An Intelligent Method for Comparing Shapes of Three-Dimensional Objects

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***Abstract*—The article discusses a method for comparing the shape of three-dimensional objects, which was developed at the Peter the Great St. Petersburg Polytechnic University. The main method is the consecutive double conversion of the original three-dimensional object to a vector of parameters that are invariant to the initial position, rotation, and size of the explored object. At the first step, three-dimensional objects are transformed by a three-dimensional version of the chord method into a matrix of distances between random points on the surface of the object. Then the matrix is converted into a vector that forms a histogram of distances. The resulting vector is summed up with additional indicators and compared in pairs with objects stored in the database using the Siamese neural network, which at the output gives an estimate for the proximity of the three-dimensional objects' shape. The developed method is the basis of a system for comparing the shape of three-dimensional industrial designs, and is also used in the detection of cancer of the lungs and prostate gland.**

I. Introduction

The article discusses methods for comparing the shape of three-dimensional models. Initially, Peter the Great St. Petersburg Polytechnic University was tasked to develop methods, algorithms and software for the automation of Federal Institute of Industrial Property (FIPS) in terms of receiving, storing and processing applications for registration of intellectual property rights for industrial designs.

In the field of design and patent protection, only two examples of working with three-dimensional models are known:

In South Korea, three-dimensional models are accepted for registration, but in fact, the objects of protection are six flat projections that are standard for drawing. This approach has an evolutionary advantage and its implementation does not require significant restructuring of the expert work system, but the disadvantages include the fact that projection drawings are not primary information. They are generated automatically from a three-dimensional model file with the loss of some of the details contained in this file.

The principles of the legal protection system of the three-dimensional model, similar to South Korean, are declared in Japan, but they also consider flat projections as primary information and have the same drawbacks.

The volume of three-dimensional information is much larger than the traditional two-dimensional one and its storage, transmission, as well as operational analysis are impossible without the use of automated tools for search and comparison of three-dimensional models.

Currently, there are a number of significant problems associated with large amounts of data and the lack of developed general algorithms for the analysis and comparison of three-dimensional models. Moreover, the problem in general cannot be solved by reducing the three-dimensional model to a set of two-dimensional projections. In the literature there are only a few backlogs in specific areas in this field:

A. Simplification of the shape of objects

Simplification of the shape of three-dimensional objects is used in computer-aided design systems to increase productivity. In various software packages for construction some methods are used to simplify the polygonal (a set of faces connected by edges and vertices) shape of 3D objects. This simplification, in principle, makes it possible to identify and compare objects of similar shape, by eliminating the small details of the form.

Most iterative simplification algorithms for polygonal three-dimensional models can be divided into three categories: thinning vertices, folding edges, and thinning faces [1], [2]. There are different methods for determining vertices that could be brought together and moved to some point, so that the number of faces and edges of the polygonal object would reduce and it would become possible to exclude one of the peaks lying close enough to the given. In this case, all edges leading to the excluded vertex are "reconnected" to a new one.

The data presented in [3] shows that the method of thinning vertices can lead to the loss of connectivity of models, and the methods of folding edges and thinning faces can reduce the number of polygons by only 60-70% without significant surface distortions, which contributes to a threefold reduction in the amount of data. But this approach does not allow performing a generalized comparison of three-dimensional models, since the objects cannot be reduced due to the iterative reduction process to the similar comparable form. Comparison is difficult because the known reduction algorithms retain all the edges on which the angle between the normal and the face change significantly. For example, in industry design the car radiator grill will be reduced to a lesser extent than other parts of the car body, and the presence of such small details will make the reduced models uneven and completely incomparable.

B. Three-dimensional face recognition

Experience in 3D model comparison has been gained at digital face recognition. Three-dimensional facial images have a number of advantages over two-dimensional ones. Three main approaches can be distinguished: analysis of the shape's features at the 3D surface of the face, statistical approach and a parametric model of the face.

