

# Memristor-Based Chaotic Circuits for Inductive Sensing

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**Abstract**—In the current study, we investigate the possibility of constructing sensors based on chaotic circuits with memristors. The study involves different memristor models, both simple and more physically realistic. We demonstrate that changes in the inductance cause notable changes in the behavior of memristive chaotic circuits, which can be detected and converted into a measuring signal. Thus, memristors can serve as the physical basis for the creation of highly sensitive sensors of a new generation, extremely compact and with ultra-low power consumption.

## I. INTRODUCTION

Sensors exploiting dynamical chaos could have several advantages over convenient sensors, such as a higher sensitivity, exceeded selectivity, simpler design and less power consumption. However, for a long time, studies in this area were mostly academic, and only recently significant practical results have been achieved. Fiderer and Brown successfully used chaos to enhance integrable quantum sensors [1]. The authors' magnetometer demonstrated near twofold improvement of sensitivity. Chaotic sensors of simple construction also gain researchers' interest. Korneta et al. [2] showed that a well-known Chua circuit can operate as a bistable sensor with an almost linear transfer function, which is suitable for many practical applications. With that, issues of chaotic circuits design using innovative electronic components have been so far little addressed. For example, there is a possibility of constructing simple chaotic circuits using memristors – passive elements suitable for creating systems with complex behavior and ultra-low power consumption, tenfold lower than its transistor-based counterparts consume [3]. Memristor production technology is being enhanced from year to year, and the first commercial memristors are already available. Some simple chaotic circuits with the theoretical memristor and the model based on titanium oxide memristor were proposed recently [4–6]. Thus, following the idea of [2], it becomes possible to investigate these circuits for their sensitivity to external signals or intrinsic parameters changes. Besides memristors, all of these circuits contain inductive elements and thus can serve as inductive sensors. Circuit designs developed in [4], [5] and [6] are presented in Fig. 1, as well as their attractors.

The paper is organized as follows. In section II, chaotic circuits with memristors under investigation are presented. In section III, analysis methods are listed. Section IV describes the results and discusses the advantages of chaotic circuits over harmonic oscillators. Section V concludes the paper.

## II. CHAOTIC CIRCUITS WITH MEMRISTORS

### A. Memristor concept

Memristor (an acronym for “memory resistor”) is an electronic component that changes its resistance in dependence on the charge flowing through it. Whereas a resistor can be characterized by the equation

$$v = Ri,$$

a memristor is defined by Leon Chua as [4]

$$\begin{aligned} v &= R(x)i \\ \dot{x} &= f(x, i) \end{aligned}$$

where  $R(x)$  is a memristance, a function of internal state  $x$ , and  $f(x, i) \Leftrightarrow f(x, q, t)$  is an internal state function providing memory effect. In an ideal memristor definition,  $R(x) \Leftrightarrow R(\phi(t))$  states the relationship between current and flux. Here we may note that some researchers emphasize the absence of magnetic flux in all physical implementations of memristors and thus deny that contemporary physical devices called memristors are those ideal memristors predicted by Chua [7]. Nevertheless, Chua's concept is useful for characterizing the processes occurring in the  $\text{TiO}_2$  memristors of HP lab and other physical implementations.

### B. Three-element chaotic circuit

In work [4], the simplest chaotic electric circuit with a memristor is proposed, see Fig. 1 (a). It consists of only three components, one of which is an active memristor. Various memristor models may be used. Two elements, capacitor and inductance, form the first two lines of equation, and the last equation defines the internal state of a charge-controlled memristor. The circuit is described by equation:

$$\begin{cases} \dot{v}_C = \frac{i_L}{C} \\ \dot{i}_L = \frac{1}{L}(-v_C + R(x)i_L) \\ \dot{x} = f(x, i_L) \end{cases}$$

The parameter values are  $C=1$ ,  $L=3$ ,  $\beta=\overline{1..2}$  and  $\alpha=0.6$ .

