Linear and Nonlinear Chebyshev Iterative Demodulation Algorithms for MIMO Systems with Large Number of Antennas

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Abstract—**Massive MIMO technology (MIMO technology with large number of antennas) is planned for use in 5G networks for significant capacity increasing. However, on the way of Massive MIMO application in 5G systems, there are many problems. Some problems are related to the development of digital signal processing algorithms with good quality characteristics and low computational complexity. In this paper Chebyshev linear and nonlinear iterative demodulation algorithms for MIMO systems are discussed along with Zero-Forcing (ZF) algorithm and minimum mean square error (MMSE) algorithm. For the comparison of these algorithms BER performance characteristics were obtained. The nonlinear algorithm shows a gain of about 1 dB compared to MMSE and has same computational complexity for the case of 64 antennas.**

I. Introduction

In the conditions of increasing the transmitted information volume, the requirements for the throughput of radio communication systems are also increasing. MIMO (Multiple Input Multiple Output) technology [1], [2], which uses multiple transmitting and multiple receiving antennas, is widely used to significantly increase the capacity of modern radio communication systems. One of the ways to increase the throughput of radio communication systems, which does not require the use of additional resources, is the use of highly efficient digital signal processing algorithms.

Massive MIMO technology (i.e. MIMO technology with a large number of antennas) is planned for use in 5G networks in order to increase the capacity. This technology allows to obtain high spectral efficiency and high energy efficiency of the radio communication systems. However, on the way of Massive MIMO application in 5G systems, there are many problems. Some problems are related to the development of digital signal processing algorithms with good quality characteristics and low computational complexity.

In this paper Chebyshev linear and nonlinear iterative demodulation algorithms for MIMO systems are discussed along with Zero-Forcing (ZF) algorithm and minimum mean square error (MMSE) algorithm. For the comparison of these algorithms BER performance characteristics were obtained. The noise immunity characteristics of these algorithms for the MIMO system with different number of transmitting and receiving antennas were obtained – dependences of bit error ratio (BER) on the signal-to-noise ratio (SNR).

II. Model of MIMO system

Fig. 1 shows the MIMO block diagram. The model of the signal at the input of the demodulator is the following [2]:

$$\boldsymbol{y} = \boldsymbol{Hs} + \boldsymbol{n} \tag{1}$$

where $\mathbf{y}$ – received signals vector of $M \times 1$ dimension; $\boldsymbol{H}$ – complex matrix of MIMO radio channel of $M \times M$ dimension; $\boldsymbol{s}$ – vector of transmitted information symbols of $M \times 1$ dimension; $\boldsymbol{n}$ – Gaussian random vector of noise of $M \times 1$ dimension. Elements $\boldsymbol{h}_{ij}$ of MIMO channel matrix $\boldsymbol{H}$ represent the complex transmission coefficients from the *j-th* transmitting antenna to the *i-th* receiving antenna.