Methods based on the analysis of the features of the form use directly the geometry of the surface, which describes the face. These approaches can be classified into local and global. Local approaches reveal local features of the face, for example, the tip of the nose, corners of eyes, cheekbones, etc. and compare three-dimensional models according to one or more features. Global approaches work with the image as a whole.

In [4], it was proposed to use surface curvature to segment the face into features that can be used to comparison. The approach described in [5] is based on descriptors of the face surface, which are the average and Gaussian curvature of the surface. As a descriptor in [6] it is proposed to use the distances and the ratio of angles between the characteristic points of the surfaces, like the angles of the eyes, the tip of the nose, the nose bridge point, and the corners of the lips. These characteristic points are distinguished on the face, then a matrix of distances and angles is compiled between these points. These matrices are compared with each other in columns and rows by compiling various norms such as the square of the difference vector. Comparison of individual facial features using signature points is made in [7], [8]. The idea of the method is that around a singular point, for example, such as the tip of the nose, a curve is drawn that is the intersection of a circular cylinder of a given radius and face surface. Then a signature is formed as a function that associates the projection of the surface's normal at the cylinder axis to the angular coordinate of the point. The angle is counted relative to the line of symmetry of the nose and the proximity of the shape is defined as the deviation of two signatures from each other no more than a given small value.

There are also hybrid methods based on combining local surface information in the form of local descriptors or descriptor functions with a global three-dimensional grid that describes the entire face [9], in this case, local descriptors are stored in relation to the intersection lines of the grid and comparisons are made for all intersection points.

Global methods use various descriptors to shape the entire face. In [10], the face was first aligned so that it was mirror symmetric, then the face profiles along the alignment plane were selected and compared. In [11], a comparison of faces was proposed based on the maximum and minimum values of the curvature of the profiles on the plane of symmetry.

Statistical methods represent a flat tone image or the distance from surface points of a three-dimensional object to a secant plane as a random function of the coordinate vector of a point on the plane. The global descriptors of the model are various moments of a random function, for example, the mean value or standard deviation [12], [13], [14]. In a review [15], it was noted that statistical methods give the largest percentage of correct face recognition. Expansions of the function in Fourier series are also applied, the coefficients of which are descriptors for comparison.

The main disadvantage of all the methods of face recognition above is the tight binding to the object. For example, in local approaches, they search for the tip of the nose, and in global approaches they analyze only the face. In our work, when comparing three-dimensional models in which features can be not only on the front side of the model, but also on all six sides, this method loses its attractiveness, since the number of descriptors increases by an order of magnitude. Also it should be mentioned that the question of how to orient the model relative to the secant planes in our case, unlike the human head, does not have a definite answer.

C. Descriptors for comparing three-dimensional models based on sections of a model by a system of parallel planes

The authors of [16], [17] propose a general approach to constructing descriptors for comparing three-dimensional models. It is based on the so-called "hypertrace-transform". This transformation consists in dissecting a three-dimensional object by a system of parallel planes, on each of which a scalar value of a certain descriptor of the section line is determined, for example, the section area. The dependence of the descriptor on the distance of the section plane to an arbitrarily assigned zero point and two Euler angular coordinates characterizing the direction of the normal to the section plane allows us to obtain a function of three coordinates. Further, various characteristics of this function are used, for example, its expansion coefficients in a series or average values.

A similar approach with approximation of the contours of sections by sets of splines was applied in [18]. Efficiency in this case is achieved due to the specific shape of the mannequins, which are all oriented in the same way and have standard horizontal sections in which the size is set (for example, along the "waist"). This makes it possible to turn a three-dimensional object into a finite set of two-dimensional ones and obtain effective descriptors tied to the functional purpose of mannequins.

In the general case, the advantages of replacing a three-dimensional surface with a function of a three-dimensional coordinate vector are not completely clear, no reduction in dimension occurs; therefore, such functions cannot be considered descriptors. In addition, the construction of a set of sections, and approximations by splines, and the calculation of the cross-sectional area require relatively large computational costs, which in the case of a real three-dimensional model containing millions of faces do not allow the use of hyper-trace and similar transformations.

