Soft Quantization of the Production's Knowledgebases for Multi-Agent Systems

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***Abstract*—In multi-agent systems, agents communicate using message on the BlackBoards, "White" and "Yellow" pages. It is assuming that the messages contain not only data, but also knowledge. The approaches to the knowledge quantization (soft knowledge coding) are considering in this article. We have proved the statements that allow you to perform isomorphic transformations for the classical additive fuzzy models, working in the field of real numbers, to their analogs able to function in the finite Galois fields also. The homeomorphism (topological isomorphism) considered fuzzy and neuro-fuzzy models. Given author's soft coding algorithm for knowledgebase. The existence of such isomorphic and homeomorphic transformations lead to the possibility of an adequate quantization the agent's knowledgebase, presented in the form of an additive fuzzy and neuro-fuzzy systems in both metric and topological spaces.**

I. INTRODUCTION

Intensive development of wireless and mobile networks, distributed information-telecommunication systems and, in particular, open multi-agent systems (OMAS) leads to the formation of new conceptual entities, such as agents that need controlled in real time [1]. The agents communicate using message on the Blackboards, "White" and "Yellow" pages. The structure of the messages at the same time, mainly considered as a set of quantized data, not knowledge. The notion of quantum knowledge today, are poorly formalizing. For the additive fuzzy models by quantum knowledge can be a set of the basis fuzzy implications, the capacity of which is determined at the stage of knowledgebase quantization.

In this article, firstly, we prove the possibility of an adequate quantization the production's knowledgebase in principle. Second, we prove that an additive fuzzy model can be arbitrarily small given precision of the approximated polynomials over finite Galois fields. Thirdly, we show the existence of a homeomorphism between the given polynomials, approximately additive fuzzy model. These results are necessary for *solving the problem* of the *knowledge bases optimization* to the communication processes of agents and information security issues in regard to open multi-agent systems.

Optimized parameters of the knowledge quantization represented in the additive fuzzy models form can be, for example, the number of significant knowledge base's rules, the number of linguistic variables terms, the number of adaptation parameters for each linguistic terms, the number of hierarchy levels for an additive fuzzy model, the accuracy of the generalization. The selected quantization parameters of knowledge, clearly define the knowledgebase volume transported by the mobile agents, speed of adaptation and modification, the possibility to applicate concrete protection mechanisms for the structure of the knowledge base and its content.

It should also be noted that the level of knowledge protection need be above the level of data protection based on which this knowledge is formed. That is, there is a need to build hierarchical protection systems both knowledge and data. Constructive approaches to developing of hierarchical information security systems exist [2, 3], and, as a rule, are realized by algebraic mechanisms in finite Galois fields. Consequently, the structure of the transported knowledgebase and its contents have to be prepared for the adequate application of hierarchical protection mechanisms. For example, the knowledgebase after its quantization can be representing as isomorphic to conceptual entities, such as polynomials over finite Galois fields. That is, the manipulation rules can be replacing by functions over polynomials.

Given the evidence say about the existence of constructive algorithms consistent exchange of information between agents, as well as capabilities adequate replenishment of databases and knowledge of agents in their migration.

II. THE APPROXIMATION OF ADDITIVE FUZZY MODELS BY THE REAL-VALUED FUNCTIONS

In works [4–7] have proved theorems about the universal approximation of fuzzy production models of polynomials over fields of the real numbers. Proofs of these theorems are basing on the Weierstrass and Stone-Weierstrass theorems in the sense that the basis of this universal approximation is the ability of the additive fuzzy models can approximate any polynomial function, which, in turn, can approximate any continuous function.

Here we state without proof the theorem of Weierstrass and Stone-Weierstrass [8].

