Low-complexity Iterative Detector for Massive MIMO Systems

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***Abstract*—Application of Multiple-input Multiple-output (MIMO) technology provides high data rates in modern standards of wireless communication. MIMO technology is characterized by the use of dozens of antennas on the transmitting and receiving sides, or even hundreds of antennas for the so-called massive MIMO technology. Massive MIMO is often considered as a key technology for 5G New Radio (NR). However, due to the increase in the number of antennas on the receiving and transmitting sides, the computational complexity of signal processing on the receiving side also increases. We suggest a new algorithm for signal detection for massive MIMO systems with lower computational complexity than the known Minimum Mean Square Error (MMSE) equalizer.**

I. Introduction

General use of MIMO technology in modern wireless communication systems is caused by the fact that the use of this technology allows to meet the demand of users in terms of capacity and data rates of wireless data transmission systems. In contrast to standard wireless communication systems, which have one antenna on the transmitting and receiving sides, when using MIMO technology, several antennas are used on the transmitting and receiving sides [1],[2]. In the simplest implementations of MIMO technology the number of antennas can be 2 or 4 [2]. Such implementations in spatial multiplexing mode make it possible to achieve an increase by several times in data rates compared to Single-input Single-output (SISO) systems [1]. However with the development of wireless communication systems, and due to the increase in the required data transfer rate, the potential of such MIMO systems has become insufficient.

The next step in the development of MIMO technology was the appearance of the massive MIMO technology, its number of antennas can reach tens or even hundreds on both the transmitting and the receiving sides [3]. Massive MIMO technology was created to provide the ability to transfer data at higher rates, as well as to increase network capacity [6]. This determines its application in the most modern wireless communication standards.

On the other hand, in addition to the above mentioned advantages, the computational complexity of signal processing on the receiving side (detection) increases with the application of the massive MIMO technology due to the increase in the number of antennas, and, as a result, the dimension of the channel matrix containing the transmission coefficients between each transmitting and each receiving antenna [4]. Thus, the application of known detection algorithms becomes difficult or even impossible to implement to ensure the required data rates.

We offer a new detection algorithm, which will reduce the computational complexity of the signal processing on the receiving side compared to the known MMSE algorithm in wireless communication systems based on massive MIMO technology.

II. MIMO system model

Fig. 1 shows a block diagram of a communication system built using MIMO technology, which consists of M transmitting antennas and N receiving antennas [2]. The input of the modulator receives a binary stream of information bits, which passes through the communication channel after modulation.

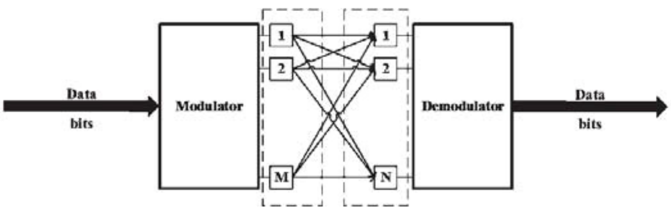

Fig. 1. MIMO structure

In spatial multiplexing mode each transmitting antenna transmits an independent signal to each of the receiving antennas [1],[2]. Thus, an increase of data rate is achieved due to the transmission of different signals by different antennas.

The signal that passed through the channel on the receiving side must be processed with one of the known detection algorithms, and then demodulated [1]-[4].

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Let us consider a mathematical model of the MIMO system. The signal observed on the receiving side can be expressed as follows:

$$\mathbf{y} = \mathbf{Hx} + \boldsymbol{\eta}, \tag{1}$$

where:

$\mathbf{y}$ – the observed vector of complex samples;

$\mathbf{H}$ – the matrix of complex transmission coefficients (channel matrix) with the dimensionality $N \times M$;

$\mathbf{x}$ – the vector of transmitted complex information symbols;

$\boldsymbol{\eta}$ – complex random Gaussian noise vector in the communication channel.

In general, the detection process is reduced to solving a System of Linear Algebraic Equations (SLAE) (1) [1]-[4]. But the presence of a random component (noise vector) may lead to errors while finding a solution Therefore, detection algorithms are used to calculate the estimate of the vector of transmitted characters on the receiving side [1]-[4].

III. MIMO SIGNAL DETECTION

In process of MIMO technology application, the signal observed during reception represents a combination of signals received from different transmitting antennas [1],[4]. After that the receiver must separate the signals received from different receiving antennas using detection algorithms.

Detection of a signal on the receiving side with a known channel matrix is one of the main issues in wireless communication systems [1]-[4], [14]. The essence of this task is to restore the transmitted signal $\mathbf{x}$ on the receiving side from the observed vector of samples $\mathbf{y}$. It is worth mentioning that the calculation of estimate of the vector of transmitted symbols on the receiving side must be processed for a period not exceeding the duration of the information symbol. Thus, as the data rate increases, this period shortens.

