

# Methods and Algorithms of Proactive control of Complex Dynamic Objects with Disturbance Compensation

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**Abstract**—Controlling a complex dynamic object in real time and satisfying multiple criteria involves constant dealing with state deviations from the constructed plan, caused by external disturbances of a stochastic nature, and the lack of knowledge about the controlled object in the process of building the plan. The present paper suggests models and algorithms for compensating such deviations in relation to the control of individual trains within the framework of the developed multi-level multi-model complex designed for group planning and coordination of trains within a single logistics network. Another covered aspect is the algorithm of parametric adaptation, involving adjustment of model parameters and improvement of planning accuracy.

## I. INTRODUCTION

Existing methods and techniques of optimal control allow us to solve a large class of complex problems in the conditions of multi-criteria, the presence of a large number of restrictions and a high dimension of the phase vector describing the state of the controlled object. Existing approaches provide high resistance to local extremes, good convergence, and resistance to changes in the implementation of the plan. However, when solving real-world problems, primarily related to real-time control, deviations from the optimal plan caused by a number of reasons, are inevitable and are to some extent present throughout the implementation of the optimal process. This effect is especially strong when trying to apply the same complex model and control algorithms for an entire class of dynamic objects, with somewhat varying quantitative and qualitative characteristics. Since the development of the model, refinement of algorithms, calibration and software automation of the control process are associated with high costs, it is sensible to reuse the same model and algorithmic support for a whole class of objects, provided the correct adaptation to the state and characteristics of the real object.

Therefore, the problem of compensation of deviations, as well as the specification of the model characteristics should be solved at each time of the considered period. In this paper, we will consider the algorithms that solve the real-time adaptive control problem, using the moving train as an example of a controlled complex dynamic object. The proposed algorithms are a part of the developed hierarchical polymodel complex intended for complex group planning and coordination of trains movement, discussed in more detail in [8].

The proper proactive control implementation is essential for robust complex plan as a whole, and reduces the probability of conflicts that could occur due to the individual train state deviations.

## II. TRAIN MATHEMATICAL MODEL

A moving train is a controlled system, which is described by the following system of differential equations:

$$\dot{x} = f(t, x, u) \tag{1}$$

The phase vector of the regarded system contains the following magnitudes:

$$x = \|t, v, u_{act}\|^T \tag{2}$$

Here  $t$  corresponds to time,  $v$  is train velocity and  $u_{act}$  is the actual position of the thrust controller, which differs from the commanded position during transitions. The train coordinate  $s$  is regarded as a free variable. The control vector is single dimensional, and the admissible control  $u \in U = [-1, 1]$ , where  $-1$  corresponds to full mechanic braking and  $1$  is the full thrust. The train movement equations for the selected phase vector are the following:

$$\left\{ \begin{array}{l} \frac{dx_1}{ds} = \frac{1}{x_2} \\ \frac{dx_2}{ds} = \frac{1}{x_2} (u_f f(x_2) - u_r r(x_2) - u_b b(x_2) - w(x_2) - g(x_1)) \\ \frac{dx_3}{ds} = \begin{cases} 0, & x_3 = u, \\ \frac{f_{sw}(x_3)}{x_2}, & x_3 \neq u \end{cases} \end{array} \right. \tag{3}$$

where  $u_f, u_r, u_b$  are control constituents for traction, regenerative and full mechanic braking correspondently,  $f(x_2), r(x_2), b(x_2)$  - are the force coefficients with account

of tractive and braking characteristics of the real train,  $w(x_2)$  - natural movement resistance,  $g(x_1)$  is the extra resistance caused by track slope,  $f_{sw}(x_3)$  is the measure of control inertia during control transitions. The natural resistance is computed via the following formula:

$$w(x_2) = k_0 + k_1 x_2 + k_2 x_2^2 \tag{4}$$

where  $k_0, k_1, k_2$  are constant, linear and quadratic components correspondently. And  $g(x_1)$  represents the track profile slope, resulting in additional resistance during ascents and lesser resistance at descending parts of the track.

