverage and decreasing energy consumption of wireless devices so-called picocells are used. Picocell is a small yet full-function base station capable to organize a network in range of few tens meters [4]. A typical areas of picocell application are: office buildings, shop centers, hotels etc. Hence, picocells are characterized with specific distribution of users location around it which is to be accounted in its analysis.

A common problem for a network developer is estimation of network capacity, e.g. number users who can simultaneously watch video content without sufficient degradation.

Obviously network occupancy depends on wireless network channel resources, bit rates of streamed videos and current radio-conditions of user equipments (UEs). Since the latest two parameter exhibit probabilistic nature, congestion is a random event either. Hence, network capacity is tightly dealt with probability of this event.

In this paper we focus on estimation of congestion probability by analysis of system bandwidth, wireless environment conditions, size of a network and available bit rates of video content.

The remainder of this paper is organized as follows: brief overview of related works is given in Section II. In Section III we introduce system model and a few formal definitions. Section IV is devoted to analysis of congestion probability and estimations of network capacity. Section V presents a few numerical examples of developed techniques. Possible use cases are given in Section VI. The paper is finalized with Conclusion in Section VII.

II. Related Works

A large number of efforts have focused on investigation of performance of wireless video streaming networks (WVSN). For modern wireless networks exist several optimization parameters such as delays, network capacity, jitter etc. However, the main criterion of WVSN system performance is user's quality of experience (QoE), this is user's subjective reaction on system parameters such as initial delays, video bit rate, number and frequency of rebufferings, smoothness of playback etc.

There are a few papers where authors propose their own interpretation of QoE, and create algorithms for improving proposed QoE metrics. For example, in paper [1] QoE is rebuffering percentage ρ, which is defined as the percentage of the total presentation time in which the user experiences rebuffering due to buffer starvation. Network capacity in this paper is defined as maximum number of UEs K for which specified part of users l_{UE} have rebuffering percentage in an appropriate level ρ_{max}:

$$N_c = \arg\max_K \left\{ \frac{\sum_{i=1}^{K} \mathbf{1}\left(\rho_i \leq \rho_{max}\right)}{K} \geq l_{app} \right\}.$$

Such definition of network capacity has clear meaning but its theoretical investigation is a quite hard problem. So, authors of the paper created dynamic system-level simulator for the LTE air-interface based on a MATLAB-based software platform with detailed abstractions of application, transport, medium access control, and physical layers.

In paper [5] quality of network operation is measured using the Structural SIMilarity index (SSIM). Congestion is interpreted as a situation of SSIM decreasing. SSIM maximization is solved as nonlinear optimization problem with the following constraint:

$$\sum_{i=1}^{N_c} \frac{R_i}{C_i} \leq 1, \tag{1}$$

where R_i – selected video bitrate in i-th transmission link and C_i - this link throughput (for communicating devices maximum achievable throughput is estimated with Shannon formula). Authors assume, that if relation (1) is satisfied than system is not in the congestion condition.

In [3] authors propose estimation of network capacity considering not only video traffic but also extra background TCP flows, passing through the same base station. Hence, the following enhancement of (1) is used:

$$\sum_{j=1}^{J} \frac{R_j^b}{C_j^b} + \sum_{i=1}^{N_c} \frac{R_i}{C_i} \leq 1.$$

Here R_j^b and C_j^b – denote the traffic rate and link throughput of j^{th} background flow. To show effectiveness of proposed solutions authors use simulations based on the ns-2.

Finally, in [2] network QoE is defined as weighted sum of video bit rate, jitter and smoothness of playback. As in previous paper, authors use ns-2 simulator, but in this work Enhanced UMTS Radio Access Network Extensions (EURANE) with three different kinds of wireless scheduling algorithm is used. Set of these algorithms includes Round-Robin, Fair Channel-Dependent Scheduling, and Maximum Carrier to Interference ratio.

In all mentioned papers, authors usually define congestion condition in wireless video streaming networks and use simulation for congestion analysis. In this paper we propose theoretical investigation for simplified system model.