Theorem 2.1 (Weierstrass approximation theorem):

Suppose $f(x)$ is a continuous real-valued function defined on the real interval $[a,b]$. For every $\varepsilon > 0$, there exists a polynomial $p(x)$ such that for all $x \in [a,b]$ we have

$$\left| f(x) - p(x) \right| < \varepsilon.$$

Theorem 2.2. (Stone-Weierstrass theorem):

Suppose X is a *locally compact* Hausdorff space and $C(X)$ is ring with given the topology of uniform convergence together with the supremum norm

$$\| f(x) \| = \max_{x \in X} | f(x) |, \quad f(x) \in C(X),$$

Suppose $C_0 \subseteq C(x)$ is a *subring*, it separates points and vanishes nowhere on *X*, i.e., for each different points $x_1 \in X$ и $x_2 \in X$ exist function $f(x) \in C_0$, which $f(x_1) \neq f(x_2)$.

Then $[C_0] = C(x)$, i.e., any continuous real-valued functions on X is limit for a uniformly convergent sequence of functions from C_0.

As well as we give the statement and proof of the theorem of Kosko [5].

Theorem 2.3. (Kosko): An additive fuzzy system F uniformly approximates function $f(X) \to Y$ if X is compact and $f(X)$ is continuous.

Proof:
Let any small constant $\varepsilon > 0$. We must show that $|F(x) - f(x)| < \varepsilon$ for all $x \in X$. X is a compact subset of R^n. $F(x)$ is the centroid of output linguistic variable for the additive fuzzy system F.

Continuity of $f(X)$ on compact X given uniform continuity. So there is a fixed distance d such that, for all x and z in X, $|f(x) - f(z)| < \frac{\varepsilon}{4}$ if $|x - z| < d$. We can construct a set of open cubes $M_1, \ldots, M_m$ that cover X and that have ordered overlap in their n coordinates so that each cube corner lies at the midpoint c_j of its neighbors M_j.

Let symmetric output fuzzy sets B_j centered on $f(c_j)$. So the centroid of B_j is $f(c_j)$. Let $u \in X$. Then by construction u lies in at most 2^n overlapping open cubes M_j. Let any w in the same set of cubes. If $u \in M_j$ and $w \in M_k$, then for all $v \in M_j \cap M_k$: $|u - v| < d$ and $|u - w| < d$. Uniform continuity implies that

$$|f(u) - f(w)| \leq |f(u) - f(v)| + |f(v) - f(w)| < \frac{\varepsilon}{2}.$$

So for cube centers c_j and c_k, $|f(c_j) - f(c_k)| < \frac{\varepsilon}{2}$.

Let $x \in X$. Then x too lies in at most 2^n open cubes with centers c_j and $|f(c_j) - f(x)| < \frac{\varepsilon}{2}$. Along the k-th coordinate of the range space R^p the k-th component of the additive system centroid $F(x)$ lies as in "ON" or "BETWEEN" the k-th components of the centroids of the B_j sets. So, since $|f(c_j) - f(c_k)| < \frac{\varepsilon}{2}$ for all $f(c_j)$, $|F(x) - f(c_j)| < \frac{\varepsilon}{2}$. Then

$$|F(x) - f(x)| \leq |F(x) - f(c_j)| + |f(c_j) - f(x)| <$$
$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon \qquad \blacksquare$$

The proof of Kosko theorem shows that we can replace the fuzzy set A_i and B_j with finite discretization or fit vectors $(a_1^i, \ldots, a_n^i)$ and $(b_1^j, \ldots, b_p^j)$. The discrete version of B_j must have a centroid at or close to the centroid of B_j. So we can always work with large-dimensional unit hypercubes and view fuzzy rules or patches as matrix mapping (or fuzzy associative memories) between hypercubes or as point in even larges hypercubes.

The result of the constructive proof of these theorems is to estimate the required number of model rules for a given accuracy of approximation, which is determined by the minimum distance between the centroids of two adjacent fuzzy sets representing the conclusions of rules, denoted as y_i and y_{i+1}:

$$|y_i - y_{i+1}| < \frac{\varepsilon}{2^g - 1}, \quad (1)$$

where ε – the accuracy of the approximation; g – the maximum number of overlapping fuzzy sets of the input variables on a compact set X (for single input variable $g = 2$).

For one input variable the required number of rules is determined by the expression

$$n \geq \frac{|X|}{\varepsilon}.$$

It is obvious that the commitment ε to zero, the number of rules is unlimited, but for a given value ε, the number of rules can be estimated using (1).