D. Using one or more sets of projections of the model on a plane

As a general approach to the analysis of these 2D projections, one can single out the construction of the descriptor function, which turns a two-dimensional array of image pixels or a polynomial approximation of the boundaries in the case of a two-dimensional vector model into a one-dimensional function of the length of the boundary line of the survey object [19], [20], [21].

This function appears to be a universal measure of form or an object handle. The proximity of the two 2D objects is understood as the small value of the module or the module square of the average difference of their descriptors.

Four important general features of the analysis of two-dimensional images should be noted, which are realized mainly by machine learning methods:

a. The selection of the descriptor function is made heuristically. Its properties, as a rule, are not proved mathematically.

b. Possible ambiguities are investigated by simple enumeration of different images.

c. The specific parameters of the descriptor function can be determined during machine learning.

d. Descriptor comparison criteria are also heuristically determined.

The main conclusion is that two-dimensional approaches are not applicable to three-dimensional models, since they are based on the analysis of a flat one-dimensional boundary of objects. In a three-dimensional model, the boundary will be a surface, and even if formally the procedures developed for two-dimensional images (for example, wavelet analysis) can be applied, the computational costs will be so big that it will not allow processing large three-dimensional data. On the other hand, with appropriate resources, it is possible to use generalized 2D methods for comparing three-dimensional objects. But the application of such approaches requires the development of an appropriate mathematical apparatus.

E. *Special descriptors of three-dimensional surfaces*

Since all the descriptor functions are obtained heuristically, they can be classified according to the construction method. The following classes can be distinguished:

- Methods based on the extraction and classification of features of the form. In fact, they are derived from the classification. Often the parameter is an isomorphism of the classification graph. An important advantage of this method is the ability to explicitly specify both the classification itself and the most important features, the disadvantage is the subjectivity of the estimates. For example, in [22], the classification is determined by the processing technology (on a turning or milling machine), material, the presence of rotation parts in the analyzed object (one, two three, etc. rotation angles, the presence and orientation of holes, etc.). In [23], the graph is based on the gradual removal of primitives from the general shape of the part. When removing the cylinder from the box, a box with a hole is obtained. In [24], on the contrary, the whole volume of the model is decomposed into primitives. In our work, we considered this comparison method, but the operation of decomposing the original object into the set of primitives, according to our estimates, is too complex and often requires a heuristic approach. Also unresolved in this case is the question of determining the initial orientation of the studied object in space. Since the comparison methods using the decomposition of the figure are sensitive to its position on orientation.
- Methods based on the functions of three-dimensional mappings of initial model, for example, mapping a surface onto an inscribed sphere, or onto a single sphere. In [25], the surface of a unit sphere is divided into hexagons, and maps of these hexagons to the surface of the object are obtained. The deformation of each hexagon is calculated when it is mapped from a unit sphere to the surface of the object, which, after converting it to a function of longitude and latitude on the surface of the sphere, become a descriptor. In [26], a surface is mapped onto an inscribed unit sphere, where for each point of the sphere a coefficient is written equal to the distance between the points of the surface and the sphere as a function of two angles (longitude and latitude). The advantage of this method is the formation of a uniform distribution of points of the descriptor function over the surface of the object. It allows tuning the accuracy of the model in proportion to the number of points on its surface. However, this method does not solve the problem of determination the initial position of the object, and also, in the general case, it is applicable only to convex figures without cavities.
- Methods based on the construction of slope diagrams. Diagrams of the angle of inclination for faces and edges are constructed when they are radially projected onto a surface of a unit sphere as a function of two angles (longitude and latitude). In [27], the object's surface is approximated by flat polygons and a descriptor is constructed in the form of a dependence on two angles of inclination of the polygon's plane to the tangent to the surface of the sphere. A complex approach was applied in [28], [29], the volume of the model is divided into primitives using Boolean algebra, each primitive is written as the angular coordinates of the vertices, when the primitive is projected onto the unit circle (actually projected into the two-dimensional space of angles), then Boolean operations are applied to the mapping results in reverse order to get the whole model displayed.
- The use of neural networks. These methods are based on the decomposition of the model into primitives or parts, for each of which the neural network contains an image in the form of a flat bitmap. The descriptor is a neural vector of primitives. Logical manipulations with images, their union and intersection are also performed using a neural network [30], [31], [32].
- Description of surface properties in the form of a graph. The surface of the model approximated by polygons is divided into faces and edges, which may have additional features (concavity, convexity, holes), the line of the graph indicates the common border of the polygons [33]. It is possible to describe the surface using the Reeb graph [34], which involves triangulating the surface (with the desired resolution of the details) and writing a graph in which one or two radii of curvature will be indicated for each triangle. The disadvantage of these approaches is the large dimension of the descriptor.
- Surface statistics. This approach uses a surface metric vector, including the ratio of surface to volume, number of holes, grooves, curvature, etc. to establish similarities in the elements of this vector [35], [36].
- Methods based on a descriptor in the form of a histogram of the distribution of distances between