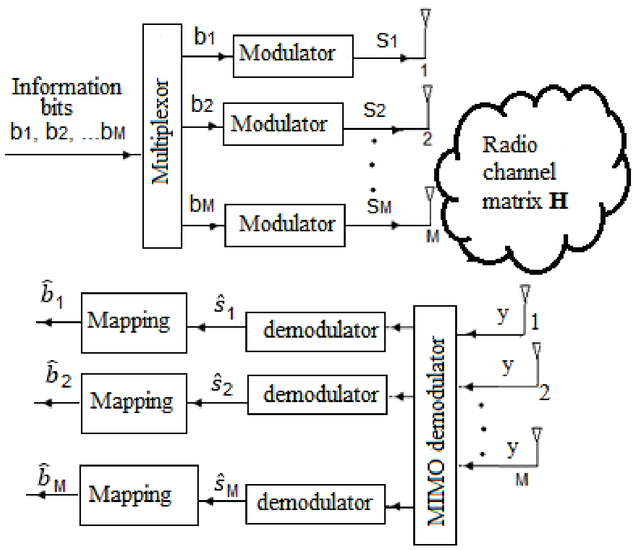

Fig. 1. MIMO system block diagram

The algorithm for MIMO system simulation has several steps: generation of bit vector $\mathbf{b}$ and information symbol vector $\boldsymbol{s}$ for all transmitting antennas, generation of MIMO channel matrix $\boldsymbol{H}$, consisting of complex coefficients $\boldsymbol{h}_{ij}$; generation of complex Gaussian noise vector $\boldsymbol{n}$ and obtaining of its mixture with signal $\boldsymbol{s}$; demodulation of signal $\mathbf{y}$ and obtaining of information symbol vector estimation $\hat{\mathbf{s}}$ and corresponding vector $\hat{\mathbf{b}}$; comparing of $\hat{\mathbf{b}}$ и $\mathbf{b}$ vectors and errors detection. Finally, bit error ratio (BER) is calculated for given signal-to-noise ratio (SNR) value. These steps are performed for a given number of experiments and SNR values to achieve average BER performance for different SNR values. A detailed description of this algorithm can be found in paper [3].

III. Linear demodulation algorithms

In MIMO systems ZF (Zero Forcing) algorithm can be used for demodulation. The ZF estimation of information symbols is the following [2], [4]:

$$\hat{\mathbf{s}}^{ZF} = \arg\min_{s\in C^I} \|\mathbf{y} - \mathbf{Hs}\|^2 = (\mathbf{H}'\mathbf{H})^{-1}\mathbf{H}'\mathbf{y} \tag{2}$$

where C^I denotes I-dimensional continuous complex space; $\hat{\mathbf{s}}$ – estimation of received information symbol vector $\mathbf{s}$; $\boldsymbol{y}$ – received signals vector which has dimension $M\times 1$; $\mathbf{H}$ – complex matrix of MIMO radio channel which has dimension $M\times M$; $(\mathbf{H}'\mathbf{H})^{-1}\mathbf{H}'$ – pseudoinverse matrix with respect to the channel matrix $\mathbf{H}$; $\mathbf{H}'$ – Hermitian conjugate matrix with respect to the channel matrix $\mathbf{H}$. It can be seen from equation (2) that Zero Forcing demodulation algorithm does not take into account the presence of noise $\mathbf{n}$, which results in a significant loss in noise immunity (Fig. 2).

Let us now consider minimum mean-square error (MMSE) demodulation algorithm. The estimation $\hat{\boldsymbol{\theta}}^{\mathbf{MMSE}}$, optimal by MMSE criteria, can be found as follows [2], [4]:

$$\hat{\boldsymbol{\theta}}^{\mathbf{MMSE}} = [\mathbf{H}'\mathbf{H} + 2\boldsymbol{\sigma}_{\boldsymbol{\eta}}^{2}\cdot\mathbf{1}]^{-1}\mathbf{H}'\mathbf{y}\,, \tag{3}$$

where $2\boldsymbol{\sigma}_{\boldsymbol{\eta}}^{2}$ – complex noise dispersion, $\mathbf{1}$ – identity matrix.

Algorithm (3) takes into account the presence of noise in radio channel and therefore it has a higher noise immunity compared to ZF algorithm. This can be seen in Fig. 2, which shows the dependences of the average BER for ZF and MMSE algorithms for different SNR values. Simulation was performed for MIMO system with 8 transmitting and 8 receiving antennas for 10,000 experiments. The same figure shows the linear Chebyshev algorithm for the case of 16 iterations, which is considered further.