In wireless telecommunication systems based on MIMO technology, there are some known algorithms used for detection [1]-[5]. Using some of them allows to get high resistance to noise, while the computational complexity of such algorithms is high. Other algorithms, requiring less number of computational operations to get estimate, on the contrary, have worse characteristics resistance to noise. Therefore, the algorithms that allow to receive higher resistance to noise, often tend to become complex even when using MIMO technology not to mention massive MIMO [14].

Among known wireless communication systems detection algorithms using MIMO technology the following algorithms can be distinguished:

1) Zero Forcing (ZF);
2) Minimum Mean Square Error (MMSE);
3) Maximum Likelihood (ML).

ML has the best characteristics of resistance to noise among the listed algorithms [1]-[4]. The essence of this method is that when calculating the estimate, the square of the residual norm is minimized [2], [13]:

$$\hat{\mathbf{x}}_{ML} = \arg \min_{\mathbf{x} \in \mathbf{X}^M} \left\| \mathbf{y} - \mathbf{Hx} \right\|^2, \tag{2}$$

where:

$\mathbf{X}^M$ – is a discrete set of values of a M-dimensional vector $\mathbf{x}$ of complex information symbols. The set of $\mathbf{X}$ is determined by the type of modulation used in the system. Finding the minimum by equation (2) implies a search for all possible combinations of the vector of transmitted complex information symbols. Thus, the computational complexity of this detection algorithm depends on two parameters: the number of transmitting antennas M and the modulation order k used in the wireless communication system. For example, using modulation with order k to calculate estimate of the vector of transmitted symbols requires completing k^M elementary arithmetic operations, which becomes difficult to implement when the values k and M are large. Taking this into consideration, the usage area of this algorithm is limited to MIMO communication systems with a low modulation order and a small number of transmitting antennas [2].

The other two algorithms, ZF and MMSE, are linear algorithms. Both of these algorithms are similar, but there is a difference: the MMSE algorithm takes into account the influence of noise in the communication channel.

When using ZF, the expression for calculating the evaluation of the vector of transmitted characters looks like this [2]:

$$\hat{\mathbf{x}}_{ZF} = \left(\mathbf{H}^H \mathbf{H}\right)^{-1} \mathbf{H}^H \mathbf{y}, \tag{3}$$

where:

^{H} is the Hermitian conjugate. In terms of computational complexity, the most difficult ZF operation is the matrix inversion $\left(\mathbf{H}^H \mathbf{H}\right)$. In this case, the computational complexity depends only on the dimension of the matrix $\mathbf{H}$, which means on the number of transmitting and receiving antennas.

Another linear detection algorithm, the MMSE, has better resistance to noise characteristics compared to the ZF, but the complexity of calculating the estimate of the vector of transmitted characters is approximately the same [2].

The expression for the MMSE detection algorithm is as follows [2], [10], [12], [13]:

$$\hat{\mathbf{x}}_{MMSE} = \left(\mathbf{H}^H \mathbf{H} + 2\sigma^2 \mathbf{1}\right)^{-1} \mathbf{H}^H \mathbf{y}, \tag{4}$$

where:

$2\sigma^2$ – is the aggregate dispersion of the real and imaginary components of the Gaussian noise vector;

$\mathbf{1}$ – identity matrix.

As can be seen from equations (3) and (4), the differences between the ZF and the MMSE consist only in the fact that when using the MMSE the aggregate dispersion of the real and imaginary components of the Gaussian noise vector is taken into account, that provides the MMSE algorithm higher resistance to noise. The most complex operation in MMSE algorithm is also the calculation of the inverse of $\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right)$.

Thus, compared to the ML, the estimates obtained using the ZF or the MMSE are the easiest to calculate. At the same time, with less computational complexity, the resistance to noise of the MMSE allows to consider it as the main algorithm for MIMO systems. But when switching to massive MIMO technology the complexity of this algorithm increases due to the increasing number of antennas and its implementation may be difficult, since the asymptotic computational complexity of this algorithm has a cubic order.

IV. Iterative detector for massive MIMO systems

To reduce the computational complexity of signal detection in massive MIMO systems, we propose an iterative detection algorithm, the computational complexity of which is lower than that of the known MMSE, while the noise immunity is close to the MMSE.

As it was mentioned above, the most complex operation in the MMSE algorithm is matrix inversion [2]. To reduce the complexity of signal detection in systems using massive technology MIMO, it is suggested to exclude the request operation of matrix inversion from the algorithm for calculating the estimate of the vector of transmitted symbols.

Next, we will use the following notation:

$$\mathbf{Y}=\mathbf{H}^H\mathbf{y},\ \mathbf{T}=\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right).$$

Now let's multiply both parts (4) by the matrix $\mathbf{T}=\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right)$ as follows:

$$\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right)\hat{\mathbf{x}}_{MMSE}=\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right)^{-1}\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right)\mathbf{H}^H\mathbf{y}$$
.

