Non-Linear Ordered Precoding with Limited Feedback for Multiuser MIMO systems

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Abstract—**Using of quantized channel state information on transmitter (CSIT) side for adaptive signal processing on base station (BS) side also called as precoding provides to reduce feedback channel bandwidth in comparison with full CSIT. There are many solutions for feedback channel quantization but the aim of this paper is to note the performance of grassmannian manifold for codebook design.**

I. Introduction

The current radio communication standards include the spatial processing of signals implying the use of MIMO (Multiple-Input-Multiple-Output) multi-antenna systems [1], [4], [11]. The use of MIMO technology in communication systems makes it possible to significantly increase the capacity, but involves the necessity of development the algorithms of signal processing. The task of adaptive signal processing for the simultaneous operation for several users on the same allocated frequency-time resources falls on the computing power of both the BS and user sides.

Precoding performed on the BS side based on the CSIT obtained with feedback channel allows to calculate the necessary weights for the most efficient transmission of information over the communication channel.

MIMO systems can work in multi-user MIMO mode (MU-MIMO) where the task of cancelling of multiuser interference is a priority. The aim of this article is increasing the noise immunity by applying new non-linear ordered precoding for wireless systems with TDD using limited feedback.

II. System model

We consider a multiuser MU-MIMO system with spatial division multiple access (SDMA) which consist of multiple antenna BS and several users where each of them has multiple antenna receivers [2], [3], [5], [6]. Let's introduce the following notations: N — number of antennas on the BS side; M — sum number of antennas on the user side; k — index number of user (k = 1, 2, …, K); K — total number of users in a multiuser system; M_k — number of antennas on k-th user side $\mathbf{H}_k$ — complex channel matrix of dimension $M{\times}N$, where each element of $\mathbf{H}_k$ is a complex channel multiplier between user antennas and BS antennas; $\mathbf{H}=\begin{bmatrix}\mathbf{H}_1^T & \mathbf{H}_2^T & \dots & \mathbf{H}_K^T\end{bmatrix}$— general matrix of MIMO channel between all users and BS of dimension $K{\times}N$, which consists of channel matrices $\mathbf{H}_k$;

The main task of a base station is simultaneous data transmission for all K users using N transmit antennas and the same time-frequency resources. The precoding procedure is performed on the BS side, where the weights for the transmitting signals for each user are given by the common precoding matrix T. The calculation of the matrix T is based on CSIT obtained by feedback channel from each user to the BS.

The signal observed at the k-th receiver can be represented as follows:

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{T}_k\mathbf{s}_k + \sum_{j=1, j\neq k}^{K} \mathbf{H}_k\mathbf{T}_j\mathbf{s}_j + \mathbf{n}_k, \tag{1}$$

where $\mathbf{y}_k$ — vector of received symbols observed at the k-th receiver, $\mathbf{s}_k$ — symbol vector which was transmitted from BS to k-th user, $\mathbf{T}j$ — precoding matrix of other j-th user, where $j \neq k$ and $\mathbf{n}_k$ represents the vector of additive noise at the k-th receiver. Second part of (1) represents interfering signals of other users of the system with respect to the signal of the k-th user.

It is important to note that each of user estimate the channel parameters, calculating the components of the complex matrix $\mathbf{H}_k$ $=\mathbf{H}_{k.est}+\mathbf{H}_{k.err}$ (taking into account the estimation error) of dimension $M_k{\times}N$.

The precoding procedure requires the feedback channel having sufficient bandwidth for periodically transmitting CSIT to BS's side. The formation of such a feedback channel may require resources (time slots or frequency bands) that are maximally used for data transmission channels.

III. Non-linear ordered precoding

The signal at the input of the base station receiver, received from the k-th subscriber, can be represented as follows [7], [8]:

$$\tilde{\mathbf{Y}}_k = \tilde{\mathbf{H}}_k\tilde{\mathbf{T}}_k\tilde{\mathbf{s}}_k + \tilde{\mathbf{n}}_k,\ k = 1, 2, ..., N,\ K = 1, 2, ..., N, \tag{2}$$

where $\tilde{\mathbf{H}}_k$ — matrix of the reversed communication channel from the subscriber terminal to the base station of dimension $N \times M_k$, $\tilde{\mathbf{T}}_k$ — precoding matrix of dimension $M_k \times R_{ch,k}$, $\tilde{\mathbf{s}}_k$ — transmit symbol vector, transmitted by the k-th user, dimension of $K_{ch,i} \times 1$, $\tilde{\mathbf{n}}_k$ — complex vector of noise in the channel between the user equipment and the base station, having zero expectation and correlation matrix $\mathbf{R}_{\tilde{\mathbf{n}}} = 2\sigma_{\tilde{\mathbf{n}}}^2\mathbf{I}$.