III. System Model

A. General Description

Investigated system includes wireless base station (BS) and a set of N users, whose equipment is able to playback multimedia content (UEs). These UEs continuously download video stream from content servers located in Internet through the BS (see fig. 1).

Each i-th UE is characterized with both bit rate of requested video stream R_i and maximum achievable throughput C_i which depends on radio-conditions in point of UE location. Hence, total number of channel resources, required for all UEs is equal to $\sum_{i=1}^{N} \frac{R_i}{C_i}$. If this sum exceeds 1 then lack of resources brings to QoE degradation.

This allows introducing the following formal definitions.

Definition 1: **Congestion** is an event, when total amount of required resources is greater than one.

Definition 2: **Network capacity** N_c is a maximum number of users, for which the probability of congestion is less than given level p_c. I. e.

$$N_c = \arg\max_{N} \left\{ Pr\left\{ \sum_{i=1}^{N} \frac{R_i}{C_i} \geq 1 \right\} \leq p_c \right\}.$$

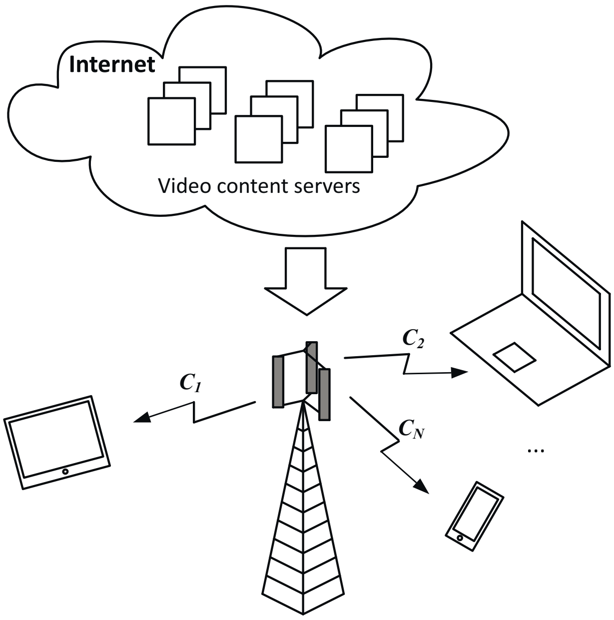

Fig. 1. System Structure

B. Channel Model

In the paper we use standard log-distance radio propagation model for signal-to-noise ratio (SNR) prediction [7]. According to it path loss is characterized with the following expression:

$$L(d) = \frac{L_0}{d^\gamma}.$$

Here L_0 is a path loss factor, γ - path loss exponent. Path loss exponent varies in range $2-6$ depending on environment [6]. For typical building $\gamma = 3$ and this value will be used in further derivations.

Note, that despite the fact that path loss exponent ”3” was obtained here from log-distance model, it can appear in different other models, such as, for example, in COST-Hata model for metropolitan area [8]. According to it for metropolitan areas path loss dependency on distance between BS and UE can be described with a following expression:

$$\begin{aligned} L_{dB}(d) = 49.3 + 33.9\lg f_0 - 13.82\lg H_{BS} + \\ + (44.9 - 6.55\lg H_{UE})\lg\left(10^{-3}d\right). \end{aligned}$$

Here L_{dB} denotes loss in dB’s, f_0 - carrier frequency (in MHz), H_{BS} and H_{UE} - heights of BS and UE respectively, d - distance in meters. For typical parameters ($H_{BS} = 180m$, $H_{UE} = 2m$, $f_0 = 2GHz$):

$$L_{dB}(d) = 39.8 + 30\lg(d).$$

This expression can be also rewritten for finding loss in times:

$$L(d) = \frac{L_0}{d^3}.$$

Here L_0 is approximately equal to 10^4. So further derivations can be applied for such models either.