However, these results do not give answers to the questions: what specific fuzzy model it is necessary to choose and how much should be rules for approximating the given function; what are the mechanisms of regulation of approximation accuracy and still have not solved the problem for compact packing of an additive fuzzy model in a limited address space.

III. APPROXIMATION OF ADDITIVE FUZZY MODELS BY POLYNOMIALS OVER FINITE FIELDS

In this section we prove the main statements on which to base and implement mechanisms for soft hierarchical consolidation of agent's knowledge and storage agents in a telecommunications OMAS. This should justify the possibility of creating a unique fuzzy and neuro-fuzzy structures functioning adequately in finite fields.

We can show that:

Theorem 3.1: Additive fuzzy system F with an arbitrarily small accuracy $\varepsilon > 0$ approximates polynomial with real-valued coefficients $p(x) \to Y$, if the set X is compact.

Proof:

Because of the faithfulness Kosko theorem 2.3 we have additive fuzzy system uniform approximates continuous function $f(x) \to Y$ on a compact set X if this function is continuous on this compact set, i.e.

$$\left|F(x) - f(x)\right| < \varepsilon_1,$$

where $\varepsilon_1 > 0$ is an arbitrarily small quantity, $x \in X$.

On the Weierstrass theorem 2.1, any continuous function $f(x)$ defined on a compact X can be approximating with accuracy ε_2 by a polynomial $p(x)$ with real-valued coefficients, i.e. for $\forall x \in X$ the condition $\left|f(x) - p(x)\right| < \varepsilon_2$ is true.

Then

$$\left|F(x) - f(x)\right| = \left|F(x) - (p(x) \pm \varepsilon_2)\right| =$$
$$= \left|F(x) - p(x) \mp \varepsilon_2\right| < \varepsilon_1,$$
$$\left|F(x) - p(x)\right| < \left|\varepsilon_1 \pm \varepsilon_2\right|.$$

Puts $\varepsilon = \left|\varepsilon_1 - \varepsilon_2\right|$ we obtain the required statement

$$\left|F(x) - p(x)\right| < \varepsilon. \ \blacksquare$$

To prove other properties approximating additive fuzzy and adaptive neuro-fuzzy systems will require a number of known statements (3.1–3.7). The need in these statements arises the requirement of a transition from the real-valued fields to calculations over finite fields. Note that the usual discretization of real-valued numbers to the set of integers the problem of scale knowledgebase to not solving because the set of integers is not a field (it does not have a multiplicative inverse element). Computation in finite fields are commonly used in coding theory and in cryptography, without which, in turn, it is impossible to justify the parameters of reliable information transmission in communication channels OMAS.

A finite field is a field with the finite number of elements. If the number of elements is a degree q^m – some prime number q, which is characteristic of this field, such a field is called Galois field and denoted $GF(q^m)$.

It is known [9] that for $\forall q$ and $\forall m \in N$, where N is the set of natural numbers, there exists a unique (up to isomorphism) field of q^m the elements. The number of elements is called *the order* of this field and denote $card\left(GF(q^m)\right)$.

Also known approval 3.1 – 3.7 [9]:

Statement 3.1:
The field $GF(q^n)$ contains a subfield $GF(q^m)$ in fact and only if $n \mid m$ (m divides n). In particular, in any $GF(q^m)$ contains $GF(q)$, called *simple* field.

Statement 3.2:
Field $GF(q^m)$ isomorphic to the field $Z \mid (q^m)$ – residue class ring of integers modulo q.

Any finite extension fields are algebraic.

Statement 3.3:
Field of real-valued numbers R is the algebraic closure Ω Galois field, as just a non-constant polynomial with coefficients from $GF(q)$ has at least one root in the field of real-valued numbers R.

Statement 3.4:
In any fixed algebraic closure Ω of the field, $GF(q)$ there is only one subfield $GF(q^m)$ for each m. The correspondence $m \leftrightarrow GF(q^m)$ is an isomorphism between the natural numbers mesh (which is a subset of real numbers) relative to the operation of divisibility and a mesh of finite algebraic field's extensions $GF(q)$, lying on Ω the inclusion.