For convenience, we introduce that $\tilde{\mathbf{H}}_{\mathbf{T},k} = \dfrac{1}{2\sigma_{n,k}^2}\tilde{\mathbf{H}}_k\tilde{\mathbf{T}}_k$:

$$\tilde{\mathbf{Y}}_k = \tilde{\mathbf{H}}_{\mathbf{T},k}\tilde{\mathbf{s}}_k + \mathbf{n}_k, \quad (3)$$

где $\mathbf{n}_k$ — the observation noise in the communication channel of the k-th user, represented by a random Gaussian value with a unit variance and zero mean.

The signal received by the base station from all users can be represented as follows:

$$\tilde{\mathbf{Y}} = \tilde{\mathbf{H}}_{\mathbf{T}}\tilde{\mathbf{S}} + \mathbf{n}, \quad (4)$$

where $\tilde{\mathbf{S}} = \begin{bmatrix}\tilde{s}_1 & \tilde{s}_2 & \dots & \tilde{s}_k\end{bmatrix}^T$ — column of users symbols, $R_{ch} = \sum_{k=1}^{K} R_{ch,i}$, $\tilde{\mathbf{H}}_{\mathbf{T}} = \begin{bmatrix}\tilde{\mathbf{H}}_{\mathbf{T},1} & \tilde{\mathbf{H}}_{\mathbf{T},2} & \dots & \tilde{\mathbf{H}}_{\mathbf{T},k}\end{bmatrix}$ — general matrix of inverse MIMO-channel of dimension $N \times R_{ch}$.

The signal demodulation at the base station side can be performed using the MMSE linear algorithm, however, the estimation accuracy can be improved by applying the OSIC procedure (Ordered Successive Interference Cancellation. The MMSE-OSIC procedure is iterative, non-linear in general, but linear at each step, allowing for improved accuracy of estimation on the receiving side compared to the conventional MMSE linear algorithm.

To detect the received signal, it is necessary to calculate the transformation matrix $\tilde{\mathbf{G}}$:

$$\hat{\mathbf{S}} = \tilde{\mathbf{G}}\tilde{\mathbf{Y}}. \quad (5)$$

The calculation of the matrix $\tilde{\mathbf{G}}$ occurs iteratively. At the first iteration, the strongest received signal is evaluated, after which the interference introduced by it is suppressed. At the next iteration, the next, strongest of the received signals is re-searched, and it is evaluated, taking into account that the interference of the signal, assessed in the first step, no longer affects the received signals. This procedure is repeated until all signals received by the base station are evaluated.

Let us consider in more detail the iterative procedure MMSE-OSIC using the example of a multi-user MIMO system consisting of a base station equipped with 4 receiving antennas and two users, each of which has 2 transmitting antennas.

Step 1. The overall matrix of the reversed channel has a dimension of 4×4. The transformation matrix $\tilde{\mathbf{G}}(i)$, where i – iteration number, has a dimension of 4 × 4 and is calculated as follows:

$$\tilde{\mathbf{G}}(i) = \left(\tilde{\mathbf{H}}(i)_{\mathbf{T}}'\tilde{\mathbf{H}}(i)_{\mathbf{T}} + \mathbf{I}\right)^{-1}\tilde{\mathbf{H}}(i)_{\mathbf{T}}'. \quad (6)$$

Step 2. Search for the strongest of the received signals:

$$\alpha_i = \arg\min_p \left\| \left(\tilde{\mathbf{G}}(i)\right)_p \right\|^2 \quad (7)$$

where $\left(\tilde{\mathbf{G}}(i)\right)_p$ is p-th row of the matrix $\tilde{\mathbf{G}}(i)$ and the line with the minimum norm corresponding to the transmitted signal with the highest signal-to-noise ratio.