According to these equations the full time for reaching coordinate  $S_f$  from coordinate  $S_0$  is

$$T = \int_{S_0}^{S_f} \frac{1}{x_2} ds \tag{5}$$

The control vector is single dimensional  $u \in U = [-2, 1]$ , where -2 corresponds to full mechanic braking and 1 corresponds to full thrust. The control  $u$  has the following components: traction  $u_f, 0 \leq u_f \leq 1$ , regenerative braking  $u_r, 0 \leq u_r \leq 1$  and mechanic braking  $u_b, 0 \leq u_b \leq 1$ , and the following constraints take place:  $u_f u_r = 0, u_f u_b = 0$ , as the regimes of simultaneous traction and braking do not exist.

Let us consider the interval  $[s_0, s_f]$  with known phase and isoperimetric constraints, the latter related to the due passage time. The control quality indicator and the loss function is the minimum electric power expenditure along the interval  $[s_0, s_f]$ :

$$E = \xi \eta_T \int_{S_0}^{S_f} (u_f f(x_2) - \eta_R u_r r(x_2)) ds \tag{6}$$

where  $\xi$  corresponds to train mass and space characteristics,  $\eta_T$  - is the traction energy conversion efficiency and  $\eta_R$  is the energy resurgence coefficient for regenerative braking mode.

With regard to the equations, constraints and loss function the resource-intensive optimization methods, in particular the Pontryagin's maximum principle, allow to compute the optimal plan for the interval  $[s_0, s_f]$ , consisting of different movement regimes. This proposed approach is discussed in more detail in [7]. The result is the optimal control  $\tilde{u}(s)$  and the corresponding optimal trajectory  $\tilde{x}(s)$ , which together form an optimal process and secure the best possible integral quality indicator value on the regarded interval. The optimality

criteria demand that the train movement should be carried out in the following regimes:

- **full thrust** mode, used for acceleration,  $u_f = 1, u_r = u_b = 0$
- **stabilization** mode with limited thrust, used for maintaining a steady velocity,  $0 < u_f < 1, u_r = u_b = 0$
- **neutral coasting** mode with no thrust, resulting in slow deceleration due to natural resistance,  $u_f = u_r = u_b = 0$
- **regenerative braking** mode with partial power recovery, as the electric engine operates in generator mode,  $u_f = 0, u_r = 1, u_b = 0$
- **full mechanical braking** mode, mainly used for final deceleration or in case of prohibiting semaphore signals approach,  $u_f = 0, u_r = 1, u_b = 1$

So in any case the optimal commanded control is a piecewise constant function, and each value corresponds to one of the regimes. These regimes follow each other in the optimal plan, interleaved by control transitions, but some of them might be missing in some cases. e.g. stabilization mode is considered ineffective and causing engine wear, so it is commonly used only for maintaining a low velocity under strict limits. In other cases, the leg passage is a combination of acceleration and coasting segments. Fig. 1 shows an example of such optimal plan, restricted by velocity limits, with several regimes following each other.

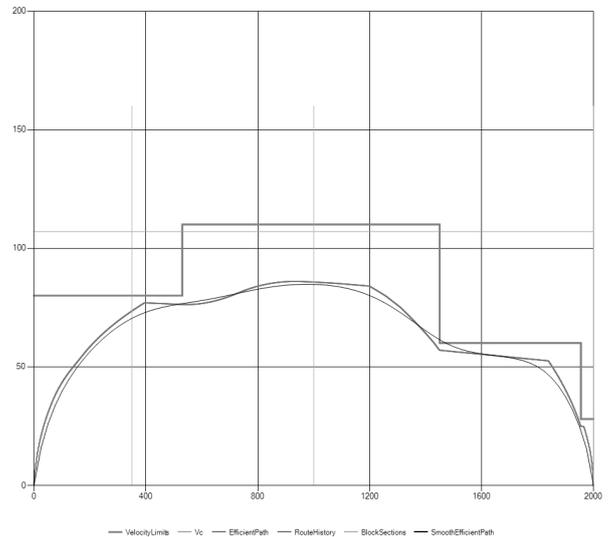


Fig. 1. The optimal plan

During the passage of the interval the plan might be recomputed to reflect the present state and amend the commanded control so that the integral quality indicators remain optimal.

However, the computation of the optimal plan alone is not sufficient for effective real time control. The correct implementation of the planned optimal process requires extra effort, related to disturbance compensation.

