Methods of Complete Surface Reconstruction through Merging of Point Clouds According to Stereo Vision Data

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Abstract–**This article concerns the stack of algorithms which can be applied to the task of 3d reconstruction using stereo vision techniques. Stereo vision data represented as a set of single-valued surfaces of the point clouds that have overlapping areas. They may differ in density and regularity, but the whole reconstructed object model consists of a subset of single-valued surfaces. Thus, the task is to find a method of transformation of these original surfaces by minimizing some functional characterizing the degree of matching. Considered the methods of representation of the point cloud as two-dimensional discrete functions, suggested variants of the measure of differences between source datasets, as well as methods of mutual localization of surfaces.**

I. Introduction

Algorithms of three-dimensional scanning reproduce an observed scene as the discrete point cloud. The position of each point is characterized by the three coordinates *x, y* and *z*. For such set of points common concepts of connectivity, topology, or a surface continuity can't be defined. Based on this set of points, we can move to the triangular piecewise linear surface model, which is already a continuous model.

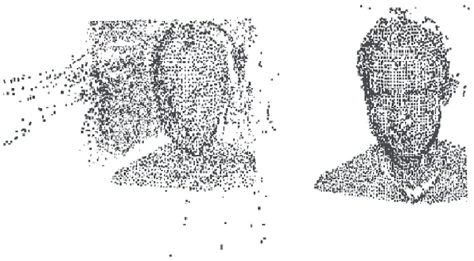

Fig. 1. Result of the process of visual stereo scanning: raw (left) and filtered point clouds

This point cloud can be described by a single-valued function of two variables defined on a discrete set of values. Unambiguity of function means that, there exists axis *a*, such that any line parallel to this axis will cross the surface no more than single way. That is, single-valued surface will not have self-intersections.

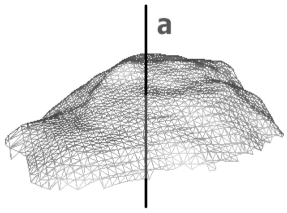

Fig. 2. Example of the single-valued surface

Such surfaces are also called 2.5-dimensional. They can be considered as a function of height that defined on a sets of points in the image plane, which is perpendicular to the axis *a*.

II. The Task of Comparing Surfaces

Stereo camera shots with a single point of view allow to reproduce only single-valued surfaces, as in this case there are no overlapping areas. Thus, the whole reconstructed object model consists of a subset of surfaces that unambiguous relative to the optical axes of these cameras.

Full reconstruction can be carried out with the aid of the initial positioning of the scanning system, as it is implemented in many modern industrial scanning systems. In this case, the subsets may not overlap. Information on the status of the individual parts of an object in a shared global coordinate system is known from the geometry of the scanning system. But originally scanned surfaces are single-valued. Each image contains information only on those points on the surface of the object, which can be seen from the position of observation.

If we are talking about the single-valued surface, the conversion from a point cloud to triangulated model could be done through the Delaunay triangulation. Then the surface will be defined by the height of the nodes in the triangular plane grid. Such surface can be seen as a set of spatial triangles composing the piecewise linear representation of the surface.

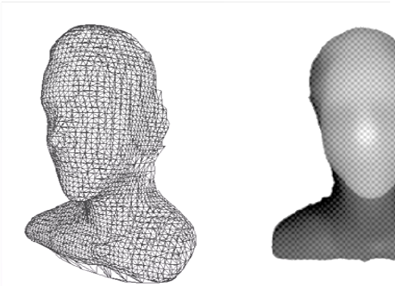

Fig. 3. Single-valued surface as a function of depth defined in the nodes of a plane grid (right)

Thus for two given single-valued surfaces it is necessary to calculate a measure of similarity between these surfaces to compare and match them. This leads to the main problem of such tasks - the correlation between the accuracy of the measure estimation and computational complexity. This ratio will determine the quality of the algorithm as a whole.

III. Methods of Specifying Surfaces

There are two main ways to specify single-valued surfaces. These are a structured surface on a regular grid and irregular chaotic one. Both methods are characterized by certain advantages and disadvantages.