Step 3. The line found in the second step is stored in the memory of the signal processor and assigned to the p-th row of the matrix $\tilde{\mathbf{G}}_{CLTD-OSIC}$, required for further signal processing on the base station side:

$$\left(\tilde{\mathbf{G}}_{CLTD-OSIC}\right)_p = \left(\tilde{\mathbf{G}}(i)\right)_p. \quad (8)$$

Step 4. Cancellation of interference introduced by the found signal with the maximum signal-to-noise ratio, removing the p-th column from the matrix $\tilde{\mathbf{H}}(i)$:

$$\tilde{\mathbf{H}}(i+1) = \mathrm{del}\left|\tilde{\mathbf{H}}(i)\right|^p, \quad (9)$$

where removes the p-th column of the matrix. The result of the first iteration is the calculation of the p-th row of the matrix. and reducing the dimension of the matrix $\tilde{\mathbf{H}}(i)$ to 4×3, and then the second iteration begins with the first step.

At the last, 4-th iteration for this example, the transformation matrix $\tilde{\mathbf{G}}(4)$ will have a dimension of 1×4 and matrix $\tilde{\mathbf{H}}(i)$ of dimension 4×1. The entire iterative process of the procedure of sequential suppression of interference is completed after the calculation of all rows of the desired matrix $\tilde{\mathbf{G}}_{CLTD-OSIC}$.

$$\tilde{\mathbf{G}}_{CLTD-OSIC} = \begin{bmatrix} \left(\tilde{\mathbf{G}}(1)\right)_{p(1)} \\ \left(\tilde{\mathbf{G}}(2)\right)_{p(2)} \\ \left(\tilde{\mathbf{G}}(3)\right)_{p(3)} \\ \left(\tilde{\mathbf{G}}(4)\right)_{p(4)} \end{bmatrix}, \quad (10)$$

where $\left(\tilde{\mathbf{G}}(i)\right)_{p(i)}$ — designation of the p-th row of the matrix $\tilde{\mathbf{G}}$, calculated at the i-th iteration. The matrix found using the iterative process $\tilde{\mathbf{G}}_{CLTD-OSIC}$ using to form a general precoding matrix for transmitting data on the downlink channel:

$$\mathbf{T} = \frac{\left(\tilde{\mathbf{G}}_{CLTD-OSIC}\right)^T}{\sqrt{tr\left(\tilde{\mathbf{G}}_{CLTD-OSIC}'\tilde{\mathbf{G}}_{CLTD-OSIC}\right)}}, \quad (11)$$

where $tr(\ )$ — the operation of calculating the trace of the matrix. The general precoding matrix consists of channel matrices for each equipment and has the following form:

$$\mathbf{T} = \begin{bmatrix}\mathbf{T}_1 & \mathbf{T}_2 & \dots & \mathbf{T}_K\end{bmatrix}. \quad (12)$$

Result of computer simulations on Fig. 1 provides to compare noise immunity of proposed technique of precoding matrix calculation and known MMSE precoding.

As shown on Fig. 1 implementation of ordered calculation of general precoding matrix on BS side provides to gain 6 dB in comparison with known MMSE precoding.

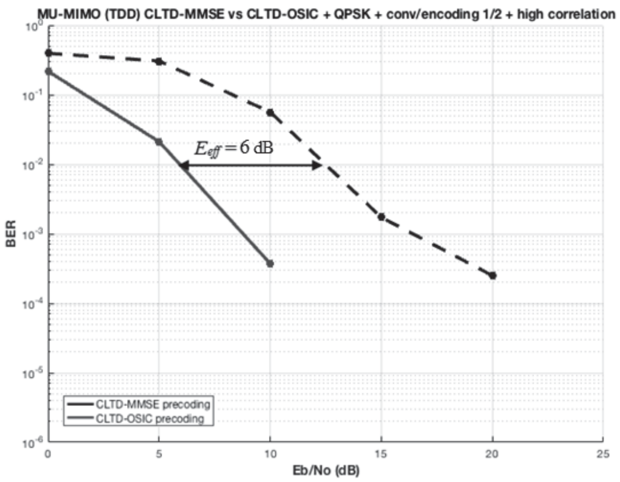

Fig. 1. MU-MIMO system with limited feedback

IV. LIMITED FEEDBACK AND FDD ADOPTATION

Precoding based on partial channel information was used in known standards of wireless communication systems such as LTE and LTE-Advanced [8], [9], [10], [12], where we can find term «codebook-based precoding». The use of such precoding technique requires a codebook, known on the BS and AT side.