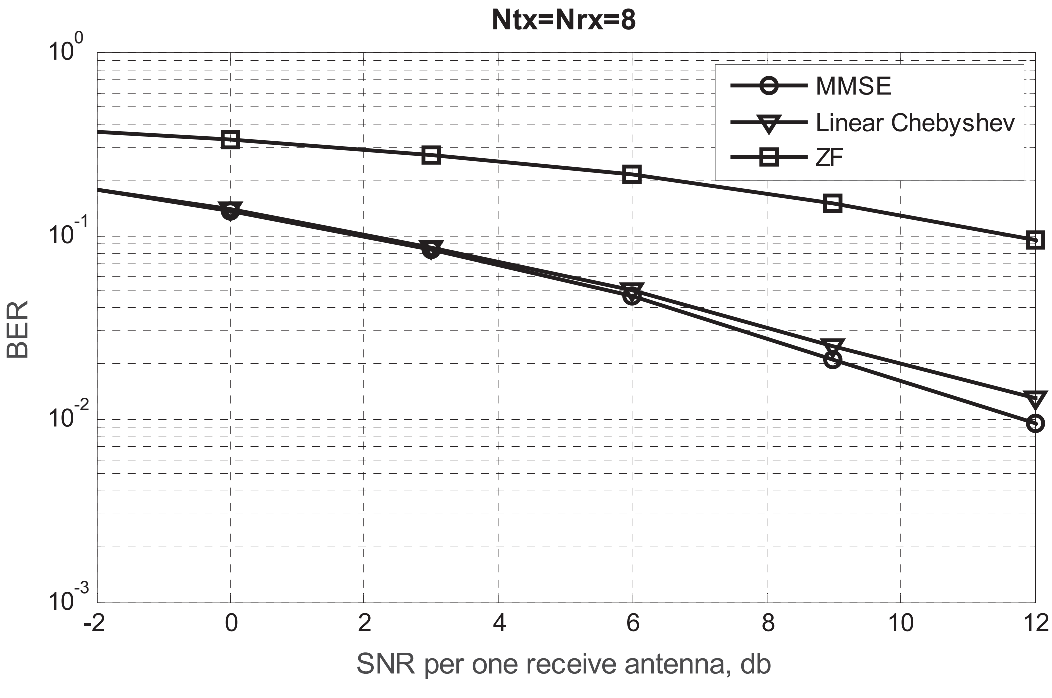

Fig. 2. BER performance for ZF and MMSE demodulation algorithms and Chebyshev linear iterative demodulation algorithm for the case of 8 transmitting and 8 receiving antennas

To implement these demodulation algorithms, it is necessary to calculate the inverse matrix. This task is a very difficult in real-time conditions for MIMO systems with large number of antennas. However, it is impossible to calculate in advance the inverse of MIMO channel matrix, which depends on the complex transmission coefficients of radio channel, since matrix $\mathbf{H}$ varies randomly. Thus, in case of large number of transmitting and receiving antennas, the use of these demodulation algorithms in practice becomes difficult for implementation.

IV. LINEAR CHEBYSHEV ITERATIVE DEMODULATION ALGORITHM

This problem can be solved using iterative methods that have less computational complexity than MMSE algorithm. These methods iteratively solve a system of linear equations $\mathbf{H}\hat{\mathbf{s}} = \mathbf{y}$, using some initial approximation $\hat{\mathbf{s}}_0$. Approximate solution is consistently calculated during several steps (iterations) [5], [6].

Let us consider Chebyshev iterative algorithm. The iteration scheme (4) with variable iteration parameters (5) is called the Chebyshev iteration scheme [5]:

$$\frac{\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_{i-1}}{\tau_i} + \mathbf{H}\hat{\mathbf{s}}_{i-1} = \mathbf{y}, \tag{4}$$

$$\gamma_1\mathbf{E} \le \mathbf{H} \le \gamma_2\mathbf{E},\ \gamma_1 > 0,$$

$$\tau_i = \frac{\tau_0}{1+\rho_0\mu_i},\ i = 1,2,...,n,\ \tau_0 = \frac{2}{\gamma_1+\gamma_2},$$

$$\rho_0 = \frac{1-\xi}{1+\xi},\quad \xi = \frac{\gamma_1}{\gamma_2}, \tag{5}$$

where γ_1, γ_2 – minimum and maximum eigenvalues of matrix $\mathbf{H}$, $\mu_i = \cos\frac{2i-1}{2n}\pi$ – the zeros of the Chebyshev polynomial $\mathrm{T}_n(x) = \cos(n \arccos x)$ on the interval $-1 \le x \le 1$. In this case, the following estimate takes place:

$$\|\mathbf{H}\hat{\mathbf{s}}_n - \mathbf{y}\| \le q_n \|\mathbf{H}\hat{\mathbf{s}}_0 - \mathbf{y}\|,\ q_n \le \varepsilon, \tag{6}$$

where $q_n = \frac{2\rho_1^n}{1+\rho_1^{2n}}$, $\rho_1 = \frac{1-\sqrt{\xi}}{1+\sqrt{\xi}}$. Let us write the expression for the parameters τ_i:

$$\tau_i = 2/[\gamma_2+\gamma_1+(\gamma_2-\gamma_1)(\cos\frac{2i-1}{2n}\pi)\ . \tag{7}$$

$$i = 1,2,...,n$$

Let us rewrite the expression (7) of a sequence of iterative parameters with a given maximum number of iterations $i_{\max}$, associated with the roots of Chebyshev polynomials, in the following form [4]:

$$\tau_i = \left[\frac{\gamma_2-\gamma_1}{2}\cos\left(\frac{i-1/2}{i_{\max}}\pi\right) + \frac{\gamma_2+\gamma_1}{2}\right]^{-1}, \tag{8}$$

where γ_2, γ_1 – maximum and minimum eigenvalues of the channel matrix, $i_{\max}$ – maximum number of iterations, which is used in the Chebyshev demodulation algorithm.

