A Robust Optimization Approach to DVB-T Network Design

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***Abstract*—We address the question of defining a robust optimization approach to model and solve a DVB-T network design problem, while taking into account the uncertainty that naturally affects the propagation of wireless signals. The robust counterpart of the Mixed Integer Linear Programming model that represents the design problem exploits multiband uncertainty, a cardinality-constrained uncertainty model that employs multiple deviation bands. Since the robust counterpart may prove challenging to solve also for state-of-the-art optimization solvers, we propose a matheuristic for its solution. The matheuristic combines a variable fixing procedure exploiting suitable (tight) linear relaxations of the model with exact large neighborhood search. Results of computational tests considering realistic instances are reported to assess the performance of the approach, showing that the matheuristic can generate solutions of higher value than a commercial optimization solver within the available time budget.**

I. Introduction

Television broadcasting still represents a major telecommunication service all around the world, despite the strong competition of a new wide range of alternative services provided through wireless and wired networks. A critical technological evolution in its history has been constituted by the passage from analogue to digital transmissions, which has taken place starting from the beginning of the new millennium: digital broadcasting has enhanced the quality and performance of the television networks, since it is able to exploit the scarce radio resources available for transmissions in a more efficient way, allowing the co-existence of a higher number of broadcasters [21], [29], [45]. Around the world, the digital television standard that has known the highest diffusion is DVB-T (Digital Video Broadcasting - Terrestrial) [21]. DVB-T is based on Orthogonal Frequency-Division Multiplexing (OFDM), a modulation technique that, thanks to orthogonality, allows to avoid interference while broadcasting on adjacent channels that are not separated by guard bands (see e.g., [44]).

In 2009, the second generation of the DVB-T standard (DVB-T2) has been released: the new standard offers improved spectral efficiency and higher data rates, by supporting more refined modulation schemes, in particular those belonging to the 256-QAM family [22], [26]. In many countries, the switch from the first to the second generation of DVB-T is currently ongoing and has required to existing companies to reset their networks. Moreover, since DVB-T2 allows the co-existence of a higher number of broadcasters, new companies have entered into the market and face the question of designing their new networks. This situation has led to a renewed interest in DVB-T design software, in particular those containing optimization tools, since they can provide (much) better design plans.

In this work, we address the question of developing a robust optimization model for designing DVB-T2 networks while taking into account the uncertainty that naturally affects wireless signal propagation. Specifically, our original contributions are:

1) We derive a robust counterpart for the Mixed Integer Linear Programming (MILP) model, based on signal-to-interference formulas, which is typically adopted to represent DVB-T design. The counterpart is defined according to the principles of Multiband Robust Optimization [10], a refined version of the classical Γ-Robust Optimization model by Bertsimas and Sim [7], which has been widely used to deal with data uncertainty in optimization problems (see e.g., [4], [6]).
2) Since the resulting robust optimization model may result very challenging even for a state-of-the art optimization software like IBM ILOG CPLEX, we define a *matheuristic* for its solution, proposing to combine a probabilistic variable fixing procedure with an exact large variable neighborhood search. The probabilistic fixing exploits the precious information that can be derived from a tight linear relaxation of the MILP model adopted to represent the DVB-T design problem, whereas the exact search consists of exploring a solution neighborhood formulating the search as an optimization problem that is solved at the optimum.
3) We highlight the performance of our new modelling and algorithmic approach by means of tests conducted on realistic DVB-T instances. The tests show that our algorithmic approach may generate solutions of much higher quality than a state-of-the-art solver.

We remark that, while the deterministic (i.e., not considering data uncertainty) optimal design of wireless networks based on signal-to-interference ratios has received wide attention, the use of optimization under uncertainty techniques, such as Robust Optimization and Stochastic Programming, has received less attention and has especially considered the effects of traffic uncertainty (as discussed in [33]) mainly considering stochastic programming approaches, such as [27] and [32] for dealing with fading uncertainty. This is also true for the case of DVB-T, in which optimization approaches have neglected data uncertainty (e.g., [16], [17], [34], [37]). To the best of our knowledge, this is the first work that discusses the adaption of a robust optimization approach to DVB-T and presents a matheuristic for the solution of the resulting complex problem.

In the remainder of the paper: Section II states the reference model that we consider for DVB-T optimal network design, while Section III discusses how to derive a multiband robust optimization for the problem; in Section IV we present the matheuristic algorithm and present results of preliminary computational tests on realistic instances. Finally, in Section VI, we conclude the paper discussing possible directions for future research.

II. DVB-T Network Design Optimization

As first step, we illustrate how to define a *deterministic* model (i.e., neglecting data uncertainty) for designing a DVB-T network. The design problem can be essentially described as that of choosing the power emissions of a set of stations that must broadcast television services to a territory discretized into a grid of small squares, commonly referred to as *testpoints* (TPs). The testpoint model is recommended to be used by telecommunications regulatory bodies (e.g., [1], [45]). Besides the power emission, it is also necessary to establish the serving station of each TP, which must be covered with a service level that satisfies quality requirements.

The DVB-T design problem represents a variant of the *Wireless Network Design Problem* (WND) (see e.g., [17], [33], [37], [43]) in which the two essential decisions that must be taken are: 1) setting the power emission of each transmitter and 2) establishing the transmitter that serves each user (user-serving transmitter association).