Denoting transmitted power as P_{TX}, Boltzmann constant as k, current temperature as T, system bandwidth as ΔF and UE receiver noise figure as N_f the following expression for i-th UE SNR can be obtained:

$$q_i = \frac{P_{TX} L_0}{k T \Delta F N_f d_i^3} = \frac{a}{d_i^3}.$$

Here $a = P_{TX} L_0 (k T \Delta F N_f)^{-1}$.

For maximum channel throughput estimation an approximation based on Shannon formula will be used:

$$C_i = \Delta F \log_2(1 + q_i). \tag{2}$$

C. Users Location in Picocell

Typically, developers of picocells try to place the BS in a center of building to maximize network coverage (see fig. 2)

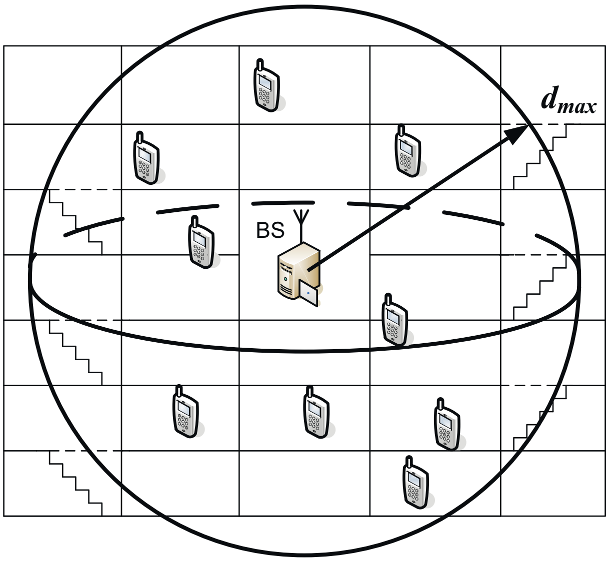

Fig. 2. Picocell Structure

Such disposition can be modeled with uniform distribution of UEs in sphere which radius corresponds to maximum distance d_{max}. Hence, cumulative distribution function of d_i is expressed as follows:

$$F_d(x) = \begin{cases} 0, & \text{if } x < 0; \\ \frac{x^3}{d_{max}^3}, & \text{if } 0 \le x \le d_{max}; \\ 1, & \text{if } x > d_{max}. \end{cases}$$

Probability distribution function in this case is as follows:

$$f_d(x) = \begin{cases} 0, & \text{if } x < 0; \\ \frac{3x^2}{d_{max}^3}, & \text{if } 0 \le x \le d_{max}; \\ 0, & \text{if } x > d_{max}. \end{cases} \tag{3}$$

D. Extra Assumptions

In the paper the following additional assumptions are used:

1) number of active picocell users is constant;
2) signal propagation conditions are constant in time, i.e.
$\forall i = \overline{1, N} : C_i(t) = C_i$;
3) location and hence maximum throughputs of UEs are independent items;
4) all users request videos in the same bit rate $R_i = R$.

IV. Analysis of Network Capacity

A. Preliminary Discussion

From definition of network capacity (Definition 2) it follows, that for estimation network capacity it is necessary to calculate the probability of congestion. Therefore the probability of congestion is as follows:

$$Pr\{\text{Congestion}\} = Pr\left\{\sum_{i=1}^{N} \frac{R_i}{C_i} \ge 1\right\} = Pr\left\{\sum_{i=1}^{N} \frac{1}{C_i} \ge \frac{1}{R}\right\},$$

where N is the number of users.

Applying approximation (2) based on Shannon formula we obtain:

$$Pr\{\text{Congestion}\} = Pr\left\{\sum_{i=1}^{N} \frac{1}{\log_2(1+q_i)} \ge \frac{\Delta F}{R}\right\}.$$

Probabilistic analysis of random variables $\log_2^{-1}(1+q_i)$ is dealt with a set of problems and, as a rule, brings to nonelementary integrals which can be solved only numerically. For significant simplification of analysis in the next subsection we propose one auxiliary inequality.