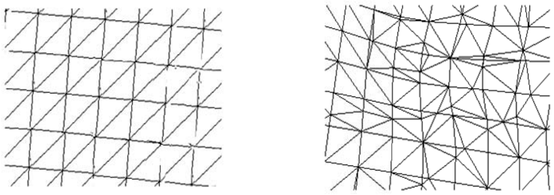

Fig. 4. Triangle grids with regular (left) and irregular structure

The facets of a regular grid are equal to each other. Regular grid nodes have a certain ordered structure (dx_i, dy_i), where *dx, dy* represent a grid spacing. Working with a regular grid does not make any specific difficulties due to the fact that the coordinates of its nodes can be calculated instead of storing them explicitly. One of the main features of regular grids is the problem of selecting the optimal cell size to achieve acceptable approximation level in a particular task.

Maximum threshold for the value of the grid spacing is selected according to the highest value of the gradient at the points of the original surface. In areas with high gradient it is necessary to use smaller grid spacing to keep as much information about the surface structure as possible. In this case the description of the entire surface with the same grid spacing is redundant because of the fact that this surface includes the low frequency regions This leads to computational and capacitive inefficiency.

The use of regular grids to compare surfaces suggests that each node of the total grid contains height values of the two surfaces. In this case there is no need to use interpolation methods for calculating the heights at intermediate points. Solution of the problem is reduced to the transformation of the original regular grid by the methods of translation and rotation. As a result of this transformation, grid in particular cases will not be a regular.

At irregular grid points can be placed arbitrary, i.e. a regular ordered structure is absent. Typically, when modeling surfaces with irregular meshes it is appropriate to use Delaunay triangulation. The main advantage of such grids is that their use does not arise the problem of surface description redundancy as in use of regular grid structures. To convert the heights of intermediate points of each surface it is necessary to use interpolation techniques. This requires the development of more sophisticated measures for comparison of surfaces defined on different grids.

IV. Common Methods of Comparing Surfaces

Research in the field of matching three-dimensional surfaces are carried out for a long time and there are several different approaches to solve this problem. Some methods involve recalculation of initial irregular grids into the composed regular grid [1, 2, 3]. Then the problem is reduced to the comparison of surfaces on regular sets of points. At the basis of the other methods are algorithms which suggest preservation of the original irregular meshes.

There are also methods that based on the comparison of feature descriptors. In such methods, the problem of comparing objects often comes down to the task of comparing the graph whose vertices are the information about the individual fragments of the surfaces of objects, and the edges contain information on the connectivity of these fragments [4].

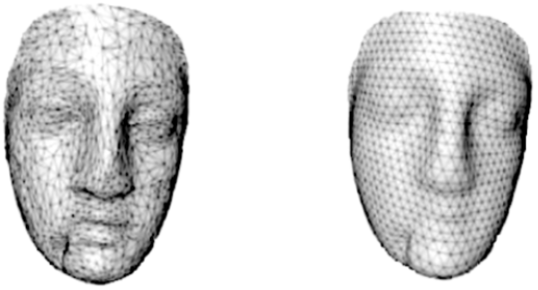

Fig. 5. Recalculation of irregular grid into a regular one

As measures to compare the surfaces so-called curvature maps may also be used. Curvature maps are isoline structures around specific points of the surfaces [5].

One of the major algorithms of combining surfaces is an iterative closest point algorithm [6, 7]. This method uses an iterative process to minimize the average distance between two clouds of points. This requires an initial evaluation of the coarse transformation of one cloud to another, which is gradually refined in the process of minimization.

For two given three-dimensional point clouds S_1 and S_2 ICP algorithm finds the best transformation that turn S_1 to S_2. The algorithm place for combining a the three-dimensional surface obtained by scanning the object from different points of view. It is assumed that in areas of overlapping surfaces are placed the two nearest points, the distance between them is less than a predetermined threshold. The main advantage of this nearest points algorithm is the simplicity of its implementation. However, he has a strong dependence on the initial approximation of objects and computational complexity.

