Air Navigation: Automation Method for Controlling the Process of Detecting Aircraft by a Radar Complex

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Abstract—**Modern air traffic control systems equipped with phased antenna arrays radar have increased performance in comparison with radars of previous generations. The radar tracking of objects starts with the detection of such objects. The earlier the object is detected, the more time is available for the selection of modes. The article solves the problem and offers a method of automated management of modern radar complexes with phased antenna arrays, which will significantly reduce the search time for objects in the view field of such complexes. The solution is based on the discretization of the object detection process by time and position. Space, where the detected objects move, is divided into cells, and each of the cells is identified with a particular position. Each object can be in each cell for a long interval of time and then move to other cells. As a result, a dynamic process with a fixed number of positions and discrete time is obtained. For optimal calculations, we chose a minimum average time for searching and detecting an object. The optimal speed problem deals with the elements of the phase space as extrapolated probabilities of the present object in the cell of the viewing area. The optimization problem is solved using a discrete analogue of the maximum principle. Its application offers sufficient conditions for optimality. For the numerical solution of the problem, the author employs a modified method of successive approximations. Based on the proposed method, the author develops an algorithm for automated management of the detection of moving objects by a radar complex, as well as an operational simulation model after objects are detected. The suggested method of automatic control significantly reduces the average search time for objects. This article is the second in a series of articles devoted to the problems of information support of the processes of navigation of aircraft and air traffic control.**

I. Introduction

The high consumer demand for air traffic leads to the growth of the air industry. Increased traffic intensity with an ever-expanding list of routes and airways filled with airplanes and helicopters of various passenger capacities, high flight speeds, new airports require improvements in safety and efficiency of flights — air traffic control systems (ATC).

National and foreign experts published many works devoted to improving the air traffic control systems of aircraft navigation. Chinese researchers are the leaders by many publications proving that a sharp and explosive growth of air traffic in China is possible only with the development of flight safety for the increased volume of passenger and cargo air transportation.

Researchers and experts from the United States, Italy, Australia, Germany, the United Kingdom, and France also actively publish on various aspects of air navigation and air traffic management. Russian authors take the fifteenth position by the volume of publications although Russain air corridors are bustling connecting Europe, Asia, America and Australia. This fact should contribute to the growth of interest of domestic researchers to the problem and increase the number of publications on this issue in future.

Most of the publications are devoted to specific technical issues of air navigation and the development of its theoretical and practical aspects.

The researchers from the Philippines [1] note that the sharp increase in air traffic in the country led to significant traffic congestion. Given the limited resources, the large airports are facing the problem and study factors leading to congestion. Researchers use the Decision Making Trial and Evaluation Laboratory Model and Analytic Network Process (DEMATEL-ANP) to determine the most critical factors in the commercial aviation industry.

The paper [2] is devoted to the analysis of research results in the field of modelling of air traffic flows. This article discusses the recent developments in models and simulations for studies of airspace structures and traffic flows. The topics discussed are new models/methods proposed by researchers for optimal flight paths/routes, optimal route structures, sectorization and so on and also new developments in the Association of Southeast Asian Nations region. An Association of Southeast Asian Nations established standard modelling and simulation function for analyses of airspace structures and traffic flows throughout the Association of Southeast Asian Nations and offered solutions for capacity and efficiency improvements. The article discusses the challenges of air traffic in the Association of Southeast Asian Nations region. The article points at the lack of research in air traffic management in the Association of Southeast Asian Nations region. The authors call for more research to meet the demands of the dynamic and fast-growing aviation market. There is a need for more research on optimal flight paths/routes, more efficient airspace structures and sectorization in the Association of Southeast Asian Nations region.

The article [3] discusses the use of machine learning methods and cognitive ergonomics for air traffic control. The authors conclude that research of air traffic management is related to AI (XAI) and computer-aided verification, so the innovations have to combine with applied AI research.

The article [4] analyses the factors contributing to the congestion of airspace at intersections of the routes and discusses the models of congestion and distribution in complex networks, predicting the distribution within the cluster depending on its capacity. The authors analyse the relationship between system parameters and the distribution of congestion, taking into account the effects of growth in the networks. The article develops the theory of dynamic models of air traffic congestion and offers a more in-depth understanding of the characteristics of congestion to achieve better congestion management. The report can be useful for airspace designers that should consider congestion for optimizing the airspace structures.