surface points that are selected using random number generators [37], [38], [39], [40]. As shown by numerous tests, this method is robust, fast in software implementation and allows adequately highlighting similar in form model. In our work, we developed a method for comparing the shapes of three-dimensional objects based on this approach

II. Method for comparing the shapes of 3D objects

Specialists of Peter the Great St. Petersburg Polytechnic University developed a method that allows calculate the degree of proximity of three-dimensional models as a numerical value. The method based on the descriptor in the form of a histogram of the distribution of distances between surface points was chosen as the base one.

An open file format STL was chosen as the main supported format for the analysis and storage of three-dimensional models. It is supported by all the main packages for working with CAD models, including FreeCAD. The STL format has a simple structure and presents data in the form of a list of coordinates of nodes of triangular faces that describe the surface of 3D model.

When a database of three-dimensional models is formed or during the loading of a new model into the system, all models are converted into the STL and JSON formats. Representation of the model in JSON format is a technical solution for use in the three.js library for displaying the model in a browser. The STL format is suitable for processing both by the python language and using the libraries of the freeware package of work with CAD models like FreeCAD.

After the transformation of the model to STL format, a random selection of points for the calculation of the distance matrix is carried out. After selecting the points, distances between all pairs of points are calculated. The result is a triangular distance matrix characterizing the studied object. The resulting matrix is invariant to the position and rotation of the initial object. Due to the fact that the points were chosen randomly, the matrix form of representation is redundant, and the next step is the aggregation of data by a histogram of distances. The largest value is selected in the matrix, all other results are located between 0 and this value. The obtained segment from 0 to the maximum value is divided into a fixed number of intervals and a calculation is made of how many distances from the matrix fell into one or another segment of the histogram. In fact, during the formation of a vector containing a histogram of distances, normalization is performed, since after the aggregation not absolute values of distances are stored, but only a vector characterizing the probability density of the distribution of these intervals.

The similarity of the two models is determined by comparing their histogram vectors. For comparison, it can be used both statistical methods, for example, the Minkowski distance between all columns of the histogram, and intelligent methods, for example, Siamese neural networks.

Fig. 1 schematically shows the procedure for transforming the model. In the second step, only 6 points are shown, since a large number of connections will make the drawing unreadable.

