A Step-Shaped Hierarchical QAM

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***Abstract*—In this paper, a step-shaped hierarchical quadrature amplitude modulation with two high-priority bits, 4/*K*-stepped *θ*-QAM, is proposed for $K = 2^{2k}$ and $k \geq 3$. We provide a construction method of the signal constellation and present a bit-to-symbol mapping for the proposed 4/*K*-stepped *θ*-QAM. Through computer simulations, we present bit error rate performance of 4/*K*-stepped *θ*-QAM in single-input single-output and multi-input multi-output systems by adopting adaptive QR decomposition-*M* detection and compare with that of the conventional hierarchical constellations.**

I. Introduction

In recent communication and broadcasting systems, as the quality of information, e.g., resolution and frame rate, increases, a large amount of data processing is needed. Moreover, development of technologies such as 5G and autonomous driving requires communication in various situations with high speed and high reliability.

In general, quadrature amplitude modulation (QAM) has been mainly adopted in communication and broadcasting systems that require large data transmission, since it can improve data rate without sacrifice in bandwidth. In particular, square QAM (SQAM) having simple modulation and demodulation methods has been used in many practical systems [1]-[3]. Although, SQAM provides low complexity in modulation and demodulation, it does not offer optimal error performance. Hence many studies have been conducted on constellation design to minimize the error probability. In [4], for instance, *θ*-QAM with the versatile structure exploiting the angle between signal points was proposed which has better error performance than SQAM. In [5], stepped *θ*-QAM with a step-shaped constellation was proposed, which provides better error performance than *θ*-QAM. Meanwhile, to guarantee the reliability of basic communication, Cover proposed to divide the transmitted data into more than two classes and to assign different degrees of protection to data according to their relative importance [6]. Motivated by [6], various studies have been conducted on hierarchical modulation which divides important data into high-priority (HP) bits and less important data into low-priority (LP) bits to achieve this idea in practical systems [7]-[10].

In addition to constellation design, multi-input multi-output (MIMO) technology is well known as one of the methods to improve the data rate and reliability of communication and broadcasting systems. In MIMO system, since the complexity and error performance change drastically depending on a detection method, many researchers have mainly focused on finding a detection method with low complexity and good error performance. Maximum likelihood (ML) detection provides optimal error performance, but its computational complexity increases exponentially as the number of antennas or modulation order increase [11]. To reduce the complexity, zero-forcing (ZF) and minimum mean square error (MMSE) detections were proposed, but they have significantly low error performance compared to ML detection [12]. Recently, in [13], adaptive QR decomposition-*M* (QRD-*M*) detection providing near ML performance with significantly low complexity was proposed.

In this paper, we propose a step-shaped hierarchical QAM with two HP bits, 4/*K*-stepped *θ*-QAM, where $K = 2^{2k}$ and $k \geq 3$, based on stepped *θ*-QAM. We provide a construction method of constellation and present bit-to-symbol mapping for 4/*K*-stepped *θ*-QAM. By adopting adaptive QRD-*M* detection, we present bit error rate (BER) performance of 4/*K*-stepped *θ*-QAM in single-input single-output (SISO) and MIMO systems and compare with that of the conventional hierarchical constellations.

II. 4/*K*-Stepped *θ*-QAM

4/*K*-stepped *θ*-QAM, $K = 2^{2k}$, $k \geq 3$, with two HP bits is constructed based on stepped *θ*-QAM. Due to the facts that QPSK constellation is embedded into hierarchical constellation which has two HP bits and the number of signal points in each quadrant is the same in stepped *θ*-QAM constellation, we first separate the signal points of stepped *θ*-QAM into four signal point sets with different HP bits according to the quadrants in which they are located.

Let $\mathbf{S}_m$ be the signal point set of *m*-th quadrant in stepped *θ*-QAM. We denote each signal point in $\mathbf{S}_m$ by s_m^1, s_m^2, ⋯, s_m^p from left to right, top to bottom where $p = K / 4$ and denote the coordinate pairs of signal point s_n^q by (x_n^q, y_n^q). As an example, Fig. 1 shows signal points of 64-ary stepped *θ*-QAM and their coordinate pairs when $\theta = 60°$. Then, 4/*K*-stepped *θ*-QAM is constructed by moving the coordinate pairs of signal point s_n^q, (x_n^q, y_n^q), by ($+\lambda$, $+\lambda$), ($-\lambda$, $+\lambda$), ($-\lambda$, $-\lambda$), and ($+\lambda$, $-\lambda$) when n = 1, 2, 3, and 4, respectively, where $\lambda \geq 0$ is a constant.