In the article [5] Chinese authors argue that in order to eliminate flight delays and risks associated with the increased air traffic, it is necessary to increase the capacity of air traffic control systems. Their approach is based on an objective evaluation of the complex situation in the airspace. The article proposes a new method to measure the complex conditions of air traffic. Their approach takes into account the indicators of airspace and traffic flow for objectively evaluation of complicated air traffic situation.

The results indicate that the new complexity index is more accurate than traffic count and reflects the number of trajectory changes as well as the high-risk situations. Additionally, analysis of potential applications reveals that this new index contributes to achieving complexity-based management, which represents an efficient method for increasing airspace system capacity.

One more article [6] of Chinese authors is devoted to the use of modelling and predicting of the situations in the airspace. The ever-increasing air traffic in China demands efficient planning, investment and management of airports and airline companies. In this context, accurate and adequate short-term air traffic forecasting is essential for the operations of those entities. In consideration of such a problem, a hybrid air traffic forecasting model based on empirical mode decomposition (EMD) and seasonal autoregressive integrated moving average (SARIMA) has been proposed in their paper. The proposed model deals with the original time series and breaks them into components and employs each element with the SARIMA forecasting model before integrating all the models into the final combined forecast result.

In the article [7] Chinese researchers offer a new approach to the calculation of air traffic complexity. Air traffic complexity is an objective metric for evaluating the operational situation in the airspace. The method has several applications in airspace design and traffic flow management. It is considered as a reliable method to measure traffic complexity accurately. Regarding many factors correlate with traffic complexity in complicated nonlinear ways, researchers have proposed several complexity evaluation methods based on machine learning models trained on large samples.

In their article [8] the authors from Serbia discuss the issue of air traffic control in the airport area as a part of the most airspace management. One of the most popular instruments of network manager's (NM) in Europe to deal with a mismatch of demand-capacity is to impose ground, i.e. Air Traffic flow management (ATFM), delays to flights. To compensate for anticipated delays and improve on-time performance, Aircraft Operators usually embed a buffer time in their schedules. The current practice for allocating ATFM delays does not take into account flights' schedule buffer to absorb ATFM delay and do not reduce delays of subsequent flights. Furthermore, the official policy aims to minimize ATFM delays to half a minute per flight, while real-time delays are approximately ten times higher. The authors discuss the ways of improving ATFM management slot allocation to minimize propagated delay and improve airport slot adherence.

II. Features of the Air Traffic Control System as a Control Object

The review mentioned above of the available scientific literature on the operational air traffic control issues proves that it is a significant factor of civil aviation safety.

The air traffic control networks include several different elements:

- radio equipment (radar stations, automatic direction finders, communications, computers);
- elements of airspace (zones, areas, sectors, airways, corridors, routes, trains);
- ATS (airfield and route control stations).

Therefore, they perform a significant role in the proper coordination, management and operation of many various elements.

The air traffic control covers multiple spaces on the ground and in the air providing air traffic safety.

To describe the complexity of air traffic control, the so-called "big air traffic control system" was adopted, which consists of three subsystems located at different levels (Fig. 1).

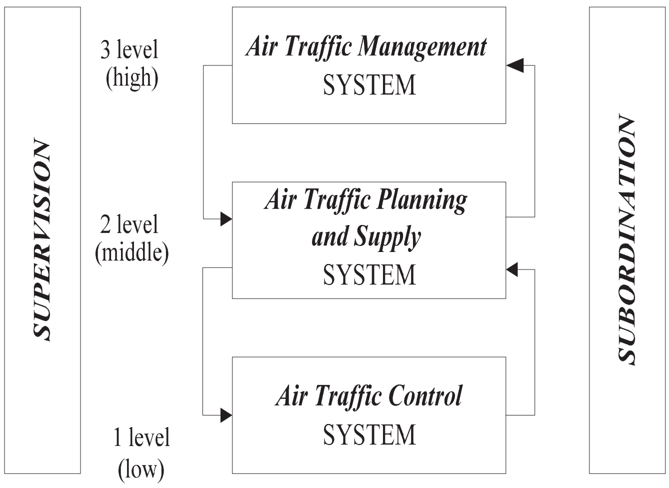

Fig. 1. Big air traffic control system

The big management system includes the comprehensive control of subsystems as closed systems for the solution of relevant problems.

All levels of the “Big ATC system” are interconnected and subordinated from bottom to top and top to bottom in terms of accountability.

Big ATC system allows solving the following tasks:

- organizing, planning, ensuring and direct regulating of air traffic;
- coordinating and interacting among air traffic control bodies and other services and other departments;
- taking timely measures to assist the aircraft in distress or in other emergencies in the air and on the ground.