B. Auxiliary Inequality

Let us consider bounding function for $\log_2^{-1}(1+x)$ in the following form.

$$\frac{1}{\log_2(1+x)} \le \log_2\left(1 + \frac{1}{x}\right) + \alpha. \tag{4}$$

We have to choose such value of summand α that inequality (4) became valid and bound was the tightest, i.e. minimal α at which $\log_2\left(1+\frac{1}{x}\right) + \alpha - \frac{1}{\log_2(1+x)} \ge 0$ for any x.

It can be found by means of finding minimum of $\log_2\left(1+\frac{1}{x}\right) + \alpha - \frac{1}{\log_2(1+x)}$. For it derivative of this function is to be taken.

$$\frac{d}{dx}\left[\log_2\left(1+\frac{1}{x}\right) + \alpha - \frac{1}{\log_2(1+x)}\right] = \\ = \frac{\log_2 e}{1+x}\left(\frac{1}{x} - \frac{1}{(\log_2(1+x))^2}\right).$$

This derivative is equal zero, only in point satisfying the following equality:

$$\sqrt{x} = \log_2(1+x).$$

Solution of this equation can be easily obtained numerically. Hence, we find value of x, substitute it into (4) and obtain α, that is 0.158. Comparison of initial function and proposed bounding function when $\alpha = 0.158$, is shown in Fig. 3.

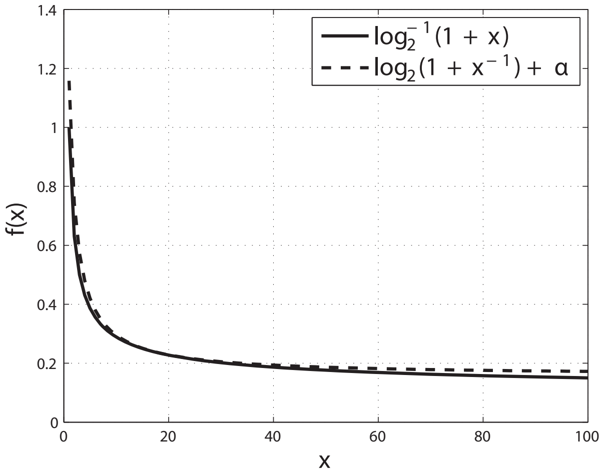

Fig. 3. Bounding function for $\log_2^{-1}(1+x)$

Hence, the following inequality can be stated:

$$Pr\{\text{Congestion}\} \leq Pr\left\{\sum_{i=1}^{N}\left(\log_2(1+q_i^{-1})+\alpha\right) \geq \frac{\Delta F}{R}\right\}.$$

For simplicity of further derivations let us denote $\log_2(1+q_i^{-1})$ as X_i and sum of N of such terms as S_N.

C. Approximate Calculation of Congestion Probability

First propose approach is based on using of Central Limit Theorem (CLT). Assuming normalization of S_N, $Pr\left\{S_N \geq \frac{\Delta F}{R}\right\}$ can be obtained from Q-function [9]:

$$Pr\{\text{Congestion}\} \approx Q\left(\frac{\Delta F - R\mu}{R\sigma}\right), \quad (5)$$

where $\mu = E[S_N]$ and $\sigma = \sqrt{Var[S_N]}$.

At first let us calculate $E[X_i]$. Substituting $\frac{a}{d_i^3}$ instead of q_i we obtain:

$$E[X_i] = \int_0^{d_{max}} \left[\log_2\left(1+\frac{x^3}{a}\right)+\alpha\right] f_d(x)dx.$$

Considering (3) and substituting t instead of x^3:

$$E[X_i] = \frac{1}{d_{max}^3}\left[\int_0^{d_{max}^3} \log_2\left(1+\frac{t}{a}\right)dt + \int_0^{d_{max}^3}\alpha dt\right] =$$
$$= \frac{k\ln m - 1}{\ln 2} + \alpha, \quad (6)$$

where $m = 1 + \frac{d_{max}^3}{a}$ and $k = 1 + \frac{a}{d_{max}^3}$.