We define $\alpha = d_h / d_l$ as the constellation parameter which controls the relative degrees of protection of data where $\alpha \geq 1$, $2d_h$ is the minimum distance between two signal points in

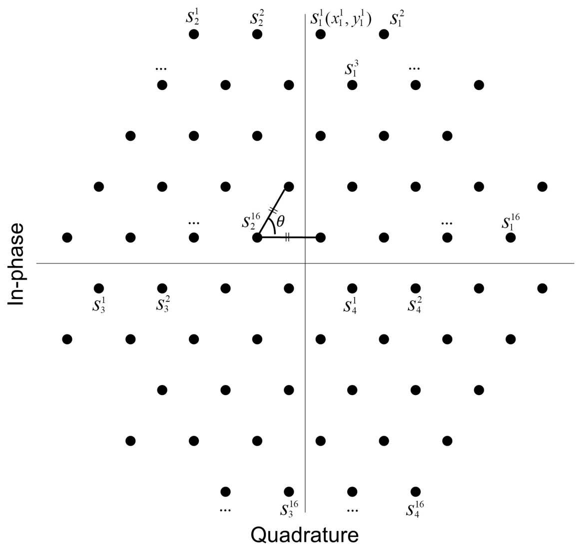

Fig. 1. Signal points of 64-ary stepped θ-QAM when $\theta = 60°$.

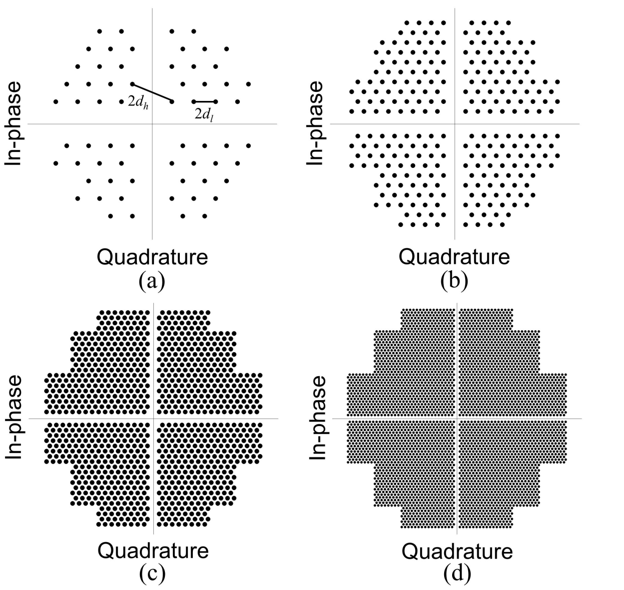

Fig. 2. Signal constellations of 4/K-stepped θ-QAM when $\alpha = 2$ and $\theta = 60°$. (a) $K = 64$. (b) $K = 256$. (c) $K = 1024$. (d) $K = 4096$.

adjacent quadrants, and $2d_l$ is the minimum distance between two signal points within one quadrant. We can formulate the relationship between α and λ as

$$\alpha = \frac{d_h}{d_l} = \frac{\sqrt{\lambda^2 + d_l\lambda + d_l^2}}{d_l}. \tag{3}$$

We depict the signal constellations of 4/K-stepped θ-QAM when $\alpha = 2$ and $\theta = 60°$ in Fig. 2.

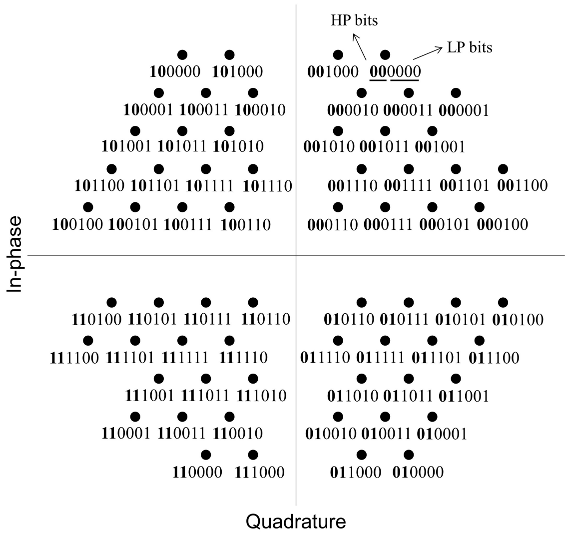

Fig. 3. Bit-to-symbol mapping of 4/64-stepped θ-QAM when $\alpha = 2$ and $\theta = 60°$.