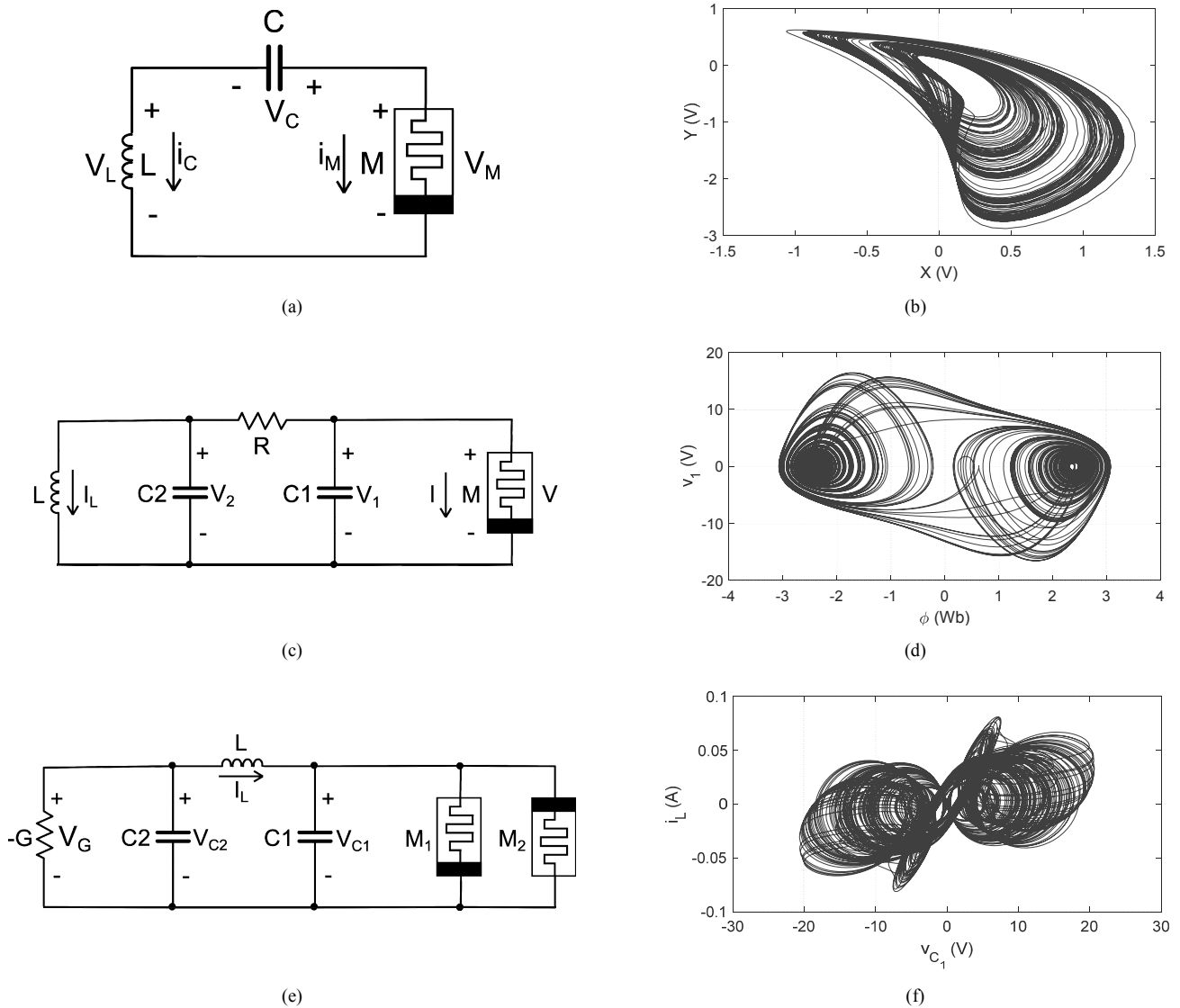


Fig. 1. Chaotic circuits and attractors: the simplest chaotic circuit with memristor by Muthuswamy and Chua [4] (a) and its attractor (b), Chua circuit with memristor by Muthuswamy [5] (c) and its attractor (d), Chua circuit with antiparallel HP memristors by Buscarino et. al. [6] (e) and its attractor (f)

For this particular case, we select  $R(x) = \beta(x^2 - 1)$ ,  $f(x, i) = i - \alpha x - ix$ . Value  $\beta = 1.7$  provides chaos. The attractor is shown in Fig. 1 (b).