The joint optimization of the power emission of transmitters and of the association user-trasnmitter gives raise to the *Scheduling and Power Assignment Problem (SPAP)*, a variant of the WND that is NP-hard [37]. The SPAP is considered a central optimization problem in wireless network design(see e.g., [37], [38]) and typically includes signal-to-interference ratios to represent service coverage of users. Besides DVB-T, it has been considered in many different technological contexts such as 5G (e.g., [46]), FTTx (e.g., [18], [41]), mesh networks (e.g, [23], [30]), UMTS (e.g., [2], [24], [25]) WiMAX (e.g, [3], [28]) and other kind of wireless network design-related problem (e.g, [13], [32], [33], [43], [47]).

If we denote by S the set of transmitting stations and by T the set of testpoints, the SPAP can be modelled by introducing the following two families of decision variables:

- continuous power variables $p_s \in [0, P_{\max}]$ $\forall s \in S$, each modelling the power emission of one station;
- binary service assignment variables $x_{ts} \in \{0,1\}$ $\forall t \in T, s \in S$ such that:

$$x_{ts} = \begin{cases} 1 & \text{if station } s \in S \text{ serves TP } t \in T \\ 0 & \text{otherwise.} \end{cases}$$

In order to assess the service quality granted by a station s to a TP t, the first observation to be done is that the power that t obtain from s is equal to the product of the power emission p_s of s and of a coefficient $a_{ts} \in [0,1]$, commonly called *fading coefficient*. The fading coefficient summarizes the power decrease to which the signal is subject while propagating from s to t [42] (i.e., the power that t receives from s is equal to $a_{ts} \cdot p_s$).

Concerning interference phenomena in DVB-T, it is important to note that, thanks to the use of OFDM, signals sent on the same frequency do not necessarily interfere: their distinction between interfering and useful to the service depends upon falling within a time window that is used by the user/testpoint for signal detection. A signal that falls within the window is useful and contributes to increase the quality of service. Instead, a signal that falls outside the window is interfering and contributes to decrease the quality of service. For a more exhaustive review of the concept of time window and its impact on designing a DVB-T, we refer the reader to [36], [37] and to [35], [39].

In modelling approaches for DVB-T design, such as [8], [36], [37], the start of a time window is commonly set equal to the time that a signal is received from a station. Thus, for each testpoint $t \in T$, in the deterministic model we consider, we have one distinction between interfering and useful signals for each station $s \in S$. We say that s is the *server* (or *serving station*) of t if t starts its detection window when receiving the signal of s. Once the server s of t is chosen, we indicate the subset of useful stations for t by $U(s,t) \subseteq S$ and the subset of interfering stations by $I(s,t) \subseteq S$ (note that it holds $S = U(s,t) \cup I(s,t)$ and $U(s,t) \cap I(s,t) = \emptyset$).

In order to establish whether a TP t is served with the desired quality of service, we rely on a canonical Signal-to-Interference Ratio (SIR):

$$SIR_{ts}(p) = \frac{\sum_{\sigma \in U(s,t)} a_{t\sigma} \cdot p_\sigma}{N + \sum_{\sigma \in I(s,t)} a_{t\sigma} \cdot p_\sigma} \geq \delta . \quad (1)$$

that considers the sum of all the useful powers over the sum of all the interfering powers. In (1), $\delta > 0$ is called SIR threshold and expresses the desired service quality, whereas $N > 0$ is the system noise. By simple operations, the previous SIR can be rewritten as:

$$\sum_{\sigma \in U(s,t)} a_{t\sigma} \cdot p_\sigma - \delta \sum_{\sigma \in I(s,t)} a_{t\sigma} \cdot p_\sigma \geq \delta \cdot N . \quad (2)$$

Now, a question that arises is that in the DVB-T design problem we do not know a priori which is the server s of a TP t, since this is part of the decision problem. So we do not even know a priori which specific SIR inequalities we should satisfy. In order to include and model the activation or deactivation of SIR inequalities, depending upon the stations chosen as servers of the TPs, we can rely on a standard optimization approach (see [40]) and define modified SIR constraints that include the binary variables x_{ts}. Specifically, inside each SIR inequality, we include a new term where a sufficiently large coefficient denoted by M (the so-called big-M coefficient) multiplies the binary service assignment decision variable x_{ts}. More in detail, for each TP t and station s, we define the following big-M SIR constraint:

$$\sum_{\sigma \in U(s,t)} a_{t\sigma} \cdot p_\sigma - \delta \sum_{\sigma \in I(s,t)} a_{t\sigma} \cdot p_\sigma + M(1 - x_{ts}) \geq \delta \cdot N \ . \quad (3)$$

When $x_{ts} = 1$, the big-M term disappears in (3) and the constraint reduces to the SIR inequality of s serving t, which must be satisfied. In contrast, when $x_{ts} = 0$, s is not the server of t, the big-M term adds a very large quantity to the left-hand-side of (3) and thus (3) is satisfied by any power emission of the variables p_σ and therefore is redundant and does not affect the feasible set.