For the modulation scheme, it is necessary to map binary bit streams into signal points in an appropriate manner. In the sense of minimum bit errors for a given symbol error, the optimum bit-to-symbol mapping can be found through a full search, but it is impracticable for the high modulation orders since it involves $K!$ possible mappings. As an alternative, [14] proposed a suboptimum bit-to-symbol mapping scheme called layer labeling algorithm which can be applied to high modulation orders, and by adopting the layer labeling algorithm, [15] presented bit-to-symbol mapping of stepped θ-QAM.

We also adopt the layer labeling algorithm and obtain bit-to-symbol mapping of 4/K-stepped θ-QAM. Since QPSK constellation is embedded in 4/K-stepped θ-QAM where two HP bits are mapped, we first assign two HP bits to the most significant bit (MSB) position, i.e., 00, 10, 11, 01 are assigned to each quadrant in sequence. Then, by using the layer labeling algorithm, the remaining bits are assigned where we assume search limit $N_e = 9$ in the algorithm of [14]. As an example, we present a bit-to-symbol mapping of 4/64-stepped θ-QAM when $\alpha = 2$ and $\theta = 60°$ obtained from the layer labeling algorithm in Fig. 3.

III. Simulation Results

In this section, we present the BER performance of 4/K-stepped θ-QAM in SISO and 2×2 MIMO systems. To detect the transmitted signal from the received signal in MIMO system, we adopt adaptive QRD-M detection proposed in [13], where the survival candidate signal points are determined at each detection layer based on a threshold to exclude unreliable candidate signal points. We consider a Rayleigh fading channel for simulation and include the results of the conventional 4/K-QAM in [9] for comparison.

Fig. 4 shows the BER versus E_b/N_0 of 4/K-QAM and 4/K-stepped θ-QAM, when $\alpha = 2$, $\theta = 60°$, and $K = 64$ and 256 in SISO system. As shown in Fig. 4, 4/K-stepped θ-QAM achieves power gains in HP bit over 4/K-QAM. Note that,

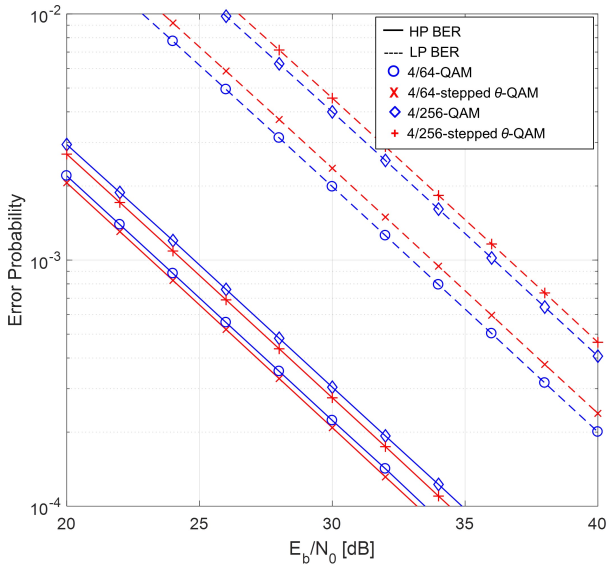

Fig. 4. BER of 4/K-QAM and 4/K-stepped θ-QAM when $\alpha = 2$, $\theta = 60°$, and $K = 64$ and 256 in SISO system.

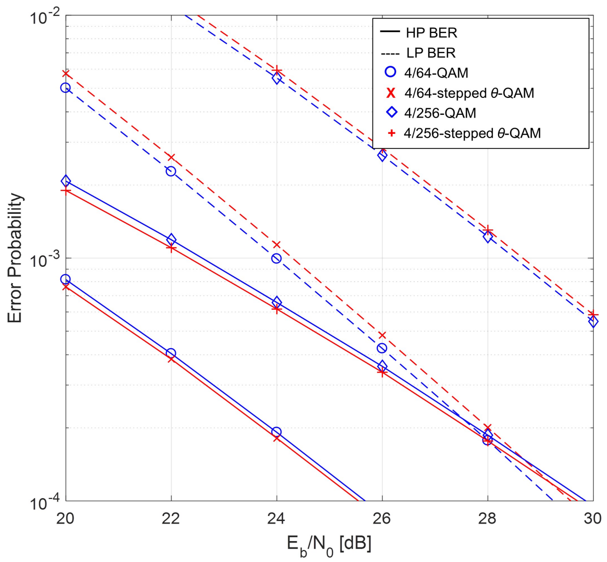

Fig. 5. BER of 4/K-QAM and 4/K-stepped θ-QAM when $\alpha = 2$, $\theta = 60°$, and $K = 64$ and 256 in 2 × 2 MIMO system.

although 4/K-stepped θ-QAM has power losses in LP bit over 4/K-QAM, the primary purpose of the hierarchical modulation is to provide better protection for HP bit.