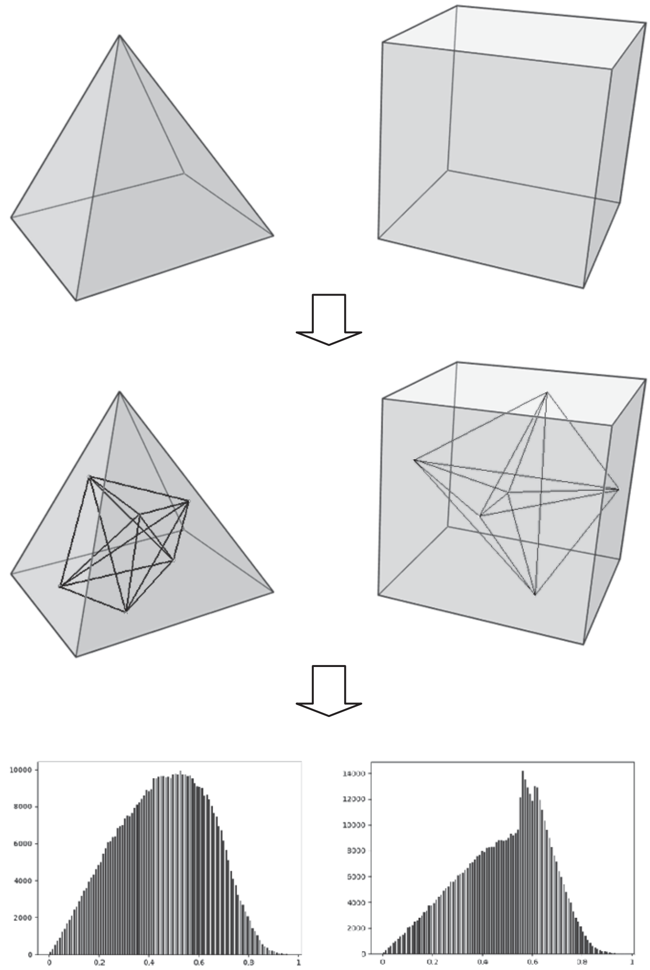

Fig. 1. The sequence of converting 3D model into histograms

When working with real industrial design objects, some adjustments and changes were made to the method described above.

The first task that arose on real data is that most 3D models come in the format of a point cloud, which initially contains a different number of points. There were models with 4 or 5 thousand points, and there were also ones with tens of millions of points. The developed method is planned for use in two areas: a comparison of industrial designs before the grant of a patent and the detection of a cancerous tumor in the images. Both of these tasks at the moment are solved by a human expert. The expert performs a visual analysis and for his work, the super-high resolution of the model is redundant. Thus, the first operation we carry out is reducing the point cloud before converting it to STL format. Reduction is performed using the voxelGrid filter of the pcl library. We conducted studies comparing models with a different number of points and as a result, it was found that with an increase in the number of points more than 5,000, the change in the proximity coefficient obtained as a result of comparison is less than 1%. And for 50,000 points it is less than 0.005%. We’ve used an iterative approach, since the reduction method depends on a given voxel size and in the general case it is impossible to predict how many points will remain. Gradually increasing the size of the voxel, we leave in the model the number of points from 5 to 50 thousand.

Further, when working with real objects, it turned out that the distribution of the points in initial model on the surface of the object is not uniform. First, to construct a histogram, we selected arbitrary points of the angles of the triangles that describe the surface. This is the simplest and least resource-intensive approach. However, with an uneven density of starting points on the surface, randomly selected points were concentrated in areas with the greatest detail or the most complex surface pattern. The reduction carried out in the previous step helped, but did not completely solve the situation. In our work, 1024 random points on the surface of the object are used. And in a number of specially prepared examples, up to 90% of the points were on less than 5% of the surface's square of the figure, and the obtained histograms mainly characterized this small part, not allowing comparison of the entire object. The transition from the angles of surface triangles to the intersection point of the medians also did not solve the problem, although, for the case of more evenly filled figures, it allowed us to form a more accurate descriptor value. An increase in the number of random points led to a slight increase in the accuracy of comparison; however, it significantly increased the computational complexity of the entire program and the amount of required memory. To speed up the calculations, a statistical approach was used, in which the procedure for selecting random points and plotting the histogram of distances between them was repeated many times, and the histograms themselves were averaged over all the data obtained.