Fig. 3 shows the noise immunity dependencies $BER = f(SNR)$, obtained by MMSE demodulation algorithm and linear Chebyshev iterative algorithm using expression (8) for the parameters τ_i for the case of 32 iterations at different SNR values. The simulation was carried out for MIMO system with 32 transmitting and 32 receiving antennas with a number of experiments equal to 10,000. The same figure shows nonlinear Chebyshev algorithm for the case of 32 iterations, which is considered further.

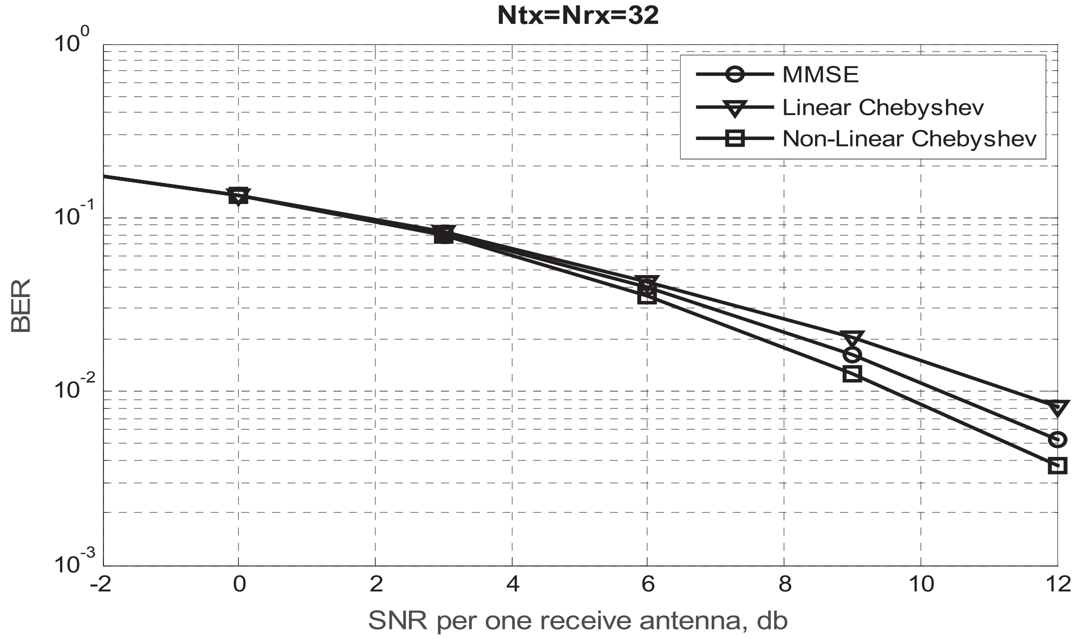

Fig. 3. BER performance of MMSE demodulation algorithm and linear and nonlinear Chebyshev iterative demodulation algorithms for the case of 32 transmitting and 32 receiving antennas

To increase the computational stability of algorithm (4) – (7) the order is important, in which the zeros of the Chebyshev polynomial are taken. The rule for constructing such a sequence of parameters $\{\tau_i\}$ of (8) is known [4],[5], for which the convergence of the iterative method is monotonic and there is no computational instability. From the sequence of parameters (8), it is necessary to form a permutation containing only the values of the iterative parameters with odd numbers and arrange them in a certain order as follows [4], [5]:

$$\theta_1 = \{1\},\ \theta_{2i-1}^{(2m)} = \theta_i^{(m)},\ \theta_{2i}^{(2m)} = 4m - \theta_{2i-1}^{(2m)} \qquad (9)$$

For example, optimal permutation for 8 iterations is the following [5]: {1; 15; 7; 9; 3; 13; 5; 11}.