Let N_1, N_2 - the number of points in the initial cloud S_1 and S_2, respectively, then in a simple implementation the value of this search complexity is $O(N_1 \cdot N_2)$, that is quadratic when $N_1 \approx N_2$. With the help of more complex data structures - such as k-d tree [8] - the search can be done in time $O(N_1 log N_2)$. Thus, the total number of operations for search pairs of nearest points with m iterations is: $O(mN_1 log N_2)$. The large number of researches have been devoted to various improvements in the iterative closest point algorithm. However, the vast majority are complex and adjust to the specific experimental data, which reduces the stability of algorithms.

V. Formulation of the Comparison Problem

Let's consider three-dimensional surface $S_N = \{(x^i,y^i,z^i)\}^N$ which is defined by a cloud consisting of N points in a coordinate system $Oxyz$. This surface can be considered as a function $z=f(x,y)$, defined on a discrete set $\{(x^i,y^i)\}^N$. For such a surface triangulation model can be built in the form of the set of oriented triangles. Each of these triangles is determined by the value z^i in the triangulation node (x^i,y^i). Thus the approximation of the original surface is carried out through a triangulation piecewise linear model.

Thus let that we have two single-valued surfaces S_1 and S_2, which are specified by point clouds $\{(x_1{}^i,y_1{}^i,z_1{}^i)\}^N$ and $\{(x_2{}^i,y_2{}^i,z_2{}^i)\}^N$, respectively, in the E^3 space. These surfaces are unambiguously projected onto the plane Oxy. Consider the space E^2. Let s_1 and s_2 - irregular two-dimensional grids, whose nodes are the projections of the initial point clouds S_1 and S_2 onto the plane Oxy.

$$s_1 = \{(x_1^i, y_1^i)\}^{N_1} \quad s_2 = \{(x_2^i, y_2^i)\}^{N_2}.$$

Consider discrete functions f_1 and f_2, specified in the mesh nodes s_1 and s_2, so that:

$$f_1^i = f_1(x_1^i, y_1^i) \quad f_2^i = f_2(x_2^i, y_2^i).$$

Thus, the task of comparing the surfaces S_1 and S_2 is reduced to the comparison of the functions defined by discrete sets of values.

Consider the set of two-dimensional grids G, which is characterized by a certain region of space, and a variety of single-valued functions F, defined on grids of the set G. For this task we need to describe a measure of functions comparison on the F set to combine the surfaces defined as grids. Let f^a - a function that approximates f on the set of grid nodes s. In this case, we can assert that $f=f^a$ in the nodes of s.

Let's look at the discrete functions f_1 and f_2 of F set. They are approximated by E^2 continuous functions $f^a{}_1$ and $f^a{}_2$ on the sets of grid nodes g_1 and g_2, respectively. If the original meshes consist of disjoint sets of nodes, number of nodes in a grid g is $N = N_1 + N_2$. The measure of differences between the original surfaces are the average distance between the two approximating functions for all points of combined grid - the *average axial distance*.

$$R(f_1, f_2) = \sum_{x,y} |f_1^a(x,y) - f_2^a(x,y)| / N.$$

In reality, the data which obtained with the help of a three-dimensional visual scanning can be significantly affected by noise effects. This is manifested in the appearance of the wrong nodal values of the grid. In order to suppress such noise can be used measurements thresholding. As an adaptive threshold α is used maximum of αN minimum axial distance.

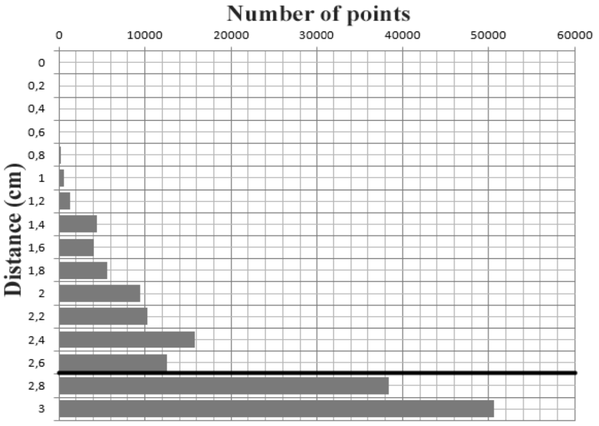

Fig. 6. Axial distances in increasing order with some threshold

Then the value of $R^\alpha(f_1,f_2)$ can be defined as follows:

$$R^\propto(f_1, f_2) = arg\ min\{R | K(R) > \propto N\},$$

where $K(R)$ - number of points at which the function values differ less than the value of R [9]. This measure is called the *truncated axial length.*