C. Chua circuit with theoretical memristor

The paper by [5] proposes an analog realization of a flux-controlled memristor and a chaotic circuit based on it, see Fig. 1 (c). The design is based on Chua's circuit and replaces Chua diode with a flux-controlled memristor. The memductance in this circuit is selected based on the cubic nonlinearity, which can be used to create chaos in the Chua circuit [5]:

$$i = W(\phi)v = (\alpha + 3\beta\phi^2)v$$

$$\dot{\phi} = (1/\zeta)v$$

Overall circuit equation can be derived using Kirchhoff's laws and is the following:

$$\begin{cases} \dot{\phi} = \frac{-v_1}{\zeta} \\ \dot{v}_1 = \frac{1}{C_1} \left( \frac{v_2 - v_1}{R} - W(\phi)v_1 \right) \\ \dot{v}_2 = \frac{1}{C_2} \left( \frac{v_2 - v_1}{R} - i_L \right) \\ \dot{i}_L = \frac{v_2}{L} \end{cases}$$

The circuit is designed using realistic values:  $L = 18$  mH,  $C_1 = 6.8$  nF,  $C_2 = 68$  nF,  $R = 2.5$  k $\Omega$ , while the chaotic mode is obtained in  $R = 2$  k $\Omega$ .

For  $\alpha = -0.667 \cdot 10^{-3}$ ,  $\beta = 0.029 \cdot 10^{-3}$ . The parameter  $\zeta$  is the rescaling factor to set realistic of voltages in the circuit. It is chosen as  $\zeta = 8200\Omega \cdot 47 \cdot 10^{-9}\text{nF}$ . Attractor is shown in Fig. 1 (d).

#### D. Chua circuit with HP memristor

In previously mentioned works ideal memristors are considered. Assuming that physical memristors will have characteristics similar to those of the memristor created in HP laboratory in 2008, Buscarino and coauthors proposed the circuit based on Chua's design that involves this type of memristors, see Fig. 1 (e) [6]. HP memristors are passive non-symmetrical elements having a nonlinearity different from cubic or piece-wise linear. HP memristor model, known as the linear drift model, is described by the following equations:

$$\begin{aligned} v &= (R_{ON}x + R_{OFF}(1-x))i \\ \dot{x} &= \frac{\mu_v R_{ON}}{D^2} F(x, i) \end{aligned}$$

Typical values are  $R_{ON} = 100\ \Omega$ ,  $R_{OFF} = 10\ \text{k}\Omega$ ,  $\mu_v = 10^{-14}\ \text{cm}_2\text{s}^{-1}\text{V}^{-1}$ ,  $D = 10^{-9}\ \text{nm}$ . Memristor internal state variable  $x$  is strictly limited to  $[0, 1]$ . To incorporate this into the model, a window function is introduced, for example, Biolek window:

$$F_B(x, i) = 1 - (x - \text{stp}(-i))^{2p}$$

Parameter  $p$  is usually set to 1. Note that the HP memristor model provides asymmetrical nonlinearity. Connecting two memristors in antiparallel, i.e. parallel with one memristor having negative polarity, we can obtain symmetrical characteristics required for chaos to emerge in this circuit. The full equation of the circuit is the following

$$\begin{cases} \dot{v}_{C_1} = \frac{1}{C_1}(i_L - i_{M_1} - i_{M_2}) \\ \dot{i}_L = \frac{1}{L}(v_{C_2} - v_{C_1}) \\ \dot{v}_{C_2} = \frac{1}{C_2}(Gv_{C_2} - i_L) \\ \dot{x}_1 = \frac{\mu_v R_{ON}}{D^2} F_B(x_1, i_{M_1}) i_{M_1} \\ \dot{x}_2 = -\frac{\mu_v R_{ON}}{D^2} F_B(x_1, -i_{M_2}) i_{M_2} \end{cases}$$