For finding variance of X_i the following expression is used:

$$Var[X_i] = E[X_i^2] - E[X_i]^2.$$

$E[X_i]$ was calculated earlier, so for computation of variance we have to find $E\left[X_i^2\right]$. Using the same substitution as in (6):

$$E\left[X_i^2\right] = \frac{k(\ln m - 1)^2 - k}{(\ln 2)^2} + 2\alpha E[X_i] + \alpha^2.$$

Since X_i are independent random variables: $E[S_N] = N \cdot E[X_i]$, $Var[S_N] = N \cdot Var[X_i]$.

A significant drawback of this approach is the complexity of its accuracy analysis. Indeed it is quite difficult to say if this approximation gives understated or exceeded value of congestion probability. In the next section we propose an approach, which gives strict upper bound for congestion probability.

D. Upper Bound of Probability of Congestion

As was mentioned above:

$$Pr\{\text{Congestion}\} \leq Pr\left\{S_N \geq \frac{\Delta F}{R}\right\}.$$

For finding upper bound for $Pr\left\{S_N \geq \frac{\Delta F}{R}\right\}$ Hoeffding inequality can be used [10]. According to it:

$$Pr\{S_N - E[S_N] \geq t\} \leq \begin{cases} e^{-\frac{2t^2}{N(x_{max}-x_{min})^2}}, & t > 0; \\ 1, & t \leq 0, \end{cases}$$

where $X_i \in [x_{min}, x_{max}]$. Since $x_{min} = \alpha$, and $x_{max} = \log_2 m + \alpha$:

$$Pr\{\text{Congestion}\} \leq \begin{cases} e^{-2\frac{\left(\frac{\Delta F}{R} - N\cdot E[X_i]\right)^2}{N(\log_2 m)^2}}, & N < \frac{\Delta F}{R\cdot E[X_i]}; \\ 1, & \text{otherwise}. \end{cases} \quad (7)$$

E. Network Capacity Estimation

Basing on results described in subsections IV-C and IV-D, we obtain an approximate value and lower bound for network capacity. The following equation is to be solved for estimation of network capacity based on CLT:

$$Q\left(\frac{\Delta F - RN_cE[X_i]}{R\sqrt{N_c \cdot Var[X_i]}}\right) = p_c.$$

It brings to the following quadratic equation:

$$E[X_i]N_c + \sqrt{Var[X_i]}Q^{-1}(p_c)\sqrt{N_c} - \frac{\Delta F}{R} = 0.$$

Hence,

$$N_c \approx \left[\frac{-g_2 + \sqrt{g_2^2 - 4g_1g_3}}{2g_1}\right]^2,$$

where:

$$\begin{cases} g_1 = E[X_i]; \\ g_2 = \sqrt{Var[X_i]}Q^{-1}(p_c); \\ g_3 = -\Delta F R^{-1}. \end{cases}$$

According to (7), we obtain the following lower bound for network capacity:

$$N_c \geq \left(\frac{-g_4 + \sqrt{g_4^2 - 4g_1g_3}}{2g_1}\right)^2.$$

Here $g_4 = \log_2 m\sqrt{-\frac{1}{2}\ln p_c}$

V. Numerical example

In this section we propose numerical example for comparison of proposed estimations with simulation result. The parameters of the model are presented in Table I.

TABLE I. Parameters of model for numerical example

Parameter of model	Value
TX power P_{TX}	0 dBm
BS antenna gain	0 dB
UE antenna gain	0 dB
Bandwidth ΔF	10MHz, 20 MHz, 40 MHz
Carrier frequency	2 GHz
UE noise figure N_f	10
Video bit rate, R	500 kbps
Maximum distance, d_{max}	60 m

Figures 4, 5 and 6 shows dependencies of congestion probabilities on the number of UEs for three values of system bandwidth.