Since the presence of feedback channels is necessary for using the precoding procedure, the actual task is to find ways to reduce the amount of resources (temporary or frequency) needed to organize such channels.

The codebook is a set of vectors (codewords) consisting of complex channel multipliers. With a known codebook user compares the channel estimate with each of the codebook vectors, and transmits through the feedback channel to the BS only the index number of the selected vector. Quantization of the channel state information allows the transmission of only certain index numbers through the feedback channel represented by B bits, which determine the result of the calculation of the precoding vectors on the BS side. The number of bits B depends of the codebook $\mathbb{F}$ used on the user side for quantizing the channel state information. For transmission of quantized channel information to the BS side, the CDI (Channel Direction Information) indicator [6] is used. For estimation of signal level on the user side, the CQI (Channel quality information) indicators are used. To evaluate the signal power, user terminals use the SINR — Signal Interference + Noise Ratio.

The structure of general BS's precoding matrix consisting of precoding vectors is shown in Fig. 3.

V. CODEBOOK DESIGN AND SELECTION CRITERIA

User's codebook can be represented as an $L \times N$ matrix $\mathbb{F}$ consisting of L/M_k matrices dimension of $M_k \times N$:

$$\mathbb{F} = \left[\mathbf{F}_1 \quad \mathbf{F}_2 \quad ... \quad \mathbf{F}_L\right], L = 2^B, \tag{13}$$

where $L = 2^B$, l — the index number of the codeword. An illustration of user's codebook structure is shown on Fig. 3.

As shown in Fig. 4, the user equipment codebook is an $L \times N$ dimension matrix, where each row matrix of this codebook is a $L \times N$ codeword. The codewords are used by user equipment as quantized values of channel state information (Fig. 4). The size of the user's codebook depends of the number of bits B transmitted on the feedback channel from the user to BS. High-dimensional codebook requires a larger value of B, and therefore a wider feedback channel, which is not always possible.