In the vast majority of real objects, this turned out to be enough, but in some cases it was necessary to implement the method of uniform distribution of points on the surface of the object. As a result, an approach was implemented that provided the projection of points from the unit sphere onto the surface of the figure. We plan to develop this approach in the future during the next stage of our work.

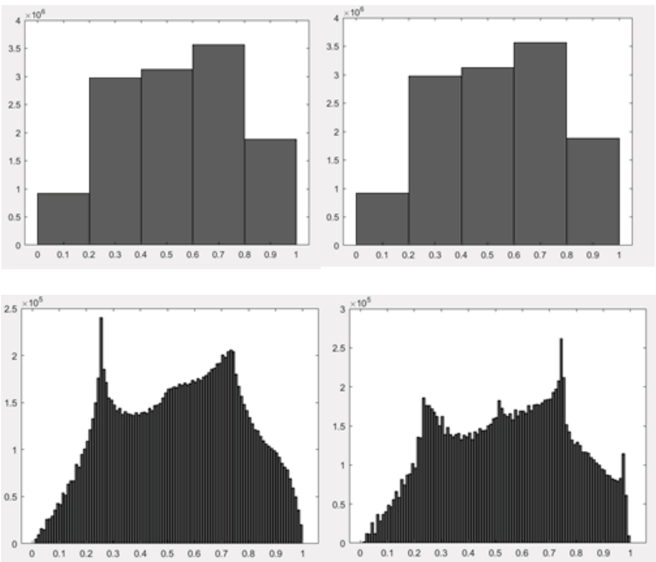

Fig. 2. Histograms with 5 and 100 segments of torus (left) and cylinder (right)

The next step of research was to determine a sufficient number of segments in the histogram. The small number of histogram segments does not allow revealing specific details of the shape of the studied object. For example, with only five segments, the histogram of a torus and the one of a flat cylinder of the same height are coinciding, i.e. the roundings and the hole in the center of the torus cease to influence on the result of the comparison (see Fig.2). It increases the number of type I errors (false positives when comparing two objects). On the other hand, when the number of segments is more than 10,000, the random nature of the choice of points on the surface begins to affect and all objects begin to differ from each other, including the object begins to differ from itself, i.e. the number of type II errors is growing (false negative). The difference between histogram with 200 and 20,000 segments is shown in Fig.3. The solution of the problem of minimizing both types of errors made it possible to determine that for various classes of objects the optimal number of segments is in the range from 50 to 200. In our research, we use 100 columns for all types of objects.

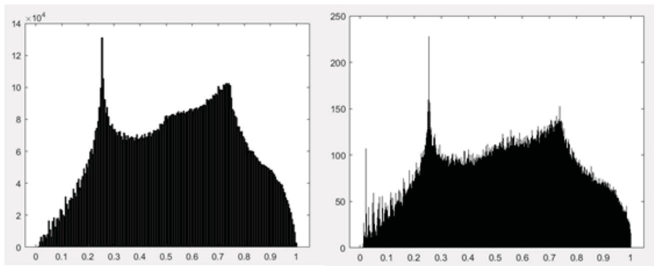

Fig. 3. Histograms with 200 (left) and 20000 (right) segments of torus

The last task in this work was the need to compare the obtained histograms in order to search for the closest objects (for patent search tasks) and classification of objects (for lung cancer detection problems). At first we used the sum of the squares of the difference between the columns of the histogram. Then we pass to the generalized measure, the Minkowski distance. However, as it was written above, we should not determine the exact coincidence of the histogram by analytical methods, but the coincidence according to the experts' opinion. It is necessary to evaluate the model in patent application as a patent specialist, and compaction in the lungs as a radiologist. To implement intelligent comparison, it was proposed to use the Siamese neural network. During the first stage, the following was carried out: 1,000 graphic objects were sent to the patent specialist for the development of a pairwise comparison matrix. Each expert had to evaluate the degree of proximity from 0 to 100, where 0 are completely different objects, and 100 is a complete coincidence. But the task had to be stopped. Almost all experts gave only 3 ratings: for 3-4% of alike objects, they rated 80 (or 90% depending on the expertise), for 80% of different objects they immediately assigned 0, the rating for the remaining ones was about 50. To get a wider range of values, the experts were given access to the results pairwise comparing models by analytical methods and asked to adjust the values. The obtained comparison matrix was used to train the Siamese neutron network. During the operation of the comparison system, it is planned to carry out further training of the neutral network with the results of the work of experts with new patent applications.