Management of modern complex technical systems depends on is the real-life information from such systems. The most illustrative example of the control of current complex technical systems is the management of measurement data: systems provide and use data on objects or processes. Examples include various monitoring radar systems, radio navigation systems, and sensors. General theoretical approaches to the construction of such automated management systems, methods and algorithms for control and information processing in such systems are considered in many articles [9], [10], [11], [12].

Radar systems employ methods of information processing and automated control at different stages of operation. The authors of the presented article and other researchers discuss the theory of automated management, including the mathematical and IT methods [13], [14], [15]; therefore, we will consider some theoretical and applied aspects of automated multifunctional control radar complexes with a control and operation parameters from the point of view of the higher level system, for example, control systems air traffic.

Modern radar systems equipped with phased antenna arrays have an increased performance potential in comparison with previous-generation radars with rotating antennas. It should be noted that the maintenance of objects located in the radar's field of view begins with the detection of such objects and the earlier the object is detected, the longer the time interval is available for its processing, such as tracking and guiding to a specified area or point in space. Longer time intervals for decision contributes to improved service and safety.

The presented article is the second in a series under the general title “Air Navigation” and devoted to various aspects of information support of air traffic control at the operational level. The primary source of information about aircraft moving in space is modern radar systems with a phased antenna array.

The first paper [16] describes the aspects of optimizing the management of such complexes during the radar tracking of aircraft on the flight routes or in the airport zone. The article suggests that the proposed management optimisation methods can significantly increase the capacity of radar systems and reduce the required number in the ATC system.

The presented article develops the ways to optimize the management of such complexes to improve their performance at the stage of detecting air objects, reduce the required energy costs for detection and allocate more efforts for tracking and other modes, which may lead to a lower required number of such radar systems in the system ATC.

III. Theoretical foundations of the method of automating the management of the detection process

One of the first papers devoted to the theory of the search for moving objects was the article [17] published in 1957. Later, more researchers [18], [19], [20] addressed relevant aspects of the topic. In later 1970s the interest to the further studies was generated by the widespread use of phased-array antenna radars. The article [21] discusses some approaches to the organization of the airspace. However, it does not address the optimization of the object search process.

Currently, there is an excellent theoretical and practical interest to the theoretical approaches to the development of automated control radar systems with phased antenna arrays and implementation of flexible (optimal or suboptimal) strategies for searching of moving objects.

Thus, the task is to develop such a method of automated management of modern radar systems with phased antenna arrays significantly reducing the search time for objects in the view field of such complexes.

The solution of the problem requires the discretization of the process of detecting objects by time and state. To do it, we apply the law of transformation of the state distribution function in the form of a recurrent algorithm of calculation of density [22]. At the same time, we apply a discrete analogue of the maximum principle method. We assume that the space in which the detected objects move is divided into cells, and each cell is identified with a particular state. Each object can be in each cell for a random interval of time before it moves to other cells. As a result, we obtain a dynamic process with a fixed number of states and discrete time.

For the mathematical description of this process, the well-known approach [23] can be used, and the expression for the extrapolated probability of finding a detectable object in a cell can be presented as follows:

$$\begin{aligned} P_{exti}(t+1) &= \hat{P}_i(t) + \\ &+\Big[\hat{P}_1(t)P(i/1)+\ldots+\hat{P}_{i-1}(t)P(i/i-1) \\ &+\hat{P}_{i+1}(t)P(i/i+1)+\ldots+\hat{P}_N(t)P(i/N)\Big]+P_{\tau i}, \end{aligned} \tag{1}$$

where $\hat{P}_i(t)$ - aposterior probability of the object presence in the cell at the moment of time t;

$P(i/j)$ – probability that in a certain interval of time (t, $t+1$) the object will move from cell j into cell i;

$P_{\tau i}$ - probability of a new object in cell i ;

$i = n\Big|_{n=1\ldots N}$, N - number of cells in the viewing field.

Let us calculate P_{ij} to describe the probability of moving objects from othe cells of the airspace in to the cell i, the formula can be presented in the following way:

$$P_{ij} = \hat{P}_1 P(i/1) + \ldots + \hat{P}_{i-1}(t)P(i/i-1) + \\ +\hat{P}_{i+1}(t)P(i/i+1) + \ldots + \hat{P}_N(t)P(i/N). \quad (2)$$

The expression (1) with the determiner c (2) looks like:

$$P_{exti}(t+1) = \hat{P}_i(t)P(i/j) + P_{ij} + P_{\tau i}. \quad (3)$$

For further simplification of the (3) we assume propability $P_{\tau i} = 0$. Such assumption does not affect the general solution of the problem and a non-zero probability can be taken into account.