The SIR constraints (3) represent a fundamental component of the MILP used to model the WND and the DVB-T network design problem. In particular, the complete model we consider for designing the DVB-T network is:

$$\max \sum_{t \in T} \sum_{s \in S} r_t \cdot x_{ts} \qquad \text{(DVB-MILP)}$$

$$\sum_{\sigma \in U(s,t)} a_{t\sigma} \cdot p_\sigma - \delta \sum_{\sigma \in I(s,t)} a_{t\sigma} \cdot p_\sigma + M(1 - x_{ts}) \geq \delta \cdot N \qquad t \in T, s \in S \quad (4)$$

$$\sum_{s \in S} x_{ts} \leq 1 \qquad t \in T \quad (5)$$

$$0 \leq p_s \leq P^{\max} \qquad s \in S$$

$$x_{ts} \in \{0, 1\} \qquad t \in T, s \in S \ .$$

In the previous model, the objective function aims at maximizing the revenue obtained from covering TPs with service (each TP generates a revenue $r_t > 0$ when covered). The constraints (4) are the big-M SIR constraints and the constraints (5) model that at most one station can provide the reference signal starting the detection window of each TP.

III. Definition of a Robust Optimization Counterpart

The fading coefficients a_{ts} that are part of the SIR constraints (4) are naturally subject to uncertainty because of the wide range of factors that influence signal propagation in a real environment (e.g., landscape, obstacles, weather, etc.) and that are hard to precisely assess [42]. These coefficients are commonly computed by (empirical) propagation models that, using extensive field propagation measurements, provide a formula for computing the coefficient values on the basis of factors like the distance between the communicating points, the portion of the spectrum adopted for transmissions, and the characteristics of the propagation environment (e.g., with many obstacles like tall buildings or in line of sight). As well-known by telecommunication professionals, the actual propagation values may be sensibly different from the values returned by the propagation models and it is thus very important to protect design solutions from possible fluctuations in these values.

Our work considers an optimization problem in which the fading coefficients represent *uncertain data*, i.e. data whose value is not exactly known when the problem is solved. The problem is thus suitable to be tackled by Robust Optimization (RO). RO is an effective and efficient methodology for dealing with data uncertainty in optimization problems, which essentially consists of including data uncertainty under the form of hard constraints that restrict the set of feasible solutions, excluding those that could become infeasible when specified input data deviations occur. We refer the reader to [5], [6] for an exhaustive introduction to RO. Here, we recall that, according to an RO methodology:

- the *actual value* of each uncertain coefficient is unknown to the decision maker;
- the decision maker has at disposal a *nominal value* of each uncertain coefficient, representing an estimation of its actual value;
- deviations against which solutions must be protected are specified by an *uncertainty set*;
- the problem that is solved is a *robust counterpart*, a modified version of the original deterministic problem, including only robust feasible solutions, namely solutions that remain feasible for all the deviations values of the uncertainty set applied to the nominal values;
- a robust optimal solution offers the best objective value under the worst data deviations;
- since the robust counterpart excludes a subset of feasible solution of the deterministic problem, the robust feasible set is a subset of the deterministic feasible set; as a consequence, a robust optimal solution grants protection against deviations, but generally presents a worse value than a deterministic optimal solution. This reduction in optimality constitutes the so-called *price of robustness* [7].

In what follows, we define a first introductive robust optimization model referring to the famous Γ-Robustness model (Γ-ROB) by [7], which is based on a *cardinality-constrained* uncertainty set combined with an *interval deviation model*. Thus, in order to formally translate the previous RO principles into mathematical terms following also Γ-ROB, we assume that the actual value of a generic uncertain fading coefficient a_{ts} belongs to the symmetric interval $[\bar{a}_{ts} - d_{ts}, \bar{a}_{ts} + d_{ts}]$ (here, $\bar{a}_{ts}$ is the nominal value of the uncertain coefficient, while d_{ts} is its maximum allowed deviation). Practically, $\bar{a}_{ts}$ could be the value provided by a propagation model, while d_{ts} could be set as the maximum deviation that the network planner wants to consider according to its risk aversion.

If we denote by p the power vector of the decision variables p_s and by Γ the number of deviations of fading coefficients for which we want to guarantee protection (here, Γ is the parameter that gives the name to Γ-ROB), we can write a first robust version of the SIR constraint as follows:

$$\sum_{\sigma \in U(s,t)} a_{t\sigma} \cdot p_\sigma - \delta \sum_{\sigma \in I(s,t)} a_{t\sigma} \cdot p_\sigma - DEV_{ts}(\Gamma, p) + M(1 - x_{ts}) \ \geq \ \delta \cdot N \ , \qquad (6)$$

where $DEV_{ts}(\Gamma, p)$ is the maximum reduction in power that is allowed by the uncertainty set obtained by allowing at most Γ fading coefficients to deviate at their upper bound d_{ts}. A distinctive feature of Γ-Rob is to impose an upper bound Γ on the number of coefficients that may deviate to their worst value in each constraint and to tackle the resulting non-linear optimization problem by duality theory. The non-linearity is due to the presence of $DEV_{ts}(\Gamma, p)$ that actually constitutes an optimization problem inside the optimization problem. The linear model is then obtained at the cost of adding additional variables and constraints (we refer the reader to [7] for a detailed description of the mathematical approach). The parameter Γ controls the robustness level of the robust counterpart: assuming m uncertain coefficients in a constraints, Γ varies in $\{0, \ldots, m\}$ and, if $\Gamma = 0$, there is no protection and the price of robustness is zero, then as Γ increases from 1 to m, also the protection increases until reaching full protection when $\Gamma = m$, for all coefficients deviating to their worst value.