For 16 iterations optimal permutation is the following [5]: {1;31;15;17;7;25;9;23;3;29;13;19;5;27;11;21}.

Using the permutations, better convergence with the same number of iterations can be obtained. The rate of convergence of the iterative process (4) - (7) with the permutation of the iterative parameters (9) can be found from the following expression [5]:

$$n = n_0(\varepsilon) = \frac{1}{2\sqrt{\xi}} \ln\frac{2}{\varepsilon}, \qquad (10)$$

where $n_0(\varepsilon)$ – number of iterations sufficient to solve the system of linear equations with a given accuracy $\varepsilon > 0$.

Linear Chebyshev method provides a high rate of convergence, but to determine the iterative parameters, a priori knowledge of the minimum and maximum eigenvalues γ_2, γ_1 of MIMO channel matrix $\mathbf{H}$ is necessary, which limits its applicability.

Instead of eigenvalues, it is advisable to use their approximate estimates. The following expressions are known for estimating the eigenvalues of the Hermitian matrix [6]:

$$\gamma_1 \approx \min_{1\le i\le M} \sqrt{\sum_{j=1}^{N} T_{ij}},\ \gamma_2 \approx \max_{1\le i\le M} \sqrt{\sum_{j=1}^{N} T_{ij}}, \qquad (11)$$

where T_{ij} - the element of matrix **T**. Matrix **T** is defined as follows:

$$\mathbf{T} = \mathbf{H}'\mathbf{H} + 2\sigma_\eta^2 \cdot \mathbf{1}, \qquad (12)$$

where $2\sigma_\eta^2$ – complex noise dispersion, $\mathbf{1}$ – identity matrix,– $\mathbf{H}'$ Hermitian conjugate matrix with respect to matrix $\mathbf{H}$.

However, the considered linear demodulation algorithms with a relatively low implementation complexity lose in terms of the noise immunity characteristics compared to the demodulator that is optimal by the maximum likelihood (ML) criteria. The expression for the demodulator, that is optimal by ML criteria is the following [2]:

$$\hat{s}^{M\Pi} = \arg\min_{s\in\Theta^I} \|\mathbf{y} - \mathbf{Hs}\| \qquad (13)$$

where Θ is a discrete set of values of a single complex information symbol and Θ^I is a discrete set of values of complex information symbols vector **s**. The difference $\mathbf{y} - \mathbf{Hs}$ is called discrepancy. Maximum likelihood estimate minimizes the square of the discrepancy norm. The computational complexity of ML algorithm grows exponentially with the number of antennas [6], [7], [8].

Thus, the algorithm that is optimal by the maximum likelihood criteria for the case with large number of antennas cannot be implemented due to the extremely high computational complexity. Simple linear algorithms — the ZF algorithm, the MMSE algorithm, and the Chebyshev linear iterative algorithm do not use important a priori information about the discreteness of the set of vector information symbols, which explains the loss in noise immunity.

It follows that the actual task is the development of demodulation algorithms taking into account a priori information about the possible values of information symbols and with acceptable computational complexity having noise immunity, close to the noise immunity of ML demodulator. In order to improve the characteristics of linear iterative demodulation algorithms and bring them closer to the potentially possible, reaching the noise immunity values of ML algorithm, it is necessary to go beyond the linear algorithms and consider nonlinear algorithms. There are approaches that allow to synthesize nonlinear iterative algorithms for demodulation of discrete signals [4], [6].

V. NONLINEAR CHEBYSHEV ITERATIVE DEMODULATION ALGORITHM

Consider the demodulation algorithm based on the non-linear Chebyshev method. The fundamental difference of this algorithm from linear Chebyshev iterative algorithm is the presence of a nonlinear function $f(\hat{\mathbf{s}})$ at each iteration. This nonlinear function depends on the type of modulation used. In our case, this is the function of hyperbolic tangent

$$f(...) = th(\frac{\hat{s}_{i-1}}{D}) \qquad (14)$$

where *D* is the parameter of the nonlinear algorithm.