This is the same mesh of finite algebraic extension set of any Galois field, which lies in its fixed algebraic closure.

Statement 3.5:
An algebraic extension $GF(q^m)/GF(q)$ is simple, i.e. $\exists \alpha \in GF(q^m)$ is a primitive element such that $GF(q^m) = GF(q)(\alpha)$. Such α a primitive element is a root of any irreducible polynomial of degree m out of the ring $GF(q)[X]$.

Statement 3.6:
Set of elements over the field $GF(q^m)$ exactly matches with roots set of a polynomial $X^{q^m} - X$ in a Ω, i.e., $GF(q^m)$ characterized as the subfield of elements Ω invariant under the automorphism $\tau : x \to x^{q^m}$, called the Frobenius automorphism.

Statement 3.7:
If $GF(q^n) \supset GF(q^m)$ the extension $GF(q^n)/GF(q^m)$ is normal and its Galois group is cyclic of order $n \mid m$. As generating groups $Gal\left(GF(q^n)/GF(q^m)\right)$ can be the automorphism τ.

Theorems 2.1-2.3, 3.1 and Statements 3.1-3.7 allow us to prove the following theorem.

Theorem 3.2 [10]:
Let the additive fuzzy system F with an arbitrarily small accuracy $\varepsilon_1 > 0$ approximates polynomial $p(x)$ with real-valued coefficients on the compact X. Then $\exists q$ and

$\exists$ polynomial $g(\widetilde{X})$ over $GF(q^m)$ ($\widetilde{x} \in \widetilde{X} = \{0,1,...,q-1\}$, $m \in N, m > 0$) is isomorphic to $p(x)$, which F is also approximates with arbitrarily small accuracy

$$\varepsilon_2 > 0, \varepsilon_2 = \varepsilon_1 \pm o(\varepsilon_1).$$

Proof:

Because of the faithfulness of the assertions (3.3) - (3.4) is an isomorphism between compact X real-valued fields and $GF(q^m)$.

So, on statement (3.6) we have the existence of lexicographic order of elements $GF(q^m)$ invariant under the Frobenius automorphism from which it follows exist $g(\widetilde{X})$ over $GF(q^m)$, and the isomorphism $p(X) \leftrightarrow g(\widetilde{X})$.

■

Theorem 3.2. show the existence of a polynomial $g(\widetilde{X})$ over $GF(q^m)$, which with a given accuracy approximates additive fuzzy system F, but not type of this polynomial. To specify this polynomial, we introduce into consideration a number of designations and constraints.

Let $x = (x_1, x_2, ..., x_m)$ a vector of fuzzy input variables $x \in X$ and y fuzzy output variable of fuzzy additive system F, $y \in Y$.

Let q some a priori specified prime number.

Constraint 3.1:

Let $A^{(i)} = \{A_1^{(i)}, A_2^{(i)}, ..., A_q^{(i)}\}$ a set of linguistic terms of a fuzzy input variable x_i defined on X the corresponding fuzzy set with membership functions $\mu_{A_l}(x_i) \in [0,1]$, $l = \overline{1,q}$. A – the set of linguistic terms of the vector $x = (x_1, x_2, ..., x_m)$:

$$A = A^{(1)} \times A^{(2)} \times ... \times A^{(m)}.$$

Constraint 3.2:

Let $B = \{B_1, B_2, ..., B_q\}$ a set of linguistic terms defined on Y the corresponding fuzzy set with membership functions $\mu_{B_z}(y) \in [0,1]$, $z = \overline{1,q}$.

Constraint 3.3:

Let us assume that each membership functions $\mu_{A_l}(x_i) \in [0,1], l = \overline{1,q}$ and $\mu_{B_z}(y) \in [0,1]$, $z = \overline{1,q}$ are symmetric and have the centroids $x_i^{(l)}$ with the vertices at the points with abscissa $\frac{l-1}{q}$ and bases:

$\left[0, \frac{1}{2q}\right]$ if $l = 1$, $\left[1 - \frac{1}{2q}, 1\right]$ if $l = q$ and $\left[\frac{l-1}{q} - \frac{1}{2q}, \frac{l-1}{q} + \frac{1}{2q}\right]$ when $l \neq 1$ and $l \neq q$.