V. MEASURE OF DIFFERENCES BETWEEN SURFACES IN THE DELAUNAY TRIANGULATION

Let T_1 = $DT(g_1)$, T_2 = $DT(g_2)$ and T = $DT(g)$ are Delaunay triangulations constructed on grids g_1, g_2 and unified grid g, respectively. Consider the triangular area bounded by points A, B and C. The volume of weighted difference between the surfaces can determined as follows:

$$V(A,B,V,f_1,f_2) = \iint |f_1^a(x,y) - f_2^a(x,y)| \mu(x,y) dxdy,$$

where $\mu(x,y) \geq 0$ is the function which determines the weight of difference between surfaces at the point (x, y). We denote by $S_C(g)$ the area of the convex hull of g, which is the sum of the areas of all triangles of the set g.

$$S_C(g) = \sum\nolimits_{\Delta ABC} S_{\Delta ABC}.$$

Then we can introduce a measure for the f_1 and f_2 as:

$$R^V(f_1,f_2) = \sum\nolimits_{\Delta ABC} V(A,B,C,f_1,f_2) / S_C(g).$$

The summation is over all triangles obtained in the process of triangulation. Thus this measure R^V specifies the function of distance between the two triangulation models of surfaces. Let's define this one as the measure of differences in the unified Delaunay triangulation.

However, the objective nature of the data is implying that the grids of the compared functions are not always uniform. Therefore, the combined grid will contain significant regions consisting of the nodes of only one initial mesh. One of the main disadvantages of the measure R^V is that it takes into account with the same weight all differences. It does not account for the presence degree of one or another surface nodes. It does not account for the presence degree of nodes of one or another surface.

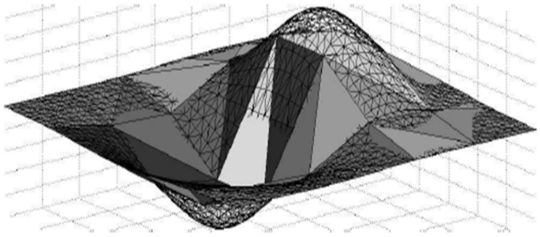

Fig. 7. Models of the same surface defined on the grids with different densities

In practice, often at the intersection of these surfaces point grids have different densities. This problem can appear in the task when we need to compare the objects surfaces obtained by different scanners, which may differ in the density of points in a cloud by several orders. In the case of a highly rarefied grid it can cause significant error of linear interpolation. This is due to the fact that within one face of the first original grid may be placed several closely lying nodes of the second one.

In the case where original grids strongly differ in density, the error of linear interpolation will be very large. Therefore, the selection of points for comparing surfaces must take into account their relative positioning. Moreover, the measure will be determined by the proximity of nodes in the unified Delaunay triangulation. It is necessary to modify the measure of R^V so that it takes into account only representative data - the areas where focused points of both grids, i.e. those triangles of T, which are not included in any of the triangulations T_1 and T_2 of source grids. The total area of these triangles denote as:

$$S_I(g) = \sum\nolimits_{\substack{\Delta ABC \in T \\ \Delta ABC \in T_1 \\ \Delta ABC \in T_2}} S_{\Delta ABC}.$$

Then, a new measure will be calculated as follows:

$$R^I(f_{1,}f_2) = \sum\nolimits_{\substack{\Delta ABC \in T \\ \Delta ABC \in T_1 \\ \Delta ABC \in T_2}} V(A,B,C,f_1,f_2) / S_I(g).$$

Index means that we consider the edges connecting nodes of different source grids - the *interface edges*.