Nonlinear Chebyshev iterative demodulation algorithm can be written in the following form:

$$\hat{s}_i = \hat{s}_{i-1} + \tau_{i-1}(\mathbf{H}'\mathbf{Y} - (\mathbf{H}'\mathbf{H} + 2\sigma_\eta^2 \cdot \mathbf{1})th(\frac{\hat{s}_{i-1}}{D}) \qquad (15)$$

Fig. 4 shows the noise immunity dependencies obtained for MMSE algorithm, linear and nonlinear Chebyshev iterative algorithms for MIMO system with 64 transmitting and 64 receiving antennas for the case of 32 iterations and 10,000 experiments. As can be seen from the noise immunity characteristics, the gain for the nonlinear Chebyshev algorithm is about 1 dB compared to the MMSE algorithm and about 2 dB compared to the linear Chebyshev algorithm (with an SNR of more than 8 dB). It should be noted that computational complexity for these algorithms is about the same. Fig. 4 shows that with an increase in SNR, an increase in noise immunity is observed.

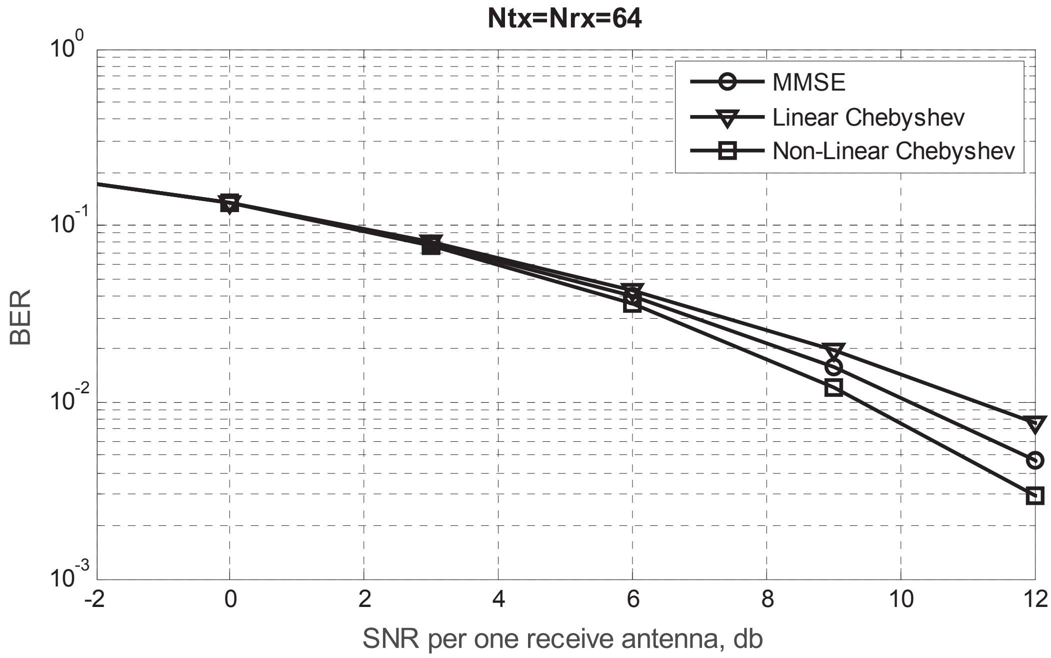

Fig. 4. BER performance of MMSE demodulation algorithm and linear and nonlinear Chebyshev iterative demodulation algorithms for the case of 64 transmitting and 64 receiving antennas

VI. SIMULATION OF CONSIDERED ALGORITHMS

For the analysis of digital signal processing algorithms along with the theoretical analysis computer simulation is often used [2], [4]. In this paper, the results of the analysis of the considered algorithms are obtained by computer simulation. Radio communication systems are systems with a large number of elements and functional links between them and various random effects on these elements. At the same time, many different signal conversions with high speed digital signal processing take place.

With the rapid increase in the capabilities of computer technology due to the tremendous growth of the complexity of modern radio communication systems, simulation models are becoming extremely important. The simulation process is a convenient, flexible, powerful tool for developing and analyzing new digital signal processing algorithms. However, for successful simulation of radio communication systems, it is necessary to solve problems related to algorithms optimization — effective methods are needed to form mathematical models of radio communication systems and their elements which have minimal complexity.

During the simulation a radio communication system is usually considered as several functional blocks, each of which is described using a separate program. Thus, a large and complex system is a collection of simple blocks, the use of which makes it possible to study them in detail and obtain the desired characteristics for analyzing the system.