where the memristors' currents are

$$i_{M_k} = \frac{v_{C_1}}{R_k(x_k)}, k = \{1, 2\}.$$

The parameters providing chaotic behavior are the following:  $R_G = G^{-1} = 714\ \Omega$ ,  $L = 1.6\ \text{H}$ ,  $C_1 = 25\ \mu\text{F}$ ,  $C_2 = 33\ \mu\text{F}$ . Resistor  $R_G$  is actually a negative resistor, i.e. a negative impedance converter which powers the circuit. Attractor of the circuit is presented in Fig. 1 (e).

### III. CHAOTIC CIRCUITS RESPONSE TO INDUCTANCE CHANGES

In order to study the presented circuits, we created their computer models first. The simulation was performed in MATLAB environment. We used custom ODE solver implementing the semi-implicit composition numerical method that is well suited for simulating memristive systems [8]. Classic fast solvers, such as Runge-Kutta 2 method, could lead to false results [9]. During the main part of the study, the values of inductances in the considered models were varied near the nominal value ( $\delta L = L / L_{nom}$ ). We used four methods of analysis, presented in Fig. 2 and Fig. 3.

1) To visualize dynamics changes in the time domain, a standard nonlinear dynamics tool was used, namely the bifurcation diagram (BD).

2) Another type of plot was used to analyze changes in the frequency domain. We called it a *bifurcation spectrogram* (BS). Unlike conventional spectrograms used to analyze non-stationary signals, in this type of diagrams a bifurcation parameter (inductance) is plotted along the horizontal axis instead of conventional time. Also, BS can reveal useful properties of the system. As in ordinary bifurcation diagrams, BS clearly shows chaotic and periodic regimes of systems in dependence on the bifurcation parameter.

3) Geometric parameters analysis. As was shown in [10], the geometric parameters of the attractor depend directly on the  $L$  and thus by calculating the distance between the attractor external points relative to one of the state variables provides valuable information about sensor response.

4) Mean and median frequency analysis. Algorithms can be found in [11].

Mean frequency is calculated from the frequencies of the power spectra by the equation:

$$\bar{f} = \frac{\sum_{n=0}^{n=N} P_n f_n}{\sum_{n=0}^{n=N} P_n},$$

where  $N$  is the number of points of Fourier transform,  $f_n$  is the frequency  $n$  and  $P_n$  is the power of frequency  $n$ .

### IV. RESULTS AND DISCUSSION

The analysis shows that the dynamics of all three systems changes significantly with the inductance change. It was discovered that the geometric parameters of the three-element chaotic circuit attractor are monotonically related to inductance, in contrast to the other two systems, on the plots of which there both monotonic and strongly non-linear sections exist. In the frequency domain, high monotonicity is found in Chua systems. The change of mode from chaotic to periodic

