

# A Study of Teager-Kaiser Energy Operator Pertinence for R Peak Detection in ECG Recordings

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**Abstract**—Continuous health monitoring holds promise for early detection of health status impairment and plays significant role in forward-looking applications and services related to fitness, well-being, chronic diseases treatment and independent living for elderly.

A number of arrhythmia detection algorithms are being developed within a CardiaCare project that is aimed at continuous monitoring of heart function in real-time and analyzing electrocardiograms on a smartphone. Arrhythmia detection algorithms are heavily rely on features extracted from electrocardiogram recordings. In particular, robust detection of so-called R peak is important. There is a variety of approaches to extract the R peaks. Nevertheless, widespread algorithms often demand of heavy computations. If these algorithms were run in mobile devices, the battery would quickly drained. Since this situation is not acceptable in continuous health monitoring, appropriate R peak detector should be carefully selected.

In proposed approach to addressing the requirement of low power consumption, fast R peak detection algorithm based on Teager-Kaiser energy operator (TKEO) is utilized. The algorithm have been thoroughly verified using real electrocardiograms with abnormalities annotated by experts that are fetched from open databases. Analysis of TKEO-based R peak extraction algorithms showed high detection performance indicators (sensitivity, specificity and accuracy) comparable to widespread algorithms.

## I. INTRODUCTION

According to World Health Organization so-called cardiovascular diseases (CVDs) are the leading cause of death globally: more people die annually from CVDs than from any other cause. It is typical for Commonwealth of Independent States (CIS) as well.

TABLE I. CVD CONTRIBUTION TO MORTALITY IN CIS

Georgia	67
Ukraine	64
Azerbaijan	60
Russia	57
Moldova	56
Belorussia	53
Kazakhstan	50
Armenia	50
Kyrgyzstan	49
Tajikistan	39

Majority cardiovascular diseases can be prevented by addressing behavioural risk factors such as tobacco use, unhealthy diet and obesity, physical inactivity and harmful use of alcohol. People with cardiovascular disease or who are at

high cardiovascular risk need early detection and management using counselling and medicines, as appropriate.

Within the CardiaCare project the efforts are concentrated on the development of the continuous monitoring system aimed timely detection of rhythm abnormalities. Despite the fact that the arrhythmias are harmless in general, they can pose serious threat of complications against chronic diseases such as hypertension or diabetes. Therefore, continuous heart rhythm monitoring provides the possibility to detect the deterioration of heart function and even to save the life.

The system operates as it is shown in Fig. 1. Raw cardiogram data are received by smartphone app and passed to the analysis core.

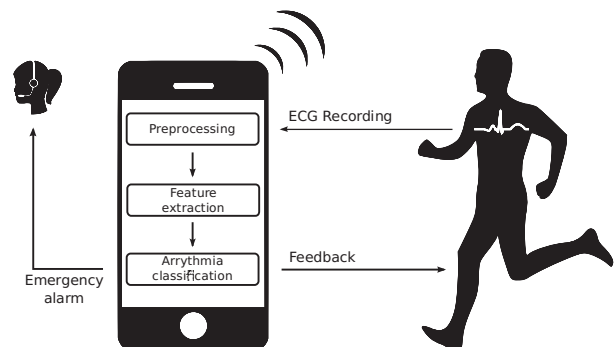


Fig. 1. Continuous monitoring system operation

Detection of cardiac abnormalities relies on electrical activity of the heart that can be registered and visualised with a plot that is known as cardiogram. Each feature of the cardiogram is related to activity of specific part of the heart from atrial contraction to ventricular relaxation. Normal sinus rhythm of the cardiac cycle consists of five typical waves: P, Q, R, S and T.

Most important in analysis is the highest point of the R wave, so-called R peak. Successful identification of R peaks allows to split the signal into segments on which other features can be estimated. Examples of different rhythm anomalies that are aimed to be caught in CardiaCare project are shown below.

In normal sinus rhythm you see the lengths of RR interval in 60-100 beats per minute (bpm). This rhythm is shown in Fig. 2 Sinus tachycardia, characterized with frequent beats (more than 100 bpm), is illustrated by Fig. 3. If sinus bradycardia occurs the heart rate is below 60 bpm. Bradycardia recording is shown in Fig. 4.



Fig. 2. Normal sinus rhythm



Fig. 3. Sinus tachycardia



Fig. 4. Sinus bradycardia

In Fig. 5, 6, 7 there are arrhythmias that require more detailed analysis but R peaks play a vital role in localization of cardiac cycles.



Fig. 5. Sinoatrial block



Fig. 6. Atrial flutter



Fig. 7. Wolff-Parkinson-White syndrome

It is obvious that in this scenario precise but complicated algorithms cannot be used since heavy computations can drain the battery of mobile device in minutes.

One of the methods was proposed by Yamamoto and Yoshida in 2013 [1]. This approach based on Teager-Kaiser energy operator widely used in speech processing [2]. This operator defines energy of a signal produced by a simple harmonic oscillator. Due to less computational requirements this method is prominent for devices with restricted energy

capabilities. In this work the one-pass algorithm based on Teager-Kaiser energy operator was constructed, tested against the real ECG recordings and implemented as a standalone library.