In this paper, we simulate demodulation algorithms for a MIMO system with different numbers of antennas to evaluate the performance of these algorithms. As a result of the simulation, the noise immunity characteristics of the MIMO system for different demodulation algorithms (the dependences of BER versus SNR) were obtained. The number of experiments was chosen to ensure the required accuracy of the simulation (L=10000).

TABLE I shows simulation algorithm for the considered linear and nonlinear Chebyshev iterative demodulator with the sequence of parameters τ_i. The simulation results are the immunity characteristics of the MIMO system for different numbers of antennas and different numbers of iterations.

TABLE I SIMULATION ALGORITHM OF LINEAR AND NONLINEAR CHEBYSHEV DEMODULATORS FOR MIMO SYSTEEM

Step number	Simulation program operations	Variables that are used in the program
1.	Start of signal-to-noise ratio cycle	*SNR*
2.	Start of experiments cycle	*L=10000*
3.	Transmitting antennas cycle	*M*
	Generation of a uniformly distributed random variable (for each antenna)	*x*
	Bit generation (1 or 0) from *x*	*b*
	Modulation (generation of information symbol for each antenna)	*s*
4.	End of the cycle (step 3)	
5.	Formation of a vector from generated information symbols for all transmitting antennas	***s***
6.	MIMO channel matrix generation, consisting of complex transmitting coefficients	***H***
7.	Generation of complex Gaussian noise vector	***n***

8.	Simulation of signal and noise mixture	$\boldsymbol{y} = \boldsymbol{Hs} + \boldsymbol{n}$
9.	Calculation of the sequence of iterative parameters. Application of the Chebyshev method for demodulation	τ_i
10.	Star of iterations cycle	i_max
11.	Obtaining estimate on the i-th iteration	$\hat{\mathbf{s}}_i$
12.	End of cycle (step 10)	
13.	Demodulation (obtaining estimate of the received vector of information symbols)	$\hat{\mathbf{s}}$
14.	Obtaining estimate of received bits vector	$\hat{\mathbf{b}}$
15.	Comparing vectors $\hat{\mathbf{b}}$ и $\mathbf{b}$ and detection of errors	*errors*
16.	Number of errors calculation	*sum*
17.	End of cycle (step 2)	
18.	Bit error ratio calculation for each SNR value	*BER*
19.	End of cycle (step 1)	
20.	Plotting bit error ratio versus signal-to-noise ratio for different numbers of antennas	$BER = f(SNR)$

VII. Conclusion

Future 5G systems [1], [9], [10] are aimed at significant increase in throughput, including using Massive MIMO technology. Iterative demodulation algorithms based on Chebyshev method are planned to be used for MIMO systems with large number of antennas, as well as for signals with higher modulation order. The main limitation of Chebyshev iteration method applicability is the need for a priori knowledge of MIMO channel matrix **H** spectrum boundaries (maximum and minimum eigenvalues), and with decreasing accuracy of these values, the convergence of the algorithm slows down. The task of finding estimates for the eigenvalues of MIMO channel matrix and applying the obtained estimates in demodulation algorithms requires additional research.

More attractive are nonlinear algorithms, which have the same order of computational complexity and allow to get closer to the potential characteristics. Chebyshev nonlinear iterative demodulation algorithm provides a gain of about 1 dB compared to the MMSE demodulator and about 2 dB compared to linear Chebyshev iterative demodulator, moreover, its characteristics do not get worse with an increase in the number of antennas. The gain in the considered nonlinear demodulation algorithm is achieved by using a nonlinear function that takes into account the discreteness of information symbols set.

However, the development of demodulation algorithms with characteristics close to ML demodulator but have much less computational complexity, requires further research [7], [9]. In addition, it is necessary to study nonlinear Chebyshev iterative demodulator for signals with high modulation order and for larger number of antennas, as well as for MIMO radio channels with spatially correlated fading.

Finally, it should be noted that this report is useful for analyzing the characteristics of MIMO with a large number of antennas and for a deeper understanding of the iterative algorithms for demodulating signals in MIMO systems. For very high requirements for 5G systems, it is important to use such demodulation algorithms that will increase the spectral efficiency and energy efficiency of such systems and have acceptable computational complexity. But on the way to the practical use of 5G systems, there are a number of difficulties [1], [9], [10], for the solution of which these research are directed.

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