In the process of the scanning the cell i in the radar viewing field, a posterior probability of the object presence in the cell can be calculated by the formula [23]:

$$P_{exti}(t+1) = \hat{P}_i(t)P(i/j) + P_{ij} + P_{\tau i}. \quad (4)$$

where $z_i(x/u,t) = \dfrac{P_{1i}(x/u,t) - P_{0i}(x/u,t)}{P_{0i}(x/u,t)}$.

Here $P_{\lambda i}(x/u,t) = \begin{cases} P_{1i}(x/u,t); \\ P_{0i}(x/u,t); \end{cases}$ – density of distribution of probability of random variable $x(t) = x$ on the output of the radar equipment. If the object is present, then $\lambda = 1$, if the object is absent $\lambda = 0$.

The improved performance of the radar complexes after the detection of the objects requires to minimize the time of detection in the field of radar complex. Therefore optimal criterion should be the minimal average object detection time [22].

Considering the above requirements to the process of object detection, the problem of managing the search for an unknown number of moving objects can be formulated as follows. At a given time interval $[0,\ L]$, it is necessary to organize the process in such a way that at the end of the interval there is a minimum average time for searching and detecting an object.

Thus, for the organization of optimal control of the detection process, it is necessary to find such a set of controlled parameters $u^0 = \{u_1(t)\}$, that satisfy the above justified criteria:

$$\bar{T} \Rightarrow \min\Big|_u \quad (5)$$

with constrains:

$$u_i(t) = \begin{cases} 1; \\ 0, i = 1, \ldots, N; \end{cases} \quad (6)$$

$$\sum_{i=1}^{N} u_i(t) = 1. \quad (7)$$

At the same time, the constrain (7) is a possibility to monitor only one cell at a time, the size of which is determined either by the physical dimensions of the radar field of the radar complex, or by predetermined dimensions, taking into account the fact that to view the cell, several attempts have to be made, and constrain (6) - as a possibility at the particular moment in time to carry out or not to carry out the scanning of a specific cell of a given radar view area.

The optimization problem (5), (1), (4), (6), (7) is a problem of optimal speed known in the optimal control theory [24], where the elements of the phase space are extrapolated probabilities of the presence of an object in the cell of the view area.

The above optimization problem can be solved using a discrete analogue of the maximum principle. It should be noted that according to [25], [26] its application provides not only necessary but also sufficient conditions for optimality.

Applying the maximum principle to the the problem in question, the equation of state for phase variables is as follows [22]:

$$P_i(t+1) = \left[P_i(t)\frac{1+u_i(t)z_i(t)}{1+P_i(t)u_i(t)z_i(t)} + P_{ij} \right] \times \\ \times \left[U_+(P_i(t)) - U_+(P_i(t) - P^*) \right], \quad (8)$$

where P^* – the threshold value of a posterior probability of the presence of an object in the cell, at which it is considered that the object is detected;

U_+ – step function [22].

Taking into account introduced in (8) Hamiltonian values into the problem in question it can be represented as follows:

$$H[P(t), u(t), \gamma(t+1)] = \\ = \sum_{i=1}^{N} \gamma_i(t+1)P_i(t+1), \quad (9)$$

where $\gamma_i(t)$ – conjugate variables .

In accordance with the requirements of the discrete analogue of the maximum principle, the canonical equation for conjugate variables is as follows [22]:

$$\gamma_i(t) = -\gamma_i(t+1)\frac{1+u_i(t)z_i(t)}{[1+P_i(t)u_i(t)z_i(t)]^2} \times \\ \times \left[U_+(P_i(t)) - U_+(P_i(t) - P^*) \right]. \quad (10)$$

The transversality condition is defined by the following relationship:

$$\gamma_i\left[L\right] = -1, \qquad i = 1,...N. \tag{11}$$

The optimal control function can be determined from the following condition:

$$\begin{aligned} &H\left[P^{opt}(t), u^{opt}(t), \gamma(t+1)\right] = \\ &= \max_{\mathbf{u}} H\left[P(t), u(t), \gamma(t+1)\right]. \end{aligned} \tag{12}$$

As a result of solving the problem (5) in accordance with condition (12), the control parameter for each point in time is obtained:

$$u_i(t) = \begin{cases} 1, \text{ if } a_i \dfrac{1-P_i(t)}{1+P_i(t)z_i(t)} = \\ = \min\limits_{j=1...N} \left\{ a_j \dfrac{1-P_j(t)}{1+P_j(t)z_j(t)} \right\}, \\ 0, \text{ for all other cases } \quad \mathrm{j} \neq \mathrm{i}, \end{cases} \tag{13}$$

where $a_{i,j} = \gamma_{i,j}(t+1)P_{i,j}z_{i,j}(t)$; $\mathrm{i} \neq \mathrm{j}$; i=1,..., N; j=1,..., N.