## II. R PEAK DETECTION BASED ON TEAGER-KAISER ENERGY OPERATOR

Let us briefly discuss the background of Teager-Kaiser Energy Operator (TKEO). Detailed explanation can be found in [2] and [3]. Consider an object with mass  $m$  suspended by a spring with a constant factor characteristic of the spring, or stiffness,  $k$ . If the mass is displaced from its equilibrium position, a restoring elastic force is exerted by the spring. This force which obeys Hooke's law and is given by

$$F = -kx \quad (1)$$

where  $x$  is the displacement from the equilibrium position. The following second order differential equation can be deduced by means of Newton's second law to describe the simple harmonic motion of considered object and is given as

$$F = \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad (2)$$

The solution to equation 2 is given by

$$x(t) = A \cos(\omega t + \phi) \quad (3)$$

where  $x(t)$  is the position of the object at time  $t$ ,  $A$  is the amplitude,  $\omega$  is the frequency, and  $\phi$  is the initial phase. The total energy of the object is given as the sum of kinetic energy of the object and the potential energy of the spring, given by

$$E = \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 \quad (4)$$

By substituting  $x(t) = A \cos(\omega t + \phi)$ , we get the following expression for the energy:

$$E = \frac{1}{2}mA^2\omega^2 \quad (5)$$

Now we consider the continuous-time form of Teager energy operator defined to be

$$\Psi_c[x(t)] = (\dot{x}(t))^2 - x(t)\ddot{x}(t) \quad (6)$$

Substituting  $x(t) = A \cos(\omega t + \phi)$ , we obtain

$$\Psi_c[x(t)] = A^2\omega^2 \quad (7)$$

Thus, the operator defined by 6 is the amplitude and frequency product squared. But from 5 the total energy is proportional to the amplitude and frequency product squared.

In order to get the discrete-time form of the operator, consider the digital signal  $x_n$  given by

$$x_n = A \cos(\Omega n + \phi) \quad (8)$$

where  $\Omega$  is the digital frequency  $\Omega = 2\pi f/F_s$ . Here  $f$  is analog frequency and  $F_s$  is the sampling frequency. By means of trigonometric identities we obtain

$$x_{n-1}x_{n+1} = A^2 \cos^2(\Omega + \phi) - A^2 \sin^2(\Omega) \quad (9)$$

Substituting  $A^2 \cos^2(\Omega + \phi)$  with  $x_n^2$  we get

$$A^2 \sin^2(\Omega) = x_n^2 - x_{n-1}x_{n+1} \quad (10)$$

Restricting  $\Omega$  to be positive and less than  $\pi/4$  and approximating the  $\sin(\Omega)$  with  $\Omega$  we get the unique solution with approximation error less than 11%. Hence, the discrete-time form of the Teager energy operator is defined by

$$\Psi_d[x_n] = x_n^2 - x_{n-1}x_{n+1} \quad (11)$$

Since R peaks have high frequency component and usually high amplitude, this approach is useful to enhance these peaks and suppress the other features.

The method proposed in [1] involves three base steps.

- 1) Estimation of the instantaneous energy of a signal

$$\Psi_d[x_n] = x_n^2 - x_{n-1}x_{n+1}$$

- 2) Emphasis of the R peaks

$$y_n = \Psi_d[x_n]^3$$

- 3) Choosing parameters  $N$ ,  $\alpha$  and  $\beta$  and computing the adaptive threshold

$$z_n = \alpha \frac{1}{N+1} \sum_{k=-N}^N y_k + \beta \sigma_y$$

Parameters  $\alpha$  and  $\beta$  depend on a signal and  $N$  should be chosen from one to doubled length of RR interval.

The algorithm can be implemented in one-pass in obvious way if we modify the sum in threshold for every sample.

### III. EXPERIMENTS

Authors of the approach did not provide the results of algorithm testing on a set of real recordings. The system aimed to verify the implemented detectors versus real electrocardiogram data with different rhythm abnormalities was developed. The architecture is shown in Fig. 8. We use MIT-BIH arrhythmia database [5], which is the part of well-known Physionet bank of open biosignals [4], as the main source of electrocardiogram episodes to be tested.

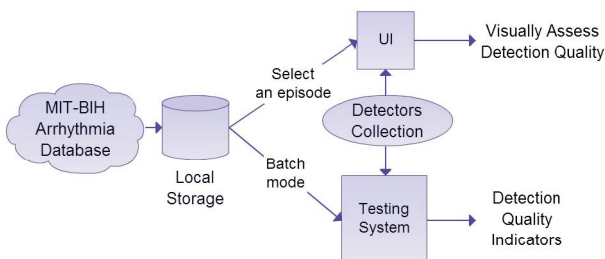


Fig. 8. An assessment system for ECG feature detectors

The system has an user interface to assess the work of detector visually. User can choose the signal from the list on the left pane and scroll it. A main view of an UI is shown in Fig. 9



Fig. 9. An user interface of the detector assessment system

### IV. CONCLUSION

During this work we have implemented the novel algorithm designed to detect R peaks on an electrocardiogram and we have developed the system to verify the detectors on a set of signals from MIT-BIH database. Average sensitivity of the TKEO-based algorithm approximately equals to 95 percents that is quite good. The worst case is the 91 percents achieved on the notoriously 207 signal that is reported to be too hard for well-known precise methods as well.

### ACKNOWLEDGMENT

This research is financially supported by the Ministry of Education and Science of the Russian Federation within project # 14.574.21.0060 (RFMEFI57414X0060) of Federal Target Program "Research and development on priority directions of scientific-technological complex of Russia for 2014–2020".

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