The developed methods and algorithms were implemented as a web platform for working with three-dimensional models. For the reduction and formation of the averaged histogram of distances for the complex 3D models, as well as for the further education of the Siamese neural network, the large computational resources are required. So the developed system operates on the basis of the “Polytechnic” supercomputer center of SPbPU, one of the most productive supercomputers in Russia.

III. Summary

The novelty of the work lies in the development, implementation and study of methods for comparing the shape of three-dimensional objects. The first level of abstraction is the representation of a figure as a matrix of distances between random points on the surface of an object. The use of such an approach ensures the invariance of the comparison method to the initial position and orientation of the compared objects in space, since distances between points on the surface of the object are absolute, not relative values. The novelty of the developed method is also the transformation of distance matrix into a histogram, which allows us to implement the requirement of invariance to the size of the compared objects. This requirement is based on the application of our algorithms: in protecting intellectual property rights, the size of an object plays a secondary role compared to form. For example, a toy car that accurately repeats the body of a real car would violate the rights of the car concern. The normalization of the histogram of distances between surface points allows us to successfully implement a comparison of objects of different initial sizes.

Another novelty of the work is methods of comparing the histograms of distances between points on the surface of objects in order to obtain a numerical estimate of the shape’s proximity of the original objects. Classical analytical methods for calculating the distance between histograms, like Minkowski distance, have several disadvantages. They do not take into account the context of the comparison problem, and some relatively small changes in source objects sometimes make it possible to obtain a fairly low estimate of proximity. To solve this problem, we used the well-known comparison method based on Siamese neural networks. The training sample was prepared by FIPS specialists. The trained Siamese network made it possible to bring closer the results of the proximity assessment given by our system to the expert assessments of

Developed methods, in contrast to comparison methods presented in 1.A, are independent of the parameters and details of the original objects. Regardless of the initial detailing, only the distance lengths between points on the surface of the compared objects are compared. Unlike the methods for comparing faces presented in 1.B, we do not use the specific characteristics of the compared objects, since we do not know in advance which objects and even classes of objects will be compared. Compared to the methods and approaches described in subsections 1.C and 1.D, our method is invariant to the position, orientation, size, and detail of the original objects. We do not stand the projections of the original objects and do not interact with any external objects relative to the figure being

The advantages of the developed methods are:

- invariance of methods to the initial position, orientation, detail and size of the compared object;
- numerical estimate of the proximity of two objects using Siamese neural networks;
- method is quite universal and allows to compare completely different objects, without any additional preliminary steps and settings.

But the developed methods also have several disadvantages:

- difficulty comparing objects with cavities and holes;
- probabilistic nature of the choice of points on the surface does not allow to obtain the equality of two identical objects;
- process of selecting points, calculating the distance matrix, building and normalizing the histogram is computationally complex and depends quadratically on the number of points;
- it is necessary to conduct training and further education of the Siamese network for each class of compared objects;
- the method does not allow searching and comparing a fragment of the initial object.

Some of disadvantages indicated above are planned to be eliminated in the near future, and some of them are a fundamental feature of the chosen comparison method and would be regulated administratively.

IV. Conclusion

The article is focused on the development of a method for comparing the shape of three-dimensional objects. There is given the analysis of the state of the art in comparison of three-dimensional objects in various fields. It is given a description of an intelligent method for comparing the shape of three-dimensional objects, as well as the results of research and implementation of the method. As a part of the development of the project, it is planned to improve the methods for describing the shape of three-dimensional objects, as well as to add additional descriptor parameters, such as color, ornament and surface texture.

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