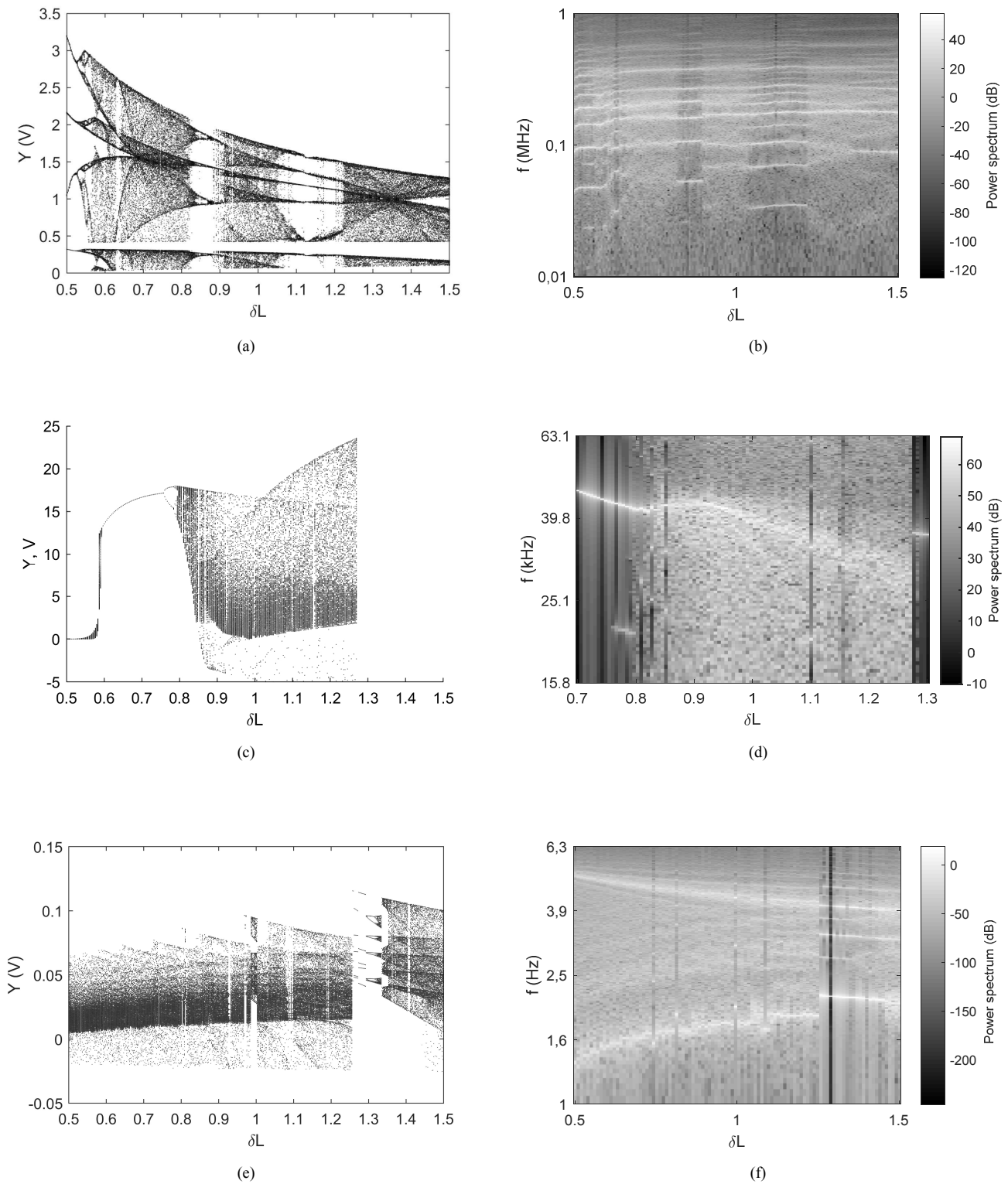


Fig. 2. Bifurcation diagrams (left) and bifurcation spectrograms (right) of the proposed systems: three-element chaotic circuit (a), (b), Chua circuit with theoretical memristor (c), (d), Chua circuit with HP memristor (e), (f). When the inductance  $L$  changes from 0.5 to 1.5 of its nominal value, the systems regime changes, including sections of periodic and chaotic oscillations. On the bifurcation spectrograms, the frequency maxima can be seen (light blurred strips), as well as regions of periodic behavior (light distinct strips over darker background).