As a result we get the following expression which represents a SLAE:

$$\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right)\hat{\mathbf{x}}_{ITER}=\mathbf{Y}\,. \qquad (5)$$

The solution of SLAE (5) is the estimate of the vector of transmitted symbols on the receiving side.

Exact and iterative methods can be used to solve SLAE (5). Exact methods allow us to get just the same estimate of the vector of transmitted characters as when using the MMSE (4). At the same time, the computational complexity of exact methods will not reduce the complexity of the calculation estimate of the vector of transmitted symbols. Thus, for the solution of SLAE (5) it is suggested to use an iterative method – stabilized biconjugate gradient method [7]-[10]. In case of iterative methods application, it is possible to reduce the number of arithmetic operations for finding the SLAE solution by reducing the number of iterations, and this solution for SLAE will be approximate.

To get the maximum effect from the application of iterative method of solving SLAE (5) and to reduce the computational complexity of obtaining an estimate of the vector of transmitted characters, it is necessary to fill in the condition $L << N$, where L – is the maximum number of iterations for solving SLAE. In addition, the stabilized biconjugate gradient method contains two operations for calculating the product of a matrix by a vector at each iteration [7]-[10]. To reduce the computational complexity of solution the SLAE it is suggested not to calculate the matrix $\mathbf{T}=\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right)$ in advance, but to calculate the product $\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{1}\right)$ on the vector at each iteration of the stabilized biconjugate gradient method.

Thus, the iterative detection algorithm can be described as follows:

Algorithm 1 Description of the developed iterative detection algorithm for massive MIMO systems

Input data: matrix $\mathbf{H}$ with the dimensionality $N\times M$, diagonal matrix $2\sigma^2\mathbf{1}$ with the dimensionality $N\times M$, vector $\mathbf{Y}$ with the dimensionality N.

Step 1. Selection of the initial approximation to the solution of the SLAE $\hat{\mathbf{x}}_{ITER}^{(0)}=0$. Set the maximum number of iterations L.

Step 2. To calculate the residual:

$$\mathbf{r}^{(0)}=\mathbf{Y}-\left(\mathbf{H}^H\mathbf{H}+2\sigma^2\mathbf{I}\right)\hat{\mathbf{x}}_{ITER}^{(0)}=$$
$$=\mathbf{Y}-\mathbf{H}^H\left(\mathbf{H}\hat{\mathbf{x}}_{ITER}^{(0)}\right)+2\sigma^2\left(\mathbf{I}\hat{\mathbf{x}}_{ITER}^{(0)}\right).$$

Since $\hat{\mathbf{x}}_{ITER}^{(0)}=0$, then $\mathbf{r}^{(0)}=\mathbf{Y}$.

Step 3. Set the vector $\bar{\mathbf{r}}^{(0)}$, on condition $\left(\mathbf{r}^{(0)},\bar{\mathbf{r}}^{(0)}\right)\neq 0$, as follows: $\bar{\mathbf{r}}^{(0)}=\mathbf{r}^{(0)}$.

Step 4. Set the base vector $\mathbf{p}^{(0)}=\mathbf{r}^{(0)}$.

Step 5. Set $l=0$.

Step 6. Calculate:

$$\mathbf{q}^{(l)}=\left[\mathbf{H}^H\left(\mathbf{H}\mathbf{p}^{(l)}\right)+2\sigma^2\left(\mathbf{I}\mathbf{p}^{(l)}\right)\right],$$
$$\mathbf{v}^{(l)}=\left[\mathbf{H}^H\left(\mathbf{H}\mathbf{s}^{(l)}\right)+2\sigma^2\left(\mathbf{I}\mathbf{s}^{(l)}\right)\right],$$
$$\alpha^{(l)}=\frac{\left(\mathbf{r}^{(l)},\bar{\mathbf{r}}^{(0)}\right)}{\left(\mathbf{q}^{(l)},\bar{\mathbf{r}}^{(0)}\right)},\qquad \mathbf{s}^{(l)}=\mathbf{r}^{(l)}-\alpha^{(l)}\mathbf{q}^{(l)},\qquad \omega^{(l)}=\frac{\left(\mathbf{v}^{(l)},\mathbf{s}^{(l)}\right)}{\left(\mathbf{v}^{(l)},\mathbf{v}^{(l)}\right)},$$
$$\hat{\mathbf{x}}_{ITER}^{(l+1)}=\hat{\mathbf{x}}^{(l)}+\alpha^{(l)}\mathbf{p}^{(l)}+\omega^{(l)}\mathbf{s}^{(l)}.$$

If $(l+1)<L$, go to **Step 7**.

If $(l+1)=L$, go to **Step 8**.