That is, constraints (3.1) and (3.3) indicate that the field's characteristic q determines the location and number of terms.

Then we can prove the following theorem:

Theorem 3.3:

An additive fuzzy system F with constraints (3.1) – (3.3) can be approximated with arbitrarily small accuracy $\varepsilon > 0$ given a polynomial with coefficients above $GF(q)$ (a generalization of the polynomial Zhegalkin on many-valued logic).

Proof:

The set of all possible patterns $(x, y) = (x_1, x_2, .., x_m, y)$ completely determine the state and output of an additive fuzzy system F, and, because of the constraint (3.1), the number of different vectors $x = (x_1, x_2, ..., x_m)$ is limited and equals $\|A\| = q^m$.

The constraints (3.3) the position of the centroid and the scope of the membership functions bases provides a *continuous* coverage on compact X of *ordered* linguistic terms that allows to apply the theorem of Kosko, and therefore, theorems 3.1 and 3.2.

Then the vectors $x = (x_1, x_2, ..., x_m)$ of additive fuzzy system F can be isomorphic mapped to a finite field $GF(q^m)$, and itself F, using the constraints (3.2), fully specify its table of values (truth table for $q = 2$) for a multivalued logic.

The number of rows in the value table equal to $card\left(GF(q^m)\right)$, and clearly associated with the capacity of the rule base for a given approximation accuracy, which is determined by the minimum distance between the centroids of two adjacent fuzzy sets representing the conclusions of rules.

It is known that the truth table for the Boolean functions can be specified by the Zhegalkin polynomial, and multiple-valued logic, a generalization of the polynomial Zhegalkin – polynomial with the given coefficients in $GF(q)$.

So,

$$|F(x) - p(x)| = |F(\widetilde{x}) - q(\widetilde{x})| < \varepsilon,$$

where $F(\widetilde{x})$ – isomorphic mapping F to $GF(q^m)$ for a priori given q, which determines the number of terms in its fuzzy variables,

$$g(\widetilde{x}) = a_0 \oplus \sum a_i \widetilde{x}_i \oplus \sum a_{ij} \widetilde{x}_i \widetilde{x}_j \oplus ... \oplus a_{12...m} x_1 x_2 ... x_m \quad (2)$$

where $g(\widetilde{x})$ – given a polynomial over $GF(q)$, $a_i, a_{ij}, ..., a_{12...m} \in GF(q)$, $\widetilde{x} \in GF(q), i = \overline{1, m_i}$, $\oplus$ - sum on modulo q. ■

Algorithms for construct of the given polynomials is similar to the algorithms of constructing of Zhegalkin polynomials, among the latter the most efficient are given in [11], [12], which in itself brings the solution to the problem of automatic generation of a full agent's knowledgebase.

The number of summands of a given polynomial (2) is determined by the number of different monomials (elementary conjuncts at $q = 2$), which in turn are the basic rules. We give lower and upper bounds on the complexity of the functions in the given class of polynomials.

IV. The complexity of the functions in the class gyven polynomials over finite fields

We introduce some notation for the estimation of functions complexity in the class of polynomials [11]. The set of all functions of multivalued logic with the base q will be denoted by P_q. Let $l(G)$ denotes the number of polynomial terms G and is called the *polynomial's length*. Even $f(x_1, x_2, ..., x_m) \in P_q$ has the corresponding given polynomial G_f above P_q.

Let us introduce the functional

$$l_G(f) = \min(l(G_f)),$$

specifies the polynomial's length of the polynomial G_f. The value $l_G(f)$ is called *the complexity function f in the class given polynomials*.

Also, we introduce into consideration the function

$$L_G(m) = \max_{f \in P_q(m)} (l_G(f)),$$

which characterizes the complexity, "most complex function" from the m-variables in the class of the given polynomials. Function $L_G(m)$ is *the Shannon complexity function in the class of the given polynomials*.