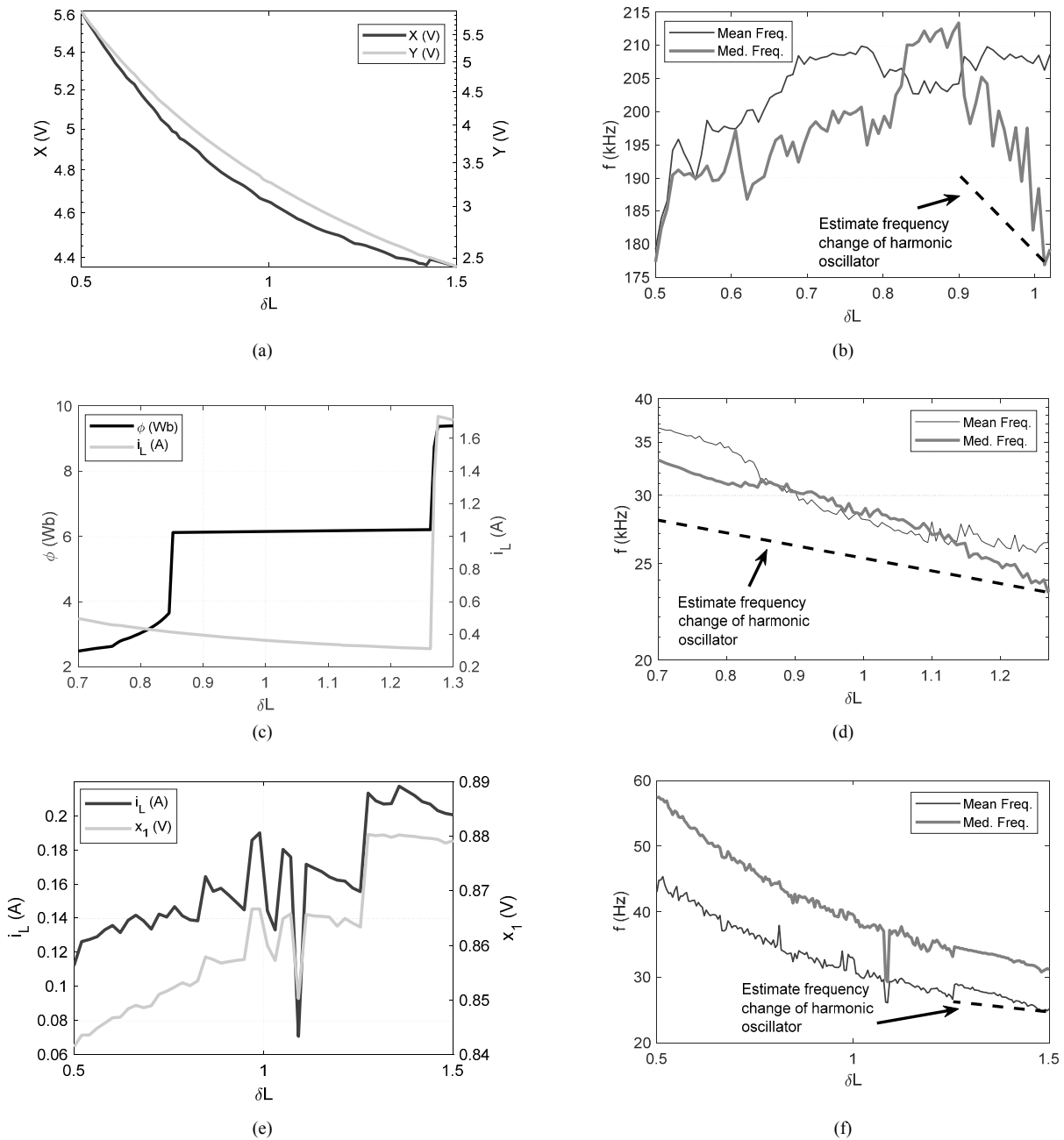


Fig. 3. Geometrical parameters analysis (left) and frequency analysis (right) of the proposed systems: three-element chaotic circuit (a), (b), Chua circuit with theoretical memristor (c), (d), Chua circuit with HP memristor (e), (f)

and vice versa can either affect the indicated parameters or not, see the bifurcation diagrams.