Instead of adopting Γ-robustness, we propose here to adopt *Multiband Robust Optimization* (MB) introduced in [10]–[12] to generalize and refine classical Γ-Rob: MB uses multiple deviation bands for better modeling arbitrary discrete distributions, under the form of histograms, which are commonly considered by professionals to analyze deviations in the input data in real-world optimization problems (as also illustrated in [4]). Following the principles of MB, for the uncertain fading coefficient we define the following Multiband Uncertainty Set (MBUS):

1) we partition the overall deviation range $[-d_{ts},\ d_{ts}]$ into K bands, defined on the basis of K deviation values: $-d_{ts} = d_{ts}^{K^-} < \cdots < d_{ts}^{-1} < d_{ts}^0 = 0 < d_{ts}^1 < \cdots < d_{ts}^{K^+} = d_{ts}$;
2) through these deviation values, K deviation bands are defined, namely: a set of positive deviation bands $k \in \{1, \ldots, K^+\}$ and a set of negative deviation bands $k \in \{K^- + 1, \ldots, -1, 0\}$, such that a band $k \in \{K^- + 1, \ldots, K^+\}$ corresponds to the range $(d_t^{k-1}, d_t^k]$, and band $k = K^-$ corresponds to the single value $d_t^{K^-}$. Note that $K = K^+ \cup K^-$;
3) we define a lower and upper bound on the number of values that may deviate in each band: for each band $k \in K$, two bounds $l_k, u_k \in \mathbb{Z}_+$: $0 \leq l_k \leq u_k \leq |T| \cdot |S|$ are introduced. Furthermore, the number of coefficients that may deviate in the zero-deviation band $k = 0$ is not limited (i.e., $u_0 = |T| \cdot |S|$) and we impose that $\sum_{k \in K} l_k \leq |T| \cdot |S|$, so as to ensure that there exists at least one feasible assignment of coefficients to deviations bands.

An MB uncertainty set is particularly suitable for modelling histograms. Furthermore, it also considers bands associated with beneficial and non-adversarial deviations: this is done since, in real-world applications, our main objective is to be protected against adversarial data deviations that lead to infeasibility, but at the same time we want to take into account also beneficial deviations which may take place and compensate the adversarial deviations, therefore reducing the price of robustness.

The linear robust counterpart of an uncertain SIR constraint defined for a couple (s, t) is obtained according to the theoretical results of Multiband Robust Optimization (in particular Theorem 1 of [10] about the mathematical form of a linear and compact multiband robust counterpart). Specifically, the single deterministic SIR constraint of (s, t) is replaced by the following set of constraints:

$$\sum_{\sigma \in U(s,t)} a_{t\sigma} \cdot p_\sigma - \delta \sum_{\sigma \in I(s,t)} a_{t\sigma} \cdot p_\sigma - \left(\sum_{k \in K} \theta_{ts}^k \cdot w_{ts}^k + \sum_{s \in S} z_{ts} \right) + M(1 - x_{ts}) \geq \delta \cdot N \qquad (7)$$

$$w_{ts}^k + z_{ts} \cdot p_\sigma \geq d_{ts}^k \, p_s \qquad k \in K \qquad (8)$$

$$w_{ts}^k \geq 0 \qquad k \in K \qquad (9)$$

$$z_{ts} \geq 0 \qquad (10)$$

which includes the additional constraints (8) and variables (9), (10) to linearly reformulate the original (non-linear) robust SIR constraints generalizing constraints (6) according to multiband robust optimization.

The robust optimization problem that we consider and that we denote by Robu-DVB-MILP is obtained by DVB-MILP substituting each SIR constraints with (7) and the auxiliary dual constraints and variables (8), (9), (10).

IV. A MATHEURISTIC FOR SOLVING THE ROBUST OPTIMIZATION PROBLEM

The multiband robust optimization model Robu-DVB-MILP constitutes a Mixed Integer Linear Programming problem and could be solved by using any optimization software. However, the presence of the robust complicated SIR constraints and of the big-M coefficients makes it a hard problem that may prove very challenging for state-of-the-art commercial solvers, even when considering instances of contained size. As a consequence, we propose a matheuristic for its solution, namely an algorithm that combines heuristic exploration of the feasible set with the adoption of exact optimization methods (i.e., guaranteeing convergence to an optimal solution) for suitable subproblems of the complete problem. Specifically, we propose a matheuristic that follows the algorithmic principles presented in [15], [18], [19], to which we refer the reader for more details. It is mainly based on a *probabilistic variable fixing* procedure integrated with an *exact large neighborhood search*.

In our case, the a-priori measure is provided by a linear relaxation of the model Robu-DVB-MILP, while the a-posteriori measure is given by a (tighter) linear relaxation of DVB-MILP (where a subset of variables has been fixed in value). At the end of each cycle of variable fixing, the a-priori fixing measure is updated, evaluating how good were the applied fixing. Once a time limit is reached, the fixing cycle stops and an exact large neighborhood search is executed for trying to improve the best solution found.