Then we can prove the following theorem for the lower and upper bounds of complexity functions.

Theorem 3.4:
A lower bound for the Shannon complexity function of the class given polynomials over $GF(q)$ is

$$L_G(m) \geq \frac{q^m}{m \log_q (q+1)}.$$

Proof:

Let $L_G(m) = L$. There are a total of $(q+1)^m$ - monomials (elementary conjuncts at $q = 2$) from m variables, so the number of polynomials of length no more L of m the variables does not exceed $\left((q+1)^m\right)^L$. The number of functions P_q from m variables is equal q^{q^m}. It is obvious that the number of polynomials may not be less than the number of functions, otherwise there is a function for which there is not an equivalent polynomial length $\leq L$, which contradicts the definition $L_G(m)$. Therefore

$$(q+1)^{mL} \geq q^{q^m}.$$

Expressing L on this inequality we get

$$L_G(m) \geq \frac{q^m}{m \log_q (q+1)} \quad \blacksquare \tag{3}$$

The upper bound for $L_G(m)$ can be obtained generalizing to the case of multivalued logics an upper bound for Boolean functions. Namely[11] $L_{P_2}(m) \leq 2 \cdot \frac{2^m}{m}(1 + \ln(m))$.

Then for a given polynomial P_q to get

$$L_G(m) \leq \frac{q^{m+1}}{m}(1 + \ln(m)). \tag{4}$$

As can be seen from expressions (3) and (4) in the approximation of additive fuzzy systems by given the polynomials above $GF(q^m)$, still, as in the classical fuzzy production models, is the exponential growth in the number of rules tends to zero of approximation error, which leads to a significant increase of computational complexity and practical inapplicability.

However, from a practical viewpoint it is sufficient to be acceptable for adequate decision-making accuracy of the approximation. In this case, the task boils down to finding a compromise between the specified precision and number of rules of the model. Approaches to finding such a compromise may be the following:

1) to modify existing well-known recursive algorithms, based on the creation of the equivalent generators for linear recurring sequences;
2) to build hierarchical structures of adaptive fuzzy systems and their isomorphic images over finite fields, using the possibility of submitting a given polynomial as a product of its irreducible factors.

Note that the constraints 3.2 can be weakened without loss of generality to soft constraints 3.4 for overlapping bases of terms, providing a touch of terms only in one point, but it's enough to keep the compact X, and therefore to leave space topological:

Constraint 3.4:

Let us assume that each membership function $\mu_{A_l}(x_i) \in [0,1], l = \overline{1,q}$ I and $\mu_{B_z}(y) \in [0,1], z = \overline{1,q}$ is symmetric and has the centroid $x_i^{(l)}$ with the vertices at the points with abscissa $\frac{l-1}{q}$ and bases:

$\left[0, \frac{1}{q}\right]$ if $l = 1$, $\left[1 - \frac{1}{q}, 1\right]$ if $l = q$ and

$\left[\frac{l-1}{q} - \frac{1}{q}, \frac{l-1}{q} + \frac{1}{q}\right]$ when $l \neq 1$ and $l \neq q$.

Constraints 3.2 and 3.4 indicate the range of possible interval settings of membership functions, if the additive fuzzy model is the adaptive (neuro-fuzzy), while maintaining its topology.

The presence of the target function, the agent allows to achieve *an adjustable balance* between number of rules and approximation accuracy is possible, forming a *hierarchical* fuzzy production model (additive *m*-input hierarchical fuzzy model), which includes $(m-1)$ – input fuzzy production models [12]. Hierarchical scheme in this case, obviously, should take into account the ranking of the sub fuzzy models (rating mechanisms in this article are not considered).

Prove that the additive m-input *hierarchical* fuzzy models are universal perceptron on polynomials. To do this, to show that the polynomial, approximating the additive fuzzy model, can be represented by some hierarchical structure. The hierarchical structure, in turn, can always be obtained from the multiplicative form of the polynomial.

Therefore, we will show that on the base of Kosko and Weierstrass theorems can be formulated in the following statement.