VI. THE ALGORITHM OF COMPARING OBJECT SURFACES

The proposed method for solving the problem of matching two surfaces consists of a series of sequential steps. On the source plane clouds we have to build Delaunay triangulations, after that the value of each of the functions must be interpolated in the nodes of the other grid i.e. construct general triangulation for two grids.

Thus, at each point of the total grid the values of the two functions are known. Now it is possible to perform operations on separate triangles of common triangulation by analyzing the mutual spatial arrangement of the triangles defined by functions relative to each other. That is the functions describing the initial surface are localized in each other.

Consider the structure of comparison algorithm to calculate the value of the measures between the surfaces represented by the functions f_1, f_2 at the nodes of networks g_1, g_2, respectively. This algorithm consists of the following steps.

1) Constructing a Delaunay triangulation T_1, T_2 on the sets of nodes of the grids g_1, g_2, respectively. The computational complexity in this case is about ~ $O(N_1 log N_1) + O(N_2 log N_2)$.

2) Localization of the nodes of each of the triangulations in the triangles of another triangulation: grid points g_1 in the triangulation T_2, g_2 grid nodes in the triangulation T_1. If we use the method of constructing the minimum spanning tree by Joseph Kruskal [10], complexity is ~ $O(N_1) + O(N_2)$.

3) Interpolation of the function f_1 in the grid g_2, interpolation of the function f_2 in the grid g_1 based on

MST method. Complexity in this case is about ~ *O(N)* for a uniform arrangement of grid nodes.

4) Construction of a general triangulation T on the combined grid g. The computational complexity in this case is also equal to ~ *O(N).*

5) Calculation of measures on separate faces of general triangulation. Complexity is ~ *O(N).*

Thus, in the case of uniform distribution of nodes of the source grids and the limited ratio between their sizes, algorithm complexity can be considered as linear to number of nodes. Except the phase of triangulations construction. This phase is defining in the terms of complexity.

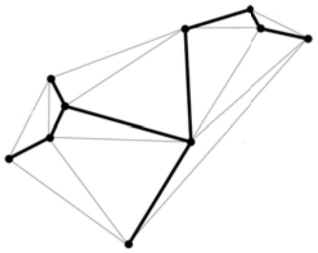

Fig. 8. Minimum spanning tree of the Delaunay triangulation

To evaluate the complexity of the proposed algorithms were held computing for large-scale cases (10 000 - 300 000). Testing values are the set of float-type points obtained by randomly choice in a rectangle with a uniform probability distribution.

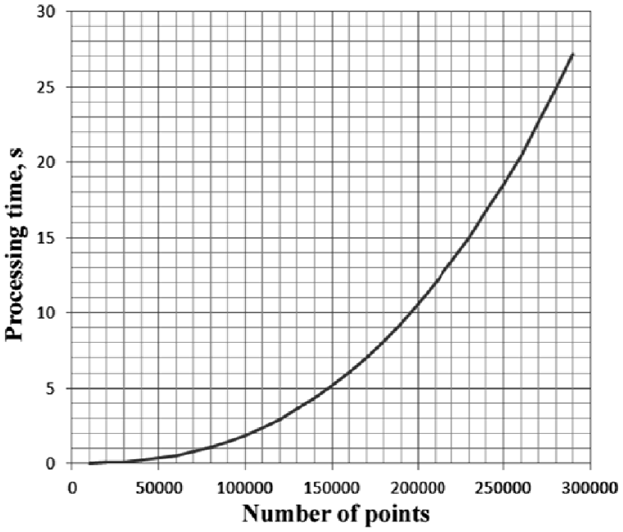

Fig. 9. Time curve for Delaunay triangulation method using "divide and conquer" implementation

TABLE I. TIME OF PROCESSING DELAUNAY TRIANGULATION USING "DIVIDE AND CONQUER" IMPLEMENTATION

Number of points	N log N / t
10000	5,664297038
50000	5,158925117
10000	4,731536691
150000	4,463141063
200000	4,275793049
250000	4,13083828
300000	4,027962598

Thus, the complexity of the algorithm to construct of the Delaunay triangulation is ($N$log$N$).