III. DEVIATION COMPENSATION ALGORITHM

In the process of real time dynamic object control with the continuous phase space, the deviation from the computed trajectory is inevitable and almost always present to a certain degree. The reason for this is, firstly, the presence of stochastic external conditions, which were not predicted at planning stage, and, secondly, the lack of knowledge about the qualitative and quantitative characteristics of the controlled object. Under such conditions, the problem of synthesis of optimal control in the presence of deviations has to be solved at each moment of time during the plan implementation. However, the problem of train controlling imposes strict limits for time to develop and implement a control decision, as the occurred deviations may not only disrupt the optimality, but also lead to fatal consequences in case of inaction. Therefore, a complete rescheduling as a response to the detection of disturbances is usually not applicable, and a less computationally intensive operation is required, which allows to keep the integral quality indicators within acceptable limits.

To solve this problem of adaptive control under perturbation conditions, the following approach is proposed. Let the free variable at the moment of control decision making have a value  $s'$ , lying within  $[s_0, s_f]$ . Due to perturbations the phase vector  $\tilde{x}(s')$  has a deviation from the optimal trajectory  $x^*(s)$  in  $s'$ . To compute the amended control and achieve optimality in the event of disturbances it is proposed to specify the interval  $[s', s'']$ ,  $s' < s'' < s_f$ , and specify the quality indicators other than the original, used for computing the optimal plan. In this case the loss function should minimize the integral discrepancy between the optimal trajectory and the actual trajectory, resulting from the amended control, in the interval  $[s', s'']$ . Thus, the loss function takes the following form:

$$J = - \int_{s'}^{s''} k(s) (\|x^d(s)\|) ds \tag{7}$$

Here  $x^d(s) = x(s) - x^*(s)$ ,  $\|x_d(s)\|$  is the weighted norm of phase deviation vector, which should be computed via the following formula:

$$\|x^d(s)\| = \sqrt{\sum_{i=1}^n w_i(s) (x_i^d(s))^2} \tag{8}$$

In this formula  $n$  is the phase vector dimension,  $w(s)$  is the vector of weight coefficients, measuring the contribution of phase vector components in the integral discrepancy. The extra nondecreasing function  $k(s)$  in the interval  $[s', s'']$  is

used to secure the convergence to the optimal plan closer to the end of the interval. The length of the interval  $[s', s'']$  is chosen as to satisfy both high computation performance requirements and precision. As the optimal trajectory  $x^*(s)$  is well known at  $[s', s''] \in [s_0, s_f]$ , this optimization is possible, as minimizing the trajectory deviation will also secure the actual integral quality indicators within admissible borders.

With regard to optimal train control this problem boils down to minimizing the deviation of the actual velocity plot from the optimal process with other phase vector constrains, mainly the piecewise constant velocity limits in the regarded interval. Since the interval is just a subset of  $[s_0, s_f]$ , the isoperimetric constraints regarding arrival due time are not considered here, and that decreases the complexity considerably.

When calculating the optimal control taking into account the limited time to minimize the integral discrepancy in the velocity plot is proposed to use a simple method of the golden section. The instantaneous control value is searched in the interval  $u \in U = [-2, 1]$  in order to minimize the integral discrepancy (7). In this case  $k(s)$  is selected for reasons of increasing the influence of the residual when approaching the end of the interval. Fig. 1 illustrates the principle of instantaneous control computation in the event of deviations.

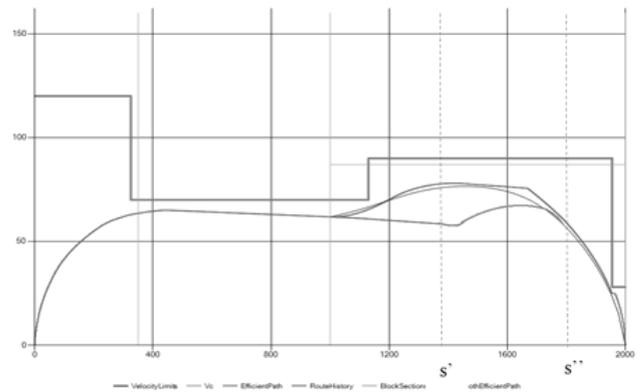


Fig 2. Control computation in the event of deviations

The figure shows that the current velocity of the train has a deviation from the plot in  $s'$ . According to the algorithm, the minimum velocity residual plot and the corresponding initial control are calculated in the interval  $[s', s'']$ . In the specific example, one control switch was used to compensate. In this formulation, when using the golden section, the problem of control calculation becomes much less computationally complex and can be applied under severe restrictions on the control decision due time. The proposed approach does not eliminate the need to reschedule and recalculate the optimal process when detecting deviations from the previously constructed plan in order to improve the integral quality

indicators for the new state of the object. However, this potentially computationally intensive procedure can be performed much less frequently in the background, and this will not affect the responsiveness of the control subsystem.