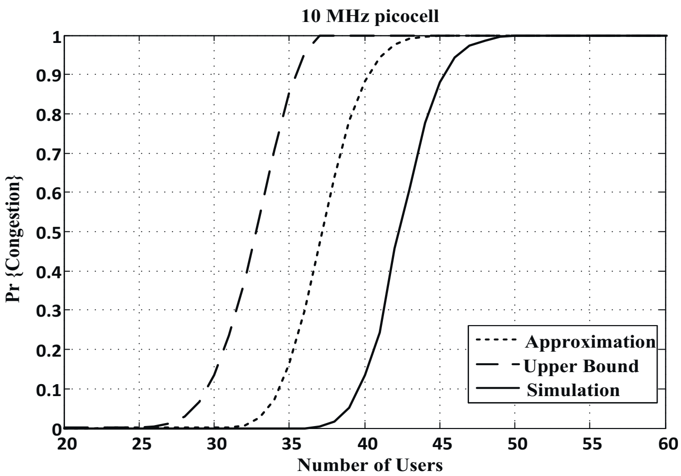

Fig. 4. Dependency of congestion probability on the number of users, $\Delta F =$ 10 MHz

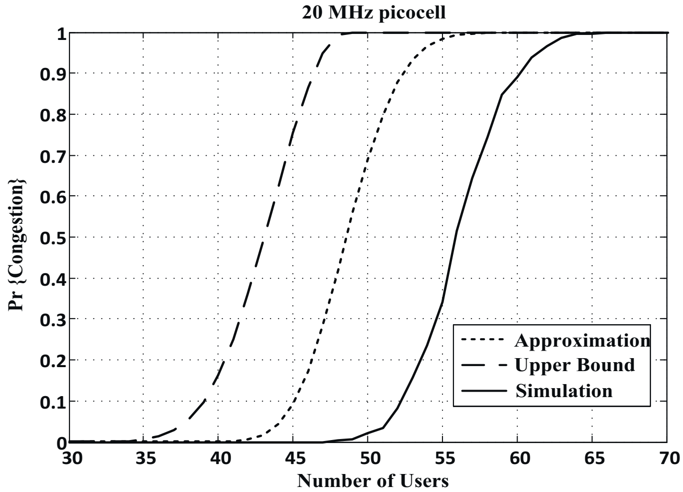

Fig. 5. Dependency of congestion probability on the number of users, $\Delta F = 20MHz$

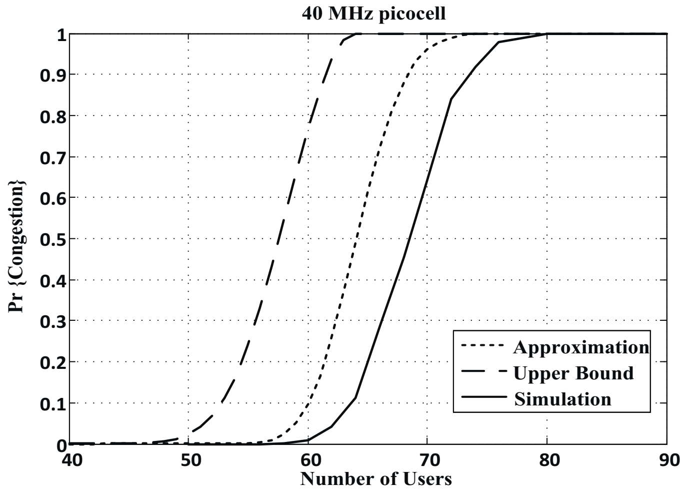

Fig. 6. Dependency of congestion probability on the number of users, $\Delta F = 40MHz$

Network capacity for investigated cases is given in Table II. For it allowable congestion probability level $p_c = 0.1$.