In the probabilistic fixing procedure, a number of solutions are built iteratively: at every iteration, a partial solution (i.e., a solution where only a subset of variables has its value fixed) is available and we can fix the value of an additional variable. Once the value of all the variables has been fixed, we obtain a complete solution whose quality is evaluated by means of its objective value. The fixing procedure is based on the observation that once the power emission variables have been fixed in value, it is possible to easily check which TPs are covered with service by some station and compute the value of the objective function. We thus base the fixing procedure on deciding the values assumed by the power variables. As preliminary step, we introduce a discretization of the power emission range $[0, P_{\max}]$ into a set $\mathcal{P} = P_1 = 0, P_2, \ldots, P_n = P_{\max}$ of discrete power values, defined according to a power discretization step ΔP. We denote by L the set of power indices $\{1, 2, \ldots, n\}$. Exploiting this discretization, the continuous power emissions are replaced by a set of binary variables $y_{sl} \in \{0, 1\}$ $\forall s \in S, l \in L$ such that:

$$y_{sl} = \begin{cases} 1 & \text{if station } s \in S \text{ emits with power } P_l \in \mathcal{P} \\ 0 & \text{otherwise,} \end{cases}$$

which must be accompanied by the constraints:

$$\sum_{l \in L} y_{sl} = 1 \qquad s \in S \tag{11}$$

expressing that each station must emit by exactly one power value. This leads to a so-called Power-Indexed optimization model, which has been proved to be very performing for dealing with SIR constraints appearing in optimal design of wireless networks [17].

At a generic iteration of the construction cycle of a feasible solution, we have at disposal a partial solution to the problem (obtained by having chosen the power emissions of a subset of stations $S^{\text{FIX}} \subseteq S$ by fixing their variables y_{sl} while respecting (11)). We probabilistically choose the next station whose power emission is fixed by means of the following formula, defined $\forall s \in S \backslash S^{\text{FIX}}, l \in L$:

$$p_{sl} = \frac{\alpha\ \tau_{sl} + (1-\alpha)\eta_{sl}}{\sum_{s \in S \backslash S^{\text{FIX}}} \sum_{\lambda \in L} [\alpha\ \tau_{\sigma\lambda} + (1-\alpha)\ \eta_{\sigma\lambda}]}\ , \tag{12}$$

which expresses the probability of fixing the power emission of station $s \in S \backslash S^{\text{FIX}}$ to power level P_l by considering all the couples $\sigma \in S \backslash S^{\text{FIX}}$, $\lambda \in L$ of stations whose emission is not yet fixed. In the formula, τ_{sl} is the a-priori attractiveness measure obtained from the optimal value of Robu-DVB-MILP including power-indexed variables, while η_{sl} is given by the value of a tight linear relaxation of DVB-MILP including fixing of variables done in previous iterations. The two measures are combined by a coefficient $\alpha \in [0, 1]$. After having fixed $y_{sl} = 1$ for some couple (s, l), because of constraint (11) we can set $y_{s\lambda} = 0$ for all $\lambda \in L : \lambda \neq l$.

After having defined the power emissions of all stations (assume this is denoted by a binary power vector $\bar{y}$), all the SIR ratios can be easily computed. On the basis of the value of these ratios, we can also easily check which testpoints are covered with service and thus derive a valorization of the server assignment variables $\bar{x}$. The resulting solution $(\bar{y}, \bar{x})$ which is feasible for DVB-MILP is accepted as robust when it maintains its feasibility also when the fading coefficients are deviating to their worst value.

Once a round of construction of feasible solutions has been operated, the a-priori measures are updated using the following formula:

$$\tau_{sl}(h) \quad = \quad \tau_{sl}(h-1) + \sum_{\text{SOL}=1}^{\gamma} \Delta\tau_{sl}^{\text{SOL}} \tag{13}$$

where:

$$\Delta\tau_{sl}^{\text{SOL}} = \tau_{sl}(0) \cdot \left(\frac{OG(v^{\text{AVG}}, u) - OG(v^{\text{SOL}}, u)}{OG(v^{\text{AVG}}, u)} \right) \tag{14}$$

where $\tau_{sl}(h)$ is the a-priori measure of fixing station s at power level P_l at the h-th execution of the cycle and $\Delta\tau_{sl}^{\text{SOL}}$ is the modification to the value of the a-priori measures, computed over a summation that considers the last γ solutions that have been constructed. Moreover, u is an upper bound on the optimal value of the problem, v^{SOL} is the value of the SOL-th feasible solution built in the last construction cycle, v^{AVG} is the average of the values of the last γ solutions that have been constructed. The optimality gap *OG*(v,u) measures how far is the value v of a solution from the upper bound u and is defined as $OG(v, u) = (u-v)/v$. The role of formula (13) is to update the a-priori measure rewarding (penalizing) those fixing that have lead to a solution with lower (higher) optimality gap in comparison to the moving average value v^{AVG}.