The plots of mean and median frequencies (Fig. 3 (b), 3 (d), 3(f)) also present a comparison of memristor chaotic circuits sensitivity with the harmonic oscillator sensitivity. The frequency of the harmonic oscillator is calculated by the equation:

$$f = (2\pi\sqrt{LC})^{-1}.$$

It leads to the expression for calculating the  $f - L$  curve:

$$f_x = f_0 \sqrt{\frac{\delta L_x}{\delta L_0}},$$

where  $(\delta L_0, f_0)$  is the reference point in the inductance-frequency plane, and  $f_x$  is the frequency at  $\delta L_x$ . Sections of these curves are presented in Fig. 3 (b), 3 (d), 3 (f). It can be seen that all three circuits have the steeper  $f - L$  curves and thus are more sensitive in the sense of frequency response.

The determination of the measured parameter (i.e. inductance) from chaotic oscillations can be performed from

the bifurcation diagram or spectrogram, geometric or frequency properties of the oscillations. The last two are of the greatest interest due to the simplicity of their calculation and can provide a high accuracy of measurements [10]. To calculate the distance from inductance one may use expressions for the specific coil. For the planar PCB coil from [12]:

$$L(d) = \frac{-2.764 \cdot 10^{-7}}{d} + 1.231 \cdot 10^{-4} \text{ H,}$$

where  $d$  is the distance to the metal plate, m. The inverse formula is:

$$d = \frac{-2.764 \cdot 10^{-7}}{1.231 \cdot 10^{-4} (1 - \delta L)}, \tag{1}$$

where  $\delta L$  is the relative inductance change from the nominal value. Consider that the circuit by Buscarino et. al. [6] is rescaled so as the value  $L_0 = 123.1 \mu\text{H}$  is observed when no target is detected. A simple technique of circuit rescaling is proposed, for example, in [12] and [13]. According to the plot in Fig. 3e and the expression (1), we may find the following distances, see Table I. Individually, geometric dimensions are not the monotonic functions from inductance but  $\delta L$  can be reliably determined by a combination of these two metrics. An example of the frequency-based range estimation for the same circuit according to Fig. 3f is given in Table II. Note that median and mean frequencies change much faster in the chaotic circuit than in the harmonic oscillator when inductance changes.

TABLE I. ESTIMATED RANGE TO THE TARGET USING GEOMETRIC ANALYSIS OF THE ATTRACTOR OF CHUA CIRCUIT WITH HP MEMRISTORS

$x_1, \text{V}$	$i_1, \text{A}$	$\delta L$	Range, m
0.866	0.19	0.99	0.22
0.857	0.144	0.95	0.044
0.856	0.153	0.9	0.022
0.852	0.138	0.8	0.011
0.848	0.133	0.7	0.008

TABLE II. ESTIMATED RANGE TO THE TARGET USING FREQUENCY ANALYSIS OF THE OSCILLATIONS IN CHUA CIRCUIT WITH HP MEMRISTORS

Mean. freq., Hz	Med. freq., Hz	$\delta L$	Range, m
33	39	0.99	0.22
31.8	40	0.95	0.044
32.7	42	0.9	0.022
34.5	44.4	0.8	0.011
37.8	47.7	0.7	0.008

V. CONCLUSIONS

In this paper, we show the fundamental possibility of chaotic circuits with memristors to serve as inductive sensors, i.e. sensors detecting the changes in inductance. The use of memristors as components may result in extremely low-consumption devices that are of practical interest for various sensing applications.

Three chaotic circuits with memristors are considered, including a circuit with the realistic HP memristor model. We show that the sensitivity of all circuits is higher than the sensitivity of a harmonic oscillator. Determination of the

inductance by oscillations signal processing can be performed by calculating the geometric parameters of the attractor or the average (median) oscillation frequency. An example of range estimation using these techniques is given, considering that the chaotic circuit serves as a proximity sensor.

The limitation of this work is that memristors are still poorly accessible for engineering applications, thus, presented results are not yet practically applicable.

In further studies, we will implement an inductive sensor based on memristor emulators. Other chaotic circuits with memristors will be considered and developed. Nonlinear models of the considered memristive sensor circuits will be used as the test nonlinear systems with complicated phase and fractal properties for studying numerical ODE solvers.

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