Theorem 3.5 :

There is a polynomial $p(X) \to Y$ with real-valued coefficients and multiplicative structure of terms, which approximates additive fuzzy system F with an arbitrarily small accuracy $\varepsilon > 0$, if the set X is compact.

Proof:

The ability to represent arbitrary polynomial in the multiplicative structure of its members derives from the existence of the interpolation formulas of Lagrange, and also directly from the statements of theorems 3.4, Weierstrass and Kosco. The Lagrange formula for the this case has the following form:

$$f(x_1, \dots, x_m) = \sum_{(a_1 a_2 \dots a_m)} f(a_1 a_2 \dots a_m)(x_1 + a_1 + 1)\dots(x_m + a_m + 1).$$

■

Both additive and multiplicative form of the given polynomial, which is a type of additive fuzzy models, allows you to distribute (and redistribute) its monomials (elementary disjunction and conjunction at $q = 2$) between the different agents are combined into one group for performing the target function. The objective function represents not that other, as a full (or full with a given accuracy of approximation) in the form given for the polynomial (additive or multiplicative).

We emphasize that any metric space is topological [13]. By varying the number of hierarchy levels on the input hierarchical neuro-fuzzy model, we essentially adjust t of the Poincare polynomial coefficients, [13] which is nothing but a number to Betty. Generalized algorithm of the soft quantization of the knowledge base at the same time will construct its architecture – building adaptive classifier, balancing between its metric and topological structure.

The basic steps of soft quantum knowledge base of the considered architectures, the following

1) For each input fuzzy variable $x_1, \dots, x_m$ knowledgebase to create your own agent $A_j,\ j = \overline{1, m}$ (assuming $x_1, \dots, x_m$ normalized).
2) Each agent is assigned is responsible for its characteristics $q_i,\ i = \overline{1, q}$, where q is a prime number.
3) The initial values for the number of terms each topology: flat $N_j^{flat} = 1$ and hierarchical $N_j^{hier} = 1$.
4) In parallel mode, each agent $A_j,\ j = \overline{1, m}$ decides for your to choice for fuzzy input topology:

While $\left| f(x_{ji}) - \hat{f}(x_{ji}) \right| = \varepsilon_i > \varepsilon$ ($\hat{f}(x_i)$ - the target during the training phase) or the number of hierarchy model's levels less q we do in parallel:

- for the flat model to increase the number of terms $N_j^{flat} = N_j^{flat} \cdot q_i$ for x_j.

- for a hierarchical model on the first layer we fix to the terms number of equal q_i. For x_j terms with the value of membership function of the second kind $\mu'(y, x_j) \geq \delta$ (δ depends on tasks and known a priori, usually $\delta = 0.5$) add a new layer, also with the number of terms $= q_i$:

$$N_j^{hier} = N_j^{hier} + q_i.$$

The exit condition from the loop suggests that it is finite.

5) To compare the number of rules in accordance with the condition:

if $N_j^{flat} \geq N_j^{hier}$ then to keep the model flat (it means that the output y depends smoothly x_j), otherwise keep the hierarchy topology.

6. To encapsulate selected in step 4, the topology of the model (to consider the m-topology generated by the candidate on the node for hierarchical neuro-fuzzy models – fix their topology, the calculated number of terms and $q_i^{(j)}$).

7) To rank $x_1,...,x_m$ according to their effect on the target y , merge the generated encapsulated nodes in the hierarchy in accordance with the decomposition number m on base $q_{opt} = \max_{j=1,m} q_i^{(j)}$. Consequently, the number of inputs to up level $m^{(+1)} = loq_{q_{opt}}(m)$. (If $m^{(+1)} \geq q_{opt}$, then the topology can be expanded up to $m^{(+2)}$, but in this case the algorithm converges to the exponential).

8) Settings the initial terms for the up-level inputs to be equal to output terms of the current level. To test the generalization error, if necessary, to adapt the parameters of the input terms of up-level.