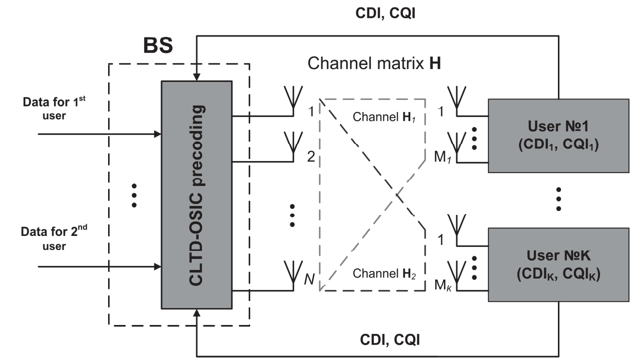

Fig. 2. MU-MIMO system with limited feedback

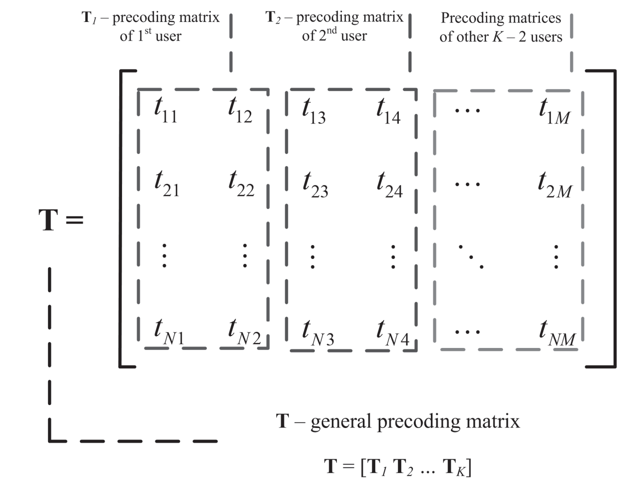

Fig. 3. Structure of general BS's precoding matrix

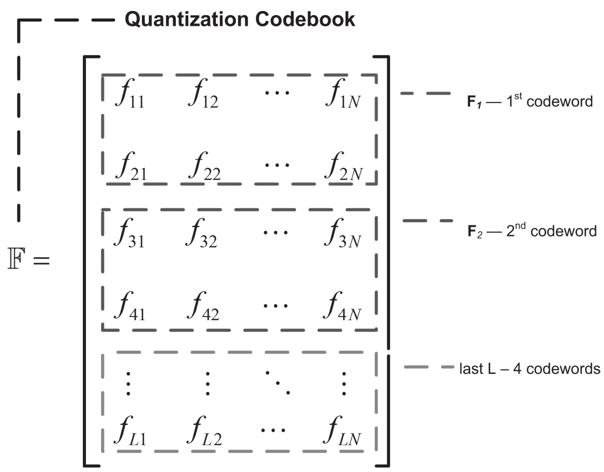

Fig. 4. User codebook structure

Availability of quantized channel vector at base station side makes it possible to calculate or select the precoding matrix. There are two approaches to feedback quantization:

— error calculation between the channel state information and codeword and deciding of precoding matrix on the BS side (Fig. 5);

— selection the precoding vector on the user side by using different criteria.

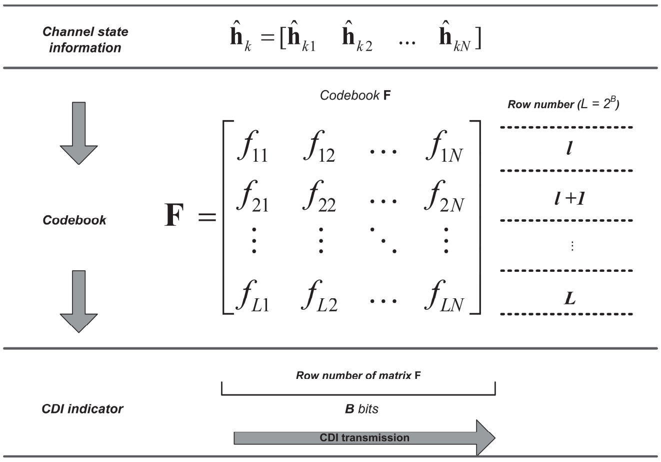

Fig. 5. Process of channel matrix quantization using first approach

In the case of the first approach, the following difference criterion is applicable:

$$\mathbf{F}_l = \arg\max_{\mathbf{F}_l \in \mathrm{F}} \left\| \mathbf{H}_k \tilde{\mathbf{F}}_l \right\|_F . \tag{14}$$

Authors in [8] propose to generate codebooks of user equipment based on grassmann manifold. Grassmannian or grassmann manifold is the set of subspaces of dimension K in N-dimensional complex space. Using of grassmann manifold packing provide to geometrically represent codewords of codebook **F** as matrices of various dimensions in a given multidimensional space. Using of grassmannian package make it possible to arrange codewords represented by subspaces in a multidimensional space using various metrics.

There are different distance metrics between subspaces, but for the case of user's equipment with multiple antenna, the codebook will consist of Grassmannian subspaces (matrices). In this case, the distance metric is applicable:

- Fubini-Study metric*:*

$$d_{FS}(\mathbf{F}_l, \mathbf{F}_m) = \arccos\left|\det\left(\mathbf{F}_m^H \mathbf{F}_l\right)\right|, \tag{15}$$

- Chordal distance metric:

$$d_{ch}(\mathbf{F}_l, \mathbf{F}_m) = \frac{1}{\sqrt{2}} \left\| \mathbf{F}_l \mathbf{F}_l^H - \mathbf{F}_m \mathbf{F}_m^H \right\|_F . \tag{16}$$

Quantizing information about the state of the channel using the Grassmannian manifold allows us to significantly reduce the amount of information transmitted via the feedback channel, therefore, reducing the required throughput of the dedicated feedback channel.

The capacity C of the MIMO system directly depends on the bandwidth of the "downlink" channel B, which can be increased by reducing the amount of information about the channel status.