Fig. 5 shows the BER versus E_b/N_0 of 4/K-QAM and 4/K-stepped θ-QAM, when $\alpha = 2$, $\theta = 60°$, and $K = 64$ and 256 in 2 × 2 MIMO system. From Fig. 5, we can see a similar tendency with the case of SISO system as shown in Fig. 4, where 4/K-stepped θ-QAM provides power gains in HP bit over 4/K-QAM.

V. Conclusion

In this paper, we proposed a step-shaped hierarchical QAM with two HP bits, 4/K-stepped θ-QAM, where $K = 2^{2k}$ and $k \geq 3$, based on stepped θ-QAM. We provided a construction method of constellation and presented bit-to-symbol mapping for 4/K-stepped θ-QAM. We then provided the BER performance of 4/K-stepped θ-QAM in SISO and MIMO systems by adopting adaptive QRD-M detection and showed that 4/K-stepped θ-QAM offers better error performance in HP BER than the conventional 4/K-QAM.

We considered the case of $\theta = 60°$ as an example. Further work is required to analyze the error performance according to θ to find the optimal angle in terms of minimum error probability.

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References

[1] *Digital Video Broadcasting (DVB); Framing structure channel coding and modulation for a second generation digital terrestrial television broadcasting system (DVB-T2)*, ETSI EN 302 755 v.1.4.1, Jul. 2015.
[2] *IEEE Standard for Information Technology—Telecommunications and Information Exchange Between Systems Local and Metropolitan Area Networks—Specific Requirements—Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, IEEE Standard 802.11-2016, 2016.
[3] *LTE; Evolved universal terrestrial radio access (E-UTRA); Physical channels and modulation*, ETSI TS 136 211 v.14.2.0, Jan. 2018.
[4] K. N. Pappi, A. S. Lioumpas, and G. K. Karagiannidis, "θ-QAM: A parametric quadrature amplitude modulation family and its performance in AWGN and fading channels," *IEEE Trans. Commun.*, vol. 58, no. 4, pp. 1014-1019, Apr. 2010.
[5] J. Lee, H. Hong, and D. Yoon, "Stepped θ-QAM for high-order modulation," *Electron. Lett.*, vol. 53, no. 25, pp. 1676-1678, Dec. 2017.
[6] T. Cover, "Broadcast channels," *IEEE Trans. On Inform. Theory*, vol. IT-18, pp. 2-14, Jan. 1972.
[7] K. Ramchandran, A. Ortega, K. M. Uz, and M. Vetterli, "Multiresolution broadcast for digital HDTV using joint source/channel coding," *IEEE J. Select. Areas Commun.*, vol. 11, Jan. 1993.
[8] L. F. Wei, "Coded modulation with unequal error protection," *IEEE Trans. Commun.*, vol. COM-41, pp. 1439-1449, Oct. 1993.
[9] P. K. Vitthaladevuni and M. S. Alouini, "BER computation of 4/*M*-QAM hierarchical constellations," *IEEE Trans. Broadcast.*, vol. 47, no. 3, pp. 228-239, Sep. 2001.
[10] J. Lee, K. Cho, and D. Yoon, "New BER Expression of Hierarchical M-ary Phase Shift Keying," *ETRI Journal*, vol. 29, no. 6, pp. 707-715, Dec. 2007.
[11] M. O. Damen, H. E. Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2389-2402, Oct. 2003.
[12] Y. Jiang, M. K. Varanasi, J. Li, "Performance Analysis of ZF and MMSE Equalizers for MIMO Systems: An In-Depth Study of the High SNR Regime," *IEEE Trans. Inform. Theory*, vol. 57, no. 4, pp. 2008-2026, Apr. 2011.
[13] H. Lim, Y. Jang, T. Li, and D. Yoon, "Improved QRD-M algorithm based on adaptive threshold for MIMO systems," in *Proc. IEEE COMSNETS*, Bangalore, India, Jan. 2014.
[14] T. G. Markiewicz, "Construction and labeling of triangular QAM," *IEEE Commun., Lett.*, vol. 21, no. 8, pp. 1751-1754, Aug. 2017.
[15] S. Ahn and D. Yoon, "BER performance of stepped θ-QAM," in *Proc. IEEE ICCIS*, Singapore, Singapore, Dec. 2018.