It should be noted that the solution of problem (5) with constraints (6) and (7) in analytical form is impossible due to the fact that there is a non-linear relationship between the values H, γ, u . In this regard, the solution of this problem is possible only numerically. The authors applied the author's modification of the method of successive approximations as a universal method for solving complex optimization problems.

The essence of the proposed method and algorithm consists in combining the discrete analogue of the maximum principle and the method of successive approximations. The main operations in the search for optimal controls are as follows:

1) Some initial approximation of the control vector is selected $\hat{\mathbf{u}}(t) = \left(\hat{u}_1(t),\, \hat{u}_2(t), ..., \hat{u}_N(t)\right)^T$, $t = 1,\ 2, \ldots, T-1$ (assuming uniform sequential scanning of all cells of the detection zone) and then calculate of extrapolated probability values using relation (8), and then determine in the reverse time all trajectories of conjugate variables (10) taking into account the transverse condition (11).

2) At each step of the calculation (12) the components of the vector are calculated and adjusted $\mathbf{u}(t)$.

3) A condition check is performed $\left|\overline{T}_{k+1} - \overline{T}_k\right| \le \varepsilon$ (ε – arbitrarily small predetermined value, k – itteration number). If the condition is not fulfilled, then return to item 1. If the condition is satisfied, then the resulting vector of controls $\mathbf{u}(t)$ is considered optimal.

In practice, however, there are cases when the proposed approach for the numerical search for the optimal control vector does not provide for the optimal solution. Perhaps, it can be explained by the presence of any local optima located in the vicinity of the global optimum. In this case, good practical performance [27] was shown by random search methods [28] used to verify the optimality of the solution obtained or to divert the optimization process from a local optimum point.

IV. Results and discussion

Based on the method proposed in the previous section, an algorithm was developed to automate the management of the detection of an unknown number of moving air objects by a multifunctional radar complex that functions as part of an air traffic control system as an information source, as well as a simulation model for detected objects.

Various cases of objects entering the detection zone (singular, in groups, with same speeds, different speeds) were investigated. The traditional method of scanning consisting of the uniform distribution of search efforts among the cells of the scanned area and its following reviews [21], as well as the proposed method of automation of control, was applied.

The simulation results showed that when using the developed automation control method, depending on the informative conditions the average search time for objects, can be significantly reduced (1.5 - 3.5 times).

These results have been achieved by moving away from the concept of uniform scanning the detection zone and concentrating large search efforts in the cells with a higher posteriori probability of object presence.

The proposed method of automatic control management requires more serious refinement before it can be used in real-time control systems. It involves the development of a suboptimal control method with lower computational complexity compared to the proposed plan.

The difficulty of the proposed method is in the nonlinearity of the controlled objects, not allowing for the final expressions for calculating the optimal control actions. It is necessary to apply numerical methods and algorithms for solving control problems with significant computational complexity.

It should be noted that the application of the proposed method in the article do not affect the value of the type I and II errors. The value of these errors is determined by the signal-to-noise ratio and methods for detecting the signal reflected from the object, which is used in the radar.

The main areas of application of the proposed method are air traffic control systems, various guidance systems for objects at a given point in space.

V. Conclusion

The method is developed for automation of the detection for moving objects in the viewing area of a multifunctional radar complex of an air traffic control system. Potentially the method can significantly reduce the time of object detection and reduce energy costs of the complex for detecting air objects.

The simulation results showed that compared with the basic methods of process of detecting objects, the developed method could significantly reduce (1.5 to 3.5 times) the average search time for objects depending on the accuracy of measurements.

The method can be effectively implemented in other operational modes of the multifunctional radar complex and significantly increase the efficiency of air traffic control in the specified service areas.

Before the practical use of the advanced control automation method, it is necessary to conduct research aimed at reducing the computational complexity of the developed algorithm for its integration into the control loop of the real-time control system.

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