We analyzed typical subscriber scenario in a vehicle moving with speed v_{user} = 120 km/h (33 m/s), what corresponds to the scenario of the user moving in the vehicle (Extended Typical Urban model - ETU). Given the high speed of user movement, we calculate the Doppler shift f_d taking into account the carrier frequency f_0 = 3 GHz:

$$f_d = f_0 \frac{v_{\text{light}}}{v_{\text{user}}} = 3 \cdot 10^9 \frac{33}{3 \cdot 10^8} = 330 \text{ Hz} . \tag{17}$$

With the 4x2x2 antenna configuration, the full information about the channel status of one user is represented by a 2 × 4 matrix, each bit of which requires 16 bits to represent.

Representing of full channel matrix of user equipped with two antenna required $B_{2\text{Rx full}}$ = 128 bits, which should be transferred in a time span Δt, much smaller than the fade correlation interval τ_{corr} (Fig. 6).

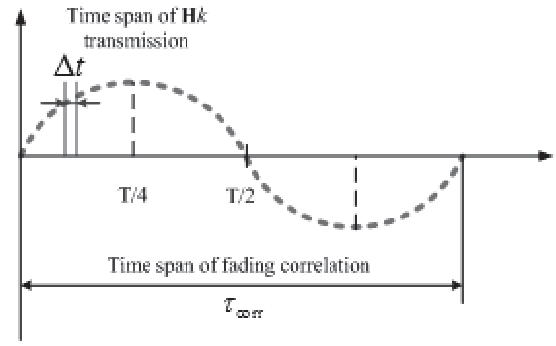

Fig. 6. Time span of channel matrix transmission

We will take the interval Δt =T/100 and calculate the required speed in the dedicated feedback channel for periodic CSI transmission to the base station side.

In the case of a multi-user system consisting of a base station equipped with four transmit antennas and two users, each of them is equipped with two antennas, it is required M_k×N×16 bits. For the 4x2x2 antenna configuration, full information about the channel status of the subscriber terminal is presented by $B_{\text{MMSE full}}$ = 128 bits. For periodic transfer of full CSI to the base station side, feedback channel capacity is required:

$$\begin{aligned} C_{\text{MMSE full}} &= B_{\text{MMSE full}} \cdot \frac{1}{\Delta t} \cdot f_d = 128 \cdot 100 \cdot 330 = \\ &= 4,22 \cdot 10^6 \text{ bps} = 4,22 \text{ Mbps}. \end{aligned} \tag{18}$$

Quantizing the channel state information using the Grassmannian manifold for systems with multi-antenna user equipment and FDD allows reducing the amount of information necessary to present information about the state of the communication channel to $B_{CLTD-OSIC}$ = 4 bits. Required transmission rate of the limited feedback channel for the CLTD-OSIC algorithm:

$$C_{CLTD-OSIC} = B_{CLTD-OSIC} \cdot \frac{1}{\Delta t} \cdot f_d = 4 \cdot 100 \cdot 330 = 132 \text{ Kbps}. \quad (19)$$

The CLTD-OSIC precoding algorithm reduces the required data transfer rate in a dedicated feedback channel more than 30 times in comparison with the case of transmitting full CSI on base station side (for 4x2x2).

VI. Simulation results

The results of computer simulation, presented in Fig. 7, 8, 9 allow to compare results of noise immunity of MU-MIMO systems with proposed non-linear precoding technique using and MMSE precoding based on full channel matrix knowledge in different conditions. Simulation parameters are shown in Table I.

TABLE I. SIMULATION PARAMETERS

Channel	MIMO rayleight fading channel
Antenna configuration	4x2x2
Number of antennas on BS side	4
Number of antennas on user side	2
Number of users	2
Transceiver architecture	V-BLAST
Channel encoding	Convolution encoding (rate 1/2)
Precoding technique	CLTD-OSIC
Number of feedback bits	6 bits
Demodulator	MMSE
Modulation	QPSK
Number of experiments	100000
Kronecker channel	High correlation model

Simulation results on Fig. 7 provide to compare noise immunity of mentioned algorithm in rayleight fading channel without channel encoding.