At the end of the construction cycle, with the aim of improving the best robust solution found, an exact neighborhood search is conducted, i.e. we explore a (very large) neighborhood of the best solution, formulating the search as an optimization problem which is optimally solved by a state-of-the-art solver (see e.g., [9], [20]). The adoption of exact searches is motivated by the fact that, while it can be difficult and long for a solver to solve the complete problem, it is instead possible to efficiently solve to optimality some subproblems. The large neighborhood that we define is built from a robust solution $(\bar{y}, \bar{x})$ allowing to change the power emission of all stations by either 1) turning off a station s (i.e., setting $y_{s0} = 1$ or 2) allowing a modification of the power emission to the adjacent power level set by $\bar{y}$ (i.e., if $y_{sl} = 1$ then it is allowed to set $y_{sl-1} = 1$ or $y_{sl+1} = 1$. The exact search is then conducted by expressing the previous conditions as linear constraints that are added to Robu-DVB-MILP and the resulting problem is solved by an exact solver.

The pseudocode of the matheuristic for solving Robu-DVB-MILP is presented in Algorithm 1. The first step consists of solving the linear relaxation of Robu-DVB-MILP including the power fixing of each couple (s, l) with $s \in S$ and $l \in L$. The obtained optimal values are employed to initialize the a-priori measures $\tau_{sl}(0)$. Then a solution construction cycle is executed until reaching a time limit. In each execution of the cycle, a number of feasible solutions are built first by fixing the power emission binary variables through formula (12), then deriving the corresponding valorization of variables x and finally checking their robustness. At the end of each execution of the cycle, the a-priori measures τ are updated on the basis formula (13). As last step, once the construction time limit is reached, the exact large neighborhood search is conducted, using as basis the best robust feasible solution defined during the construction cycle.