TABLE II. Networks capacities

Technique	10 MHz	20 MHz	40 MHz
Lower bound	30	39	51
CLT approximation	34	45	60
Simulation	40	52	62

VI. Possible Use Cases

Proposed expressions allow solving typical task of communication systems engineers. Imagine large office building which is to be equipped with a set of picocells for provision of users with wireless Internet. Users can be distributed in building arbitrary: for example, some floors in building are more populated, some floors - less (see figures 7 and 8). Hence, the optimization problem is as follows: minimize cost of chosen equipment, which guarantees normal coverage of network without congestions.

Usually, there are two opposite ways for network organization:

1) install small number of powerful picocells (see fig. 7);
2) install a large number of relatively weak picocells (see fig. 8);

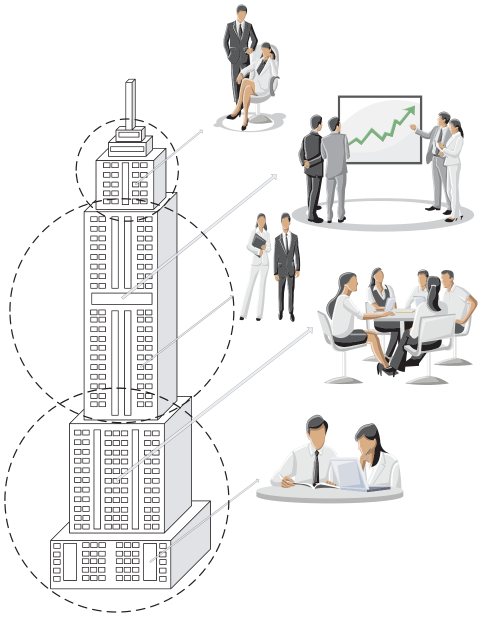

Fig. 7. Three power picocells

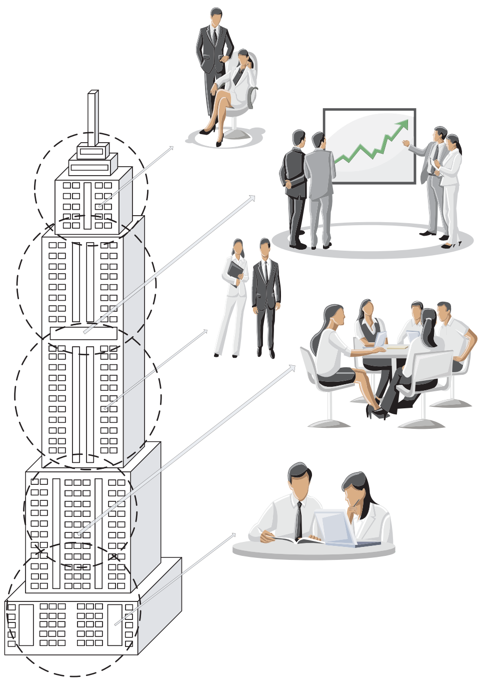

Fig. 8. Five weak picocells

For any way network developer has to be sure, that required users QoE would be satisfied (in our case - congestion probability is less than given level). Hence algorithm of network development can be as follows:

1) Choose placement and nomenclature of picocells.
2) For each picocell estimate radius of coverage and number of users served.
3) Using expression (5) or expression (7) estimate congestion probability.
4) If this probability is above required level modify the network configuration: choose more powerful picocells or add extra picocell and go to Step 2.

VII. Conclusion

In this paper we investigated congestion probability for wireless video streaming picocell network. For it we proposed convenient majorant of function $log_2^{-1}(1+x)$. This made possible analytical evaluation of both approximate solution and upper bound for congestion probability. Comparison with simulation results showed, that accuracy of proposed approaches depends on system parameters such as for example bandwidth.

Proposed expressions allows simple estimating of network capacity. However the results are applicable only for environments, where path loss factor is close to ”3”. Hence, generalization of obtained results for wider conditions may be a direction of further research.

Acknowledgment

Studies were conducted according to the work plan which has been submitted in application 14-11-00644 for grants competition held by Russian Science Foundation.

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