TABLE II. TIME OF CONSTRUCTING THE MINIMUM SPANNING TREES ACCORDING TO THE DELAUNAY TRIANGULATION DATA

Number of points	Processing time, s	N log N / t
10000	0,302234	33087
50000	1,4537	34395
100000	2,761668	36210
150000	4,18235	35865
200000	6,054002	33036
250000	6,941552	36015
300000	8,66476	34623

The complexity of the algorithm for constructing the minimum spanning tree of the Delaunay triangulation has (N) complexity. Next steps have the same dependence on the number of points. The complexity of grid nodes localization using the minimum spanning tree and the interpolation of function values at the nodes of grids have the same $O(N)$ complexity.

VII. THE TASK OF COMBINING SURFACES

In practice, usually the task of combining surfaces requires multiple operations of finding measure of the difference between the surfaces. The influence of the error in the calculation of the measures will increase with each new iteration.

The task of the spatial alignment of source surfaces means the bringing multiple images of an object into a single global coordinate system. This is accomplished by transforming objects through translations and rotations around the coordinate axes. If we consider the small values of the transformation parameters, it allows us to calculate the value of the measure between the matching surfaces in optimal time *O(mN)*, where *m* - number of iterations.

Let M denote the movement in three-dimensional space, which is carried out by serial rotations through angles $\alpha_{M,}$ β_M γ_M around the axes O_X, $O_{Y,}$ O_Z respectively, and through parallel transition with the vector $\Delta_M = \{ \Delta_X, \Delta_Y, \Delta_Z\}$. We define the condition that such a motion of the surface S does not remove it from the class of surfaces which uniquely projected onto the plane O_{XY}. Form of the function f which describes the original mesh after motion is denoted by f^M.

Thus, we can represent the optimization problem as the minimization of the functional $R(f_1,f_2^M)$. It will be implemented in the space E^6 by 6 motion parameters $\alpha_{M,}$ β_M $\gamma_{M,}$ Δ_X, Δ_Y, Δ_Z.

To solve this problem it is proposed to use the simplex method [11]. It allows us to define a local minimum of the function, without imposing restrictions on the smoothness of the functional to be minimized. This method depends on the initial approximation. Its idea is the serial deformation of the simplex and moving it around the extremum of a function. This allows us effectively compare the surface, using as a measure of their proximity the minimum distance between the approximating functions, which can be obtained by the spatial comparison.

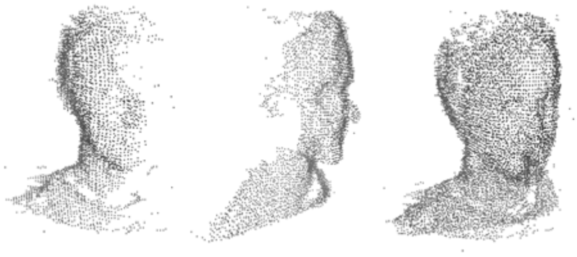

Fig. 10. Combining source point clouds into a complete model of the object using the set of proposed algorithms

VIII. CONCLUSION

This paper proposes a method of comparing the surfaces consisting of several steps. These are constructing a Delaunay triangulation for both grids and approximation each surface by the piecewise linear function of two variables; calculation the values of each of these functions at the nodal points of another grid based on the location of nodes in the first one; calculation measures of the differences by comparing the values of the two functions in the nodes of both networks.

The distance between the functions is calculated only in those parts of these functions domain of definition, where the nodes of their grids placed close to each other. The method of searching for such points is to calculate the united interface edges of the Delaunay triangulation of both grids. This is especially important when the initial grids vary greatly in the node density.

In the case where each of the source grids is uniform, the computational complexity of the localization algorithm is linear to the average number of nodes in the source grids. Assumption of a uniform distribution of the grid nodes is appropriate for most practical applications.

The combination process is reduced to the iterative comparison of approximating functions using translations and rotations of grids nodes. This allows to effectively compare the surfaces, using as a measure of proximity the minimum distance between the approximating functions obtained by consecutive fitting.

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