Algorithm 1

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compute the linear relaxation of the power-indexed version of Robu-DVB-MILP for all y_sl = 1 and initialize the values τ_sl(0) with the corresponding optimal values
let (x*, y*) be the best robust feasible solution found
while a global time limit is not reached do
    for SOL := 1 to γ do
        construct a feasible power vector ȳ using the probabilistic fixing formula (12)
        compute the set of station-testpoint couples associated with satisfied SIR quantities and derive the corresponding x̄ vector
        check the robustness of the feasible solution (x̄, ȳ)
        if the coverage granted by (x̄, ȳ) is better than that of (x*, y*) then
            update (x*, y*) with (x̄, ȳ)
        end if
    end for
    update τ according to (13)
end while
execute the exact large neighborhood search using (x*, y*) and the modified power-indexed version of Robu-DVB-MILP as basis
return (x*, y*)
```

V. Preliminary computational results

The robust optimization approach was tested on 10 instances including realistic data defined from regional DVB-T networks deployed in Italy, including up to about 300 stations and 4000 testpoints. The revenue associated with covering a testpoint is represented by the population of the testpoint, so, in what follows, the value of the best solution found by an algorithm is expressed as the percentage of the population covered with service. As optimization software, we used IBM ILOG CPLEX [14] and the algorithms were tested on a Windows machine with 2.70 GHz Intel i7 and 8 GB of RAM. Both CPLEX and the matheuristic ran with a time limit of 1 hour (in the case of the matheuristic, 50 minutes are devoted to the solution construction and 10 minutes are reserved to the execution of the exact neighborhood search). The parameters α and γ are set equal to 0.5 and 5, respectively. The robust model takes into account a deviation range that allows deviation up to 20% of the value of the fading coefficients and that is partitioned into 5 deviation bands. The preliminary results of the computational tests are presented in Table 1, where: i) ID identifies the instance; ii) *COV-CPLEX%* and *COV-MH%* are the percentage of population covered by the best robust feasible solution found by CPLEX and the best robust feasible solution found by the metaheuristic; iii) Δ*COV%* is the percentage increase in population coverage that the metaheuristic grants with respect to CPLEX.

TABLE I. EXPERIMENTAL RESULTS

ID	COV-CPLEX%	COV-MH%	ΔCOV%
DVB1	75.4	91.5	21.3
DVB2	74.0	88.4	19.4
DVB3	71.2	86.0	20.7
DVB4	66.8	64.8	26.9
DVB5	67.4	90.4	34.1
DVB6	74.7	87.5	17.1
DVB7	71.5	82.6	15.5
DVB8	79.0	88.1	11.5
DVB9	68.2	82.3	20.6
DVB10	72.6	87.5	20.5

Evaluating the results, it can be observed that CPLEX experiences difficulties to find high quality solutions, identifying robust solutions of quite low value in comparison to the matheuristic. Specifically, while CPLEX offers coverage between about 66 and 79 %, the matheuristic offers a coverage at least above 80% that may exceed 90% and offers an average coverage of 86%. Thus the matheuristic is able to offer solutions that are on average about 20% better in coverage value than those returned by CPLEX. The better performance of the matheuristic can be attributed to the fact of heuristically uncoupling the power and assignment variables, which, when linked, produce a much more complicated problem. Moreover, the variable fixing heuristic attempts at exploiting the valuable information coming from (strengthened) linear relaxations of the considered models.

VI. Conclusion and future work

In this paper, we proposed a robust optimization approach for tackling the uncertain nature of fading coefficients appearing in optimization models adopted for designing DVB-T wireless networks. The robust optimization model is based on Multiband Robust Optimization and may prove difficult to solve also for a state-of-the-art optimization solver, when considering realistic instances. In order to identify solutions of higher quality, we proposed a matheuristic that combines a variable fixing procedure based on exploiting linear relaxations of the robust and deterministic model with an exact large neighborhood search. Preliminary computational results based on realistic instances indicated that the matheuristic is able to identify solutions of sensibly better quality than those returned by a commercial optimization solver. In the future, we intend to widen the computational experience to a larger set of instances, also conducting a study about the impact of parameter tuning. Moreover, we intend to also better study the impact of different characterization of the uncertainty set on the robustness of solutions.

REFERENCES

[1] Italian Authority for Telecommunications (AGCOM). Specifications for a DVB-T planning software tool. http://www.agcom.it/Default.aspx?message=downloaddocument&DocID=3365 (in Italian), 2009. Retrieved 01.06.2020.

[2] E. Amaldi, P. Belotti, A. Capone, F. Malucelli, "Optimizing base station location and configuration in UMTS networks". Ann. Oper. Res. 146, 135–151, 2006.

[3] J. Andrews, A. Ghosh, R. Muhamed, *Fundamentals of WiMAX*, Prentice Hall, Upper Saddle River, 2007.

[4] T. Bauschert, C. Büsing, F. D'Andreagiovanni, A.M.C.A. Koster, M. Kutschka, U. Steglich: Network Planning under Demand Uncertainty with Robust Optimization. IEEE Communications Magazine 52 (2), 178-185 (2014) DOI: 10.1109/MCOM.2014.6736760

[5] A. Ben-Tal, L. El Ghaoui, A. Nemirovski: Robust Optimization. Springer, Heidelberg, 2009.

[6] D. Bertsimas, D. Brown, C. Caramanis: Theory and Applications of Robust Optimization. SIAM Review 53 (3) 464-501 (2011)

[7] D. Bertsimas, M. Sim: The Price of Robustness. Oper. Res. 52 (1), 35-53 (2004)

[8] R. Beutler, Frequency Assignment and Network Planning for Digital Terrestrial Broadcasting Systems, Springer, Heidelberg (2004)

[9] C. Blum, J. Puchinger, G. R. Raidl, A. Roli, "Hybrid metaheuristics in combinatorial optimization: A survey". Applied Soft Computing 11, 4135-4151, 2011.

[10] C. Büsing, F. D'Andreagiovanni: New Results about Multi-band Uncertainty in Robust Optimization. In: Klasing, R. (ed.) Experimental Algorithms, LNCS, vol. 7276, pp. 63-74. Springer, Heidelberg (2012) DOI: 10.1007/978-3-642-30850-5_7