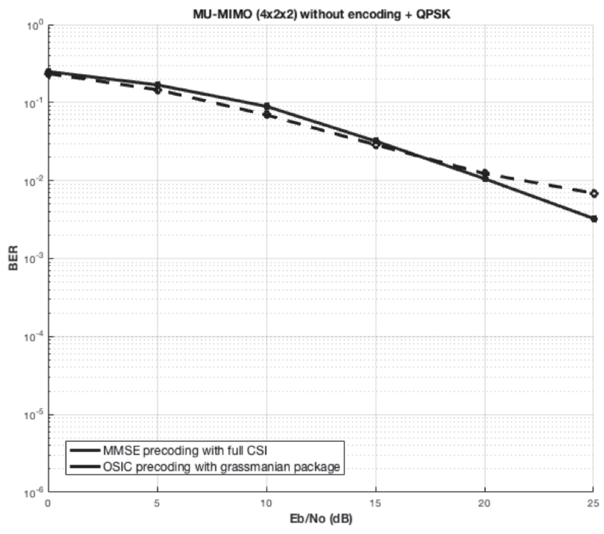

Fig. 7. Simulation results

As shown on Fig. 7 proposed non-linear precoding technique provides to save noise immunity of MMSE precoding with full CSI with loses only on high levels of E_b/N_o. Herewith capacity of feedback channel using non-linear precoding technique reduced down to 132 Kbps in comparison with 4,22 Mbps which needeed to transmit full CSI to base station side. Loses in noise immunity of channel matrix quantization aplliying are compensated by using non-linear ordered calculation of general precoding matrix on base station side.

Simulation results on Fig. 8 provide to compare noise immunity of mentioned algorithm in rayleight fading channel with convolutional encoding (rate 1/2).

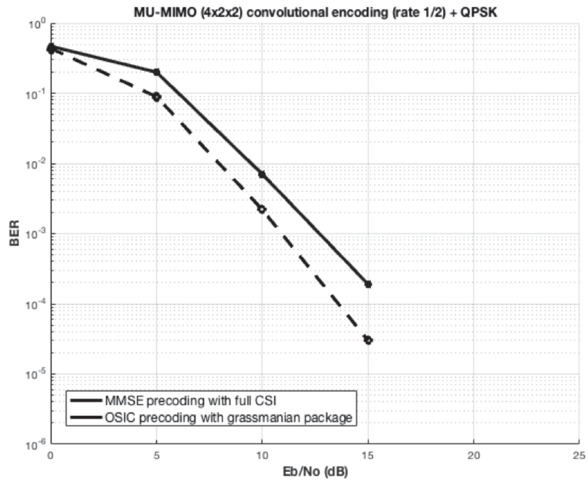

Fig. 8. Simulation results

As shown on Fig. 8 using of proposed non-linear ordered precoding technique based on grassmannian manifold [8] provides to gain 2 dB of the energy efficiency in comparison with known MMSE precoding technique based on full channel matrix knowledge.

Simulation results on Fig. 9 provide to compare noise immunity of mentioned algorithm in rayleight fading channel with convolutional encoding (rate 1/2) and Kronecker model of fading correlation.

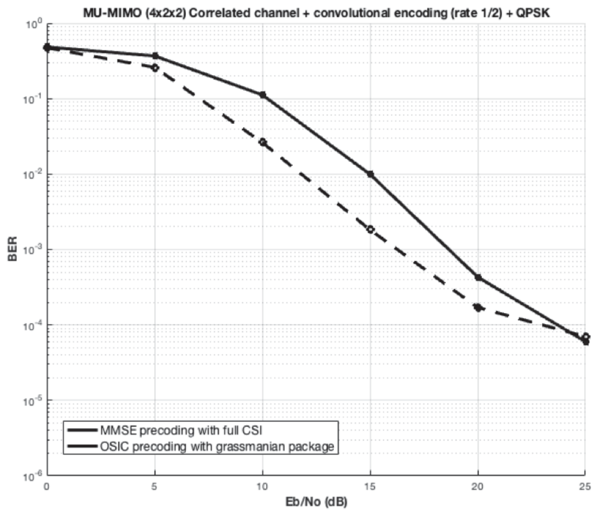

Fig. 9. Simulation results

As shown on Fig. 9 using of proposed non-linear ordered precoding technique based on grassmannian manifold [8] provides to gain 2 dB of the energy efficiency in comparison with known MMSE precoding technique based on full CSI despite high level of fading correlation.

Achieved results of implemented non-linear iterative precoding technique are interesting for further research in using of modulation alphabet based codebooks application.

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