IV. PARAMETRIC ADAPTATION ALGORITHM

As stated previously, controlling a complex dynamic object often implies not only compensating disturbances of stochastic nature, but also dealing with regular deviations from the optimal plan in the process of its implementation. These deviations are caused mainly by the lack of knowledge about the quantitative and qualitative characteristics of the original object. In relation to a train control, these parameters include mass and size, traction and braking characteristics, linear and quadratic components of the main resistance to movement, parameters that determine the additional resistance, depending on the profile of the railway and several others, mainly

affecting the  $\frac{dx_2}{ds}$  equation. As the real-time train control

involves constant monitoring of the phase vector state, the actual position and velocity and their deviation from the computed plan may be used to improve the model accuracy. The parametric adaptation allows to adjust the model, and the source of information for it is the deviation of the responses of the object and the model, the elimination of which implements the adaptation process. However, one of the main problems of implementing such management with adjustment of characteristics, called dual, is the fact that the management is not diverse enough to give information about the specific properties of the object, which should be reflected in the model. In addition to the parametric adaptation implemented in the dual control, there is a need for structural adaptation, which consists in changing the structural and functional composition of the model. An example of such a change may be taking into account the curvature of the trajectory for the rounded sections of the road that affect the dynamics of the speed, or taking into account the skid effects. However, this type of adaptation is weakly amenable to automation, especially in the conditions of real-time control.

Therefore, this paper will focus on parametric model adaptation. The movement of the train along the track described by the system of differential equations (3) can be divided into short segments corresponding to different values of the free variable, at the end of each of which the state of the control object is monitored. Since in the course of observation it is necessary to ensure the drift of the model characteristics as close as possible to the original, it is advisable to consider the system of equations as a single-layer artificial neural network (ANN), trained in the process of dual control. The input signals of this network will be the state variables (2), as well as the control action, the outputs – new vector values after a short interval of observation and the energy expended, and the activation functions of neurons – the equations of motion.

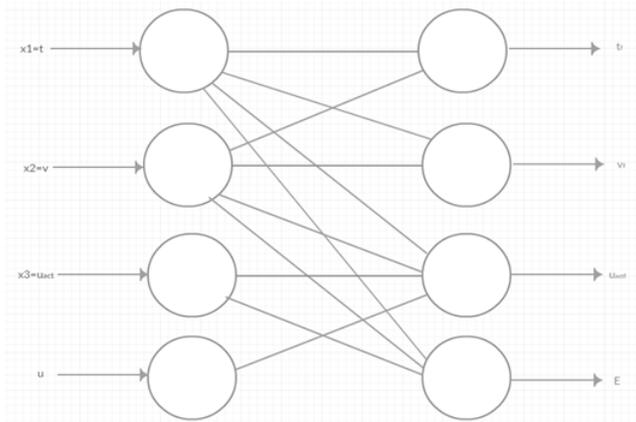


Fig. 3. The adaptation neural network

IV. CONCLUSION

The proposed models and algorithms allow to solve the problem of control of a complex dynamic object and compensate the deviations from the computed optimal process. Thanks to the simplified calculation of optimal control with perturbation compensation it is possible to combine the advantages of complex optimization methods that are resistant to local extremes and a high computing performance that is critical in a limited time for control decision-making. The algorithm, discussed in this paper, is an important part of the multi-level model-algorithmic complex designed for group planning and coordination of multiple trains within a single logistics network. It corresponds to the lower level of the model hierarchy, which is devoted to a single train movement planning and control. Effective compensation of deviations for a single train can reduce the impact of disturbances and therefore increase the robustness of the complex plan as a whole and ensure the key quality indicators remain acceptable. Moreover, the proposed algorithm of parametric adaptation clarifying the train model numeric parameters allows to reduce the deviation between real complex objects and predictions, which is also important for complex planning accuracy. The precise tuning of quantitative model characteristics is also vital for aimed deceleration, when accurate stopping at a station is obligatory.

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