[11] C. Büsing, F. D'Andreagiovanni: A New Theoretical Framework for Robust Optimization Under Multi-Band Uncertainty. In: Helber S. et al. (eds) Operations Research Proceedings 2012, pp. 115-121. Springer, Cham (2014)

[12] C. Büsing, F. D'Andreagiovanni, A. Raymond: 0-1 Multiband Robust Optimization. In: Huisman D., Louwerse I., Wagelmans A. (eds) Operations Research Proceedings 2013, pp. 89-95. Springer, Cham (2014)

[13] A. Capone, L. Chen, S. Gualandi, D. Yuan, "A New Computational Approach for Maximum Link Activation in Wireless Networks under the SINR Model". IEEE Trans. Wireless Comm. 10, 1368–1372, 2011.

[14] IBM ILOG CPLEX: http://www-01.ibm.com/software

[15] F. D'Andreagiovanni, J. Krolikowski, J. Pulaj, "A fast hybrid primal heuristic for Multiband Robust Capacitated Network Design with multiple time periods". Applied Soft Computing 26, 497-507, 2015.

[16] F. D'Andreagiovanni, H. Lakhlef, A. Nardin, "A Matheuristic for Joint Optimal Power and Scheduling Assignment in DVB-T2 Networks". *Algorithms* 13(1), 2020.

[17] F. D'Andreagiovanni, C. Mannino, A. Sassano, "GUB Covers and Power-Indexed Formulations for Wireless Network Design". *Management Science* 59 (1), 142-156, 2013.

[18] F. D'Andreagiovanni, F. Mett, A. Nardin, J. Pulaj, "Integrating LP-guided variable fixing with MIP heuristics in the robust design of hybrid wired-wireless FTTx access networks". Applied Soft Computing 61, 1074- 1087, 2017.

[19] F. D'Andreagiovanni, A. Nardin, "Towards the fast and robust optimal design of wireless body area networks". Applied Soft Computing 37, 971-982, 2015.

[20] E. Danna, E. Rothberg, C. Le Pape, "Exploring relaxation induced neighborhoods to improve MIP solutions". *Math. Progr.* 102, 71-90, 2005.

[21] DVB Project: DVB-T. https://www.dvb.org/standards/dvb-t. Retrieved 01.06.2020.

[23] P. Dely, F. D'Andreagiovanni, A. Kassler, "Fair optimization of mesh-connected WLAN hotspots". Wirel. Commun. Mob. Com. 15, 924–946, 2015.

[22] DVB Project: DVB-T2. https://www.dvb.org/standards/dvb-t2. Retrieved 01.06.2020.

[24] A. Eisenblätter, H. Geerdes, "Capacity optimization for UMTS: Bounds and benchmarks for interference reduction", PIMRC 2008, 1-6, 2008.

[25] A. Eisenblätter, A. Fügenschuh, H.Geerdes, D. Junglas, T. Koch, A. Martin. Optimisation Methods for UMTS Radio Network Planning. In: Ahr D. et al. (eds) Operations Research Proceedings 2003, pp. 31-38. Springer, Berlin, Heidelberg, 2003.

[26] European Broadcasting Union, "Frequency and Network Planning Aspects of DVB-T2", 2014. https://tech.ebu.ch/docs/tech/tech3348.pdf. Retrieved 01.06.2020.

[27] T. Heikkinen, A. Prekopa, "Optimal power control in a wireless network using a model with stochastic link coefficients". Naval Research Logistics 52 (2), 178-192, 2005.

[28] T. Hu, Y.P. Chen, W. Banzhaf, "WiMAX Network Planning Using Adaptive-Population-Size Genetic 508 Algorithm". In: C. Di Chio et al. (eds.) EvoApplications 2010. Lecture Notes in Computer Science, 509 vol. 6025, 31-40. Springer, Heidelberg, 2010.

[29] International Telecommunication Union: REPORT ITU-R BT.2140 Transition from Analogue to Digital Terrestrial Broadcasting. https://www.itu.int/dms_pub/itu-r/opb/rep/R-REP-BT.2140-2008-PDF-E.pdf. Retrieved 01.06.2020.

[30] K. Kamimura, H. Hayashi, "Optimization of power-efficient wireless mesh networks in outdoor and indoor environments in Japan". IEEJ Transactions on Electrical and Electronic Engineering 10(6), 2015.

[31] M. Kang, Y. Chung, "An Efficient Energy Saving Scheme for Base Stations in 5G Networks with Separated Data and Control Planes Using Particle Swarm Optimization". Energies 10, 2017.

[32] J. Kalvenes, J. Kennington, E. Olinick, "Base station location and service assignments in W-CDMA networks". INFORMS J. Comp. 18, 366–376, 2006.

[33] J. Kennington, E. Olinick, D. Rajan, *Wireless Network Design: Optimization Models and Solution Procedures*, Springer, Heidelberg, 2010.

[34] G. Koutitas, "Green Network Planning of Single Frequency Networks". IEEE Trans. on Broadcasting 56 (4), 541-550, 2010.

[35] M. Lanza, A. Gutierrez, J. Perez, J. Morgade, M. Domingo, L. Valle, P. Angueira, J. Basterrechea, "Coverage Optimization and Power Reduction in SFN Using Simulated Annealing". IEEE Trans. on Broadcasting 60 (3), 474-485, 2014.

[36] A. Ligeti, J. Zander, "Minimal cost coverage planning for single frequency networks". IEEE Trans. Broadcasting 45(1), 78-87, 1999.

[37] C. Mannino, F. Rossi, S. Smriglio, "The network packing problem in terrestrial broadcasting". Oper. Res. 54, 611–626, 2006.

[38] C. Mannino, F. Rossi, S. Smriglio, "A Unified view in planning broadcasting networks". DIS Tech. Rep. 08-07, Sapienza Università di Roma, Roma, Italy, 2007.

[39] G. Martinez, J. Sanchez, D. Barquero, N. Cardona, "Optimization of the Digital Terrestrial Television Transmission Mode of DVB-T2 in Colombia". IEEE Latin America Transactions 13 (7), 2144–2151, 2015.

[40] G. Nehmauser, L. Wolsey, *Integer and Combinatorial Optimization*. John Wiley & Sons, Hoboken, 1988.

[41] D. Novak, R. B. Waterhouse, A. Nirmalathas, C. Lim, P.A. Gamage, T.R. Clark, M.L. Dennis, J.A. Nanzer, "Radio-Over-Fiber Technologies for Emerging Wireless Systems". IEEE J. Quantum Electronics 52, 2016.

[42] T. Rappaport, *Wireless Communications: Principles and Practices*. Prentice Hall, Upper Saddle River, 2001.

[43] M. Resende, P. Pardalos, *Handbook of Optimization in Telecommunications*, Springer, Heidelberg, 2006.

[44] H. Schulze, C. Lüders, "Theory and Applications of OFDM and CDMA: Wideband Wireless Communications", J. Wiley & Sons, Hoboken, 2005.

[45] The Chester 1997 Multilateral Coordination Agreement. Technical criteria, coordination principles and procedures for the introduction of terrestrial digital video broadcasting. http://www.archive.ero.dk/132D67A4-8815-48CB-B482-903844887DE3?frames=no&5, 1997. Retrieved 01.06.2020.

[46] J. Wang, J. Weitzen, O. Bayat, "Interference coordination for millimeter wave communications in 5G networks for performance optimization". J. Wireless Com. Network 46, 2019.

[47] A. Zakrzewska, F. D'Andreagiovanni, S. Ruepp, M.S. Berger, "Biobjective optimization of radio access technology selection and resource allocation in heterogeneous wireless networks". 2013 11th International Symposium and Workshops on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks, WiOpt 2013, 652-658, 2013.