9) Depending on the scope of application - whether your knowledgebase of the agent does not need to publish (that is, he was quantification to get data from other agents), then the problem of quantization considered solved. If the knowledgebase need to be transported through other agents or through insecure communication channels, then perform a hard threshold, the coding of the obtained topology, saving the configuration array of membership functions for the moment the hard quantization. The volume of such a linear array in relation to the inputs number in a hierarchical neuro-fuzzy model. For each input within its node to find a given polynomial, approximates this input as a linear recurrent sequence. In this case the loss of information occurs, and the amount of transferred knowledgebase is determined by the degree of the resulting polynomial (More efficient algorithms of formation of the given polynomials, the author will publish in the near future).

In practice, when solving the tasks of Autonomous adaptive control of technological parameters, for example, in mining and metallurgy production, floating optimized settings successfully manage to control neuro-fuzzy networks (agents) with powerful stew of rules to 1000. For example, the content SiO_2 in the slag of copper-Nickel ores ($Y = SiO_2$) is controlled by the adaptive system [14] with an average power of rules 378 with 16 input parameters. The quantization is performed automatically under the "fourth leg" of fractal structure in Y (Fig.1), changing in the range from 3 to 5. Note that information on precious metals in the open channel is transmitted is protected.

In General, we note that a hierarchical adaptive neuro-fuzzy model presented is given by polynomials, allow to realize the principles of distributed multi-agent systems and their practical implementation [14] confirms the effectiveness of the operation in distributed industrial information systems

IV. Conclusion

This paper upon the theorems about uniform approximation of fuzzy production models, based on the Weierstrass, Stone-Weierstrass theorems proofs, as well as the Kosko theorem proved the author's theorem (3.2,3.3,3.5) about the existence of a fixed Prime number q , wherein the additive fuzzy system F with symmetric membership functions for the input x_i and output variable y on the compact X can be approximated with arbitrarily small accuracy $\varepsilon \geq 0$ a given polynomial (a generalization of a polynomial Zhegalkin for multivalued logic).

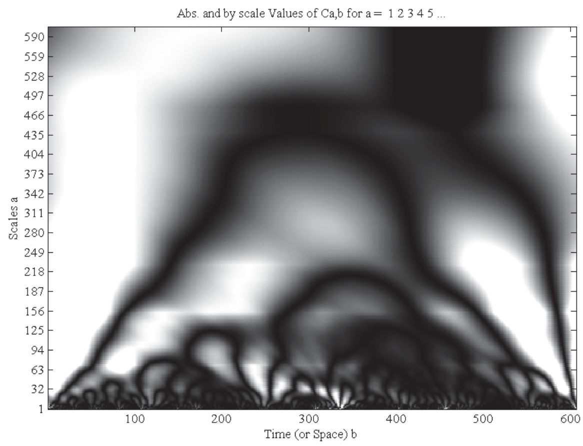

Fig.1. Wavelet decomposition of Daubechies-5 controlled parameter Y

That is, the characteristic q of the field specifies the number of terms for each linguistic variable in fuzzy implications, and for hierarchical additive models also determines the depth of its hierarchy levels. To manipulate the characteristic q agent can control the precision of approximation in an automatic mode.

Given upper and lower bounds of the Shannon complexity functions for given polynomials over finite Galois fields (theorem 3.4). The possibility of approximation by hierarchical fuzzy and neuro-fuzzy systems are by given polynomials over finite Galois fields.

Given author's soft algorithm for automatically quantification agents knowledgebase.

The article obtained evidence can be used to solve the task of automatic quantization of knowledgebase implemented in the form of an additive fuzzy and neuro-fuzzy networks, which is especially important in the development of intelligent agents OMAS for distributed information and telecommunication systems.

In particular, the solution of practical tasks presented in the paper results allow, first, automatically reduce the amount of production knowledge base (much) by building a hierarchy of rules presented in the multiplicative form of the given polynomial. If necessary, conversely, to increase the knowledge base of fuzzy implications. Second, shaping meta - knowledge as functions over the given polynomials. Third, to automatically control the level of security knowledge and data, changing it in accordance with the hierarchy level rules allocated at the stage of quantization. Fourth, to allocate and reallocate rules between mobile agents before it is transported via communication channels, thus balancing the time of its delivery to the recipient while ensuring the required level of reliability.

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