Step 7. Calculate:

$$\mathbf{r}^{(l+1)}=\mathbf{s}^{(l)}-\omega^{(l)}\mathbf{v}^{(l)},\ \beta^{(l)}=\frac{\left(\mathbf{r}^{(l+1)},\bar{\mathbf{r}}^{(0)}\right)}{\left(\mathbf{r}^{(l)},\bar{\mathbf{r}}^{(0)}\right)}\times\frac{\alpha^{(l)}}{\omega^{(l)}},$$
$$\mathbf{p}^{(l+1)}=\mathbf{r}^{(l+1)}+\beta^{(l)}\mathbf{p}^{(l)}-\beta^{(l)}\omega^{(l)}\mathbf{q}^{(l)}.$$

Set $l=(l+1)$.

Step 8. Set $\hat{\mathbf{x}}_{ITER}=\hat{\mathbf{x}}_{ITER}^{(l+1)}$ as solution.

Output data: vector $\hat{\mathbf{x}}_{ITER}$ with the dimensionality M.

The described detection algorithm can be used for massive MIMO systems, providing the reduction in computational complexity compared to the known MMSE algorithm (4). Next, we will draw the analysis of computational complexity and resistance to noise of the described iterative detection algorithm.

V. Analysis of Resistance to Noise of Proposed Iterative Detector

To evaluate the effectiveness of the developed iterative algorithm, a comparison of its resistance to noise with the MMSE (4) for massive MIMO antennas configurations is given below. Simulation conditions are in Table I.

TABLE I. Simulation conditions

Parameter	Value
Channel	MIMO
Fading	Uncorrelated Rayleigh
Number of transmitting antennas	128; 256
Number of receiving antennas	128; 256
Architecture	V-BLAST
Modulation	16-QAM
FEC	Turbo-code with rate 1/2
Number of iterations for iterative detector	8; 10; 12

Frame error rate for massive MIMO systems with 16-QAM modulation and Forward Error Correction (FEC) for antennas configuration 128×128 is shown on Fig. 2.

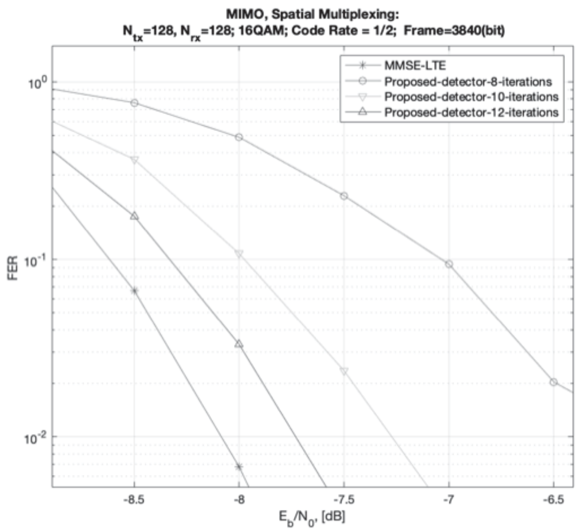

Fig. 2. Comparison resistance to noise in massive MIMO system with 16-QAM and FEC for antennas configuration 128×128

Fig. 2 shows that the resistance to noise in massive MIMO systems with 16-QAM modulation in case of application of the proposed iterative detection algorithm Algorithm 1 is 0,9 dB by $FER = 10^{-2}$ compared to the known MMSE algorithm (4).

Frame error rate for massive MIMO systems with 16-QAM modulation and FEC for antennas configuration 256×256 is shown on Fig. 3.

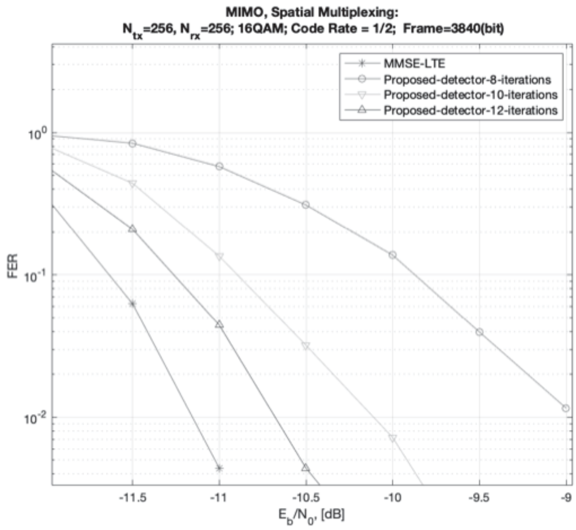

Figure 3. Comparison resistance to noise in massive MIMO system with 16-QAM and FEC for antennas configuration 256×256

Fig. 3 shows that the application of the proposed iterative detection algorithm Alg. in massive MIMO system with antennas configuration 256×256 and FEC with modulation 16-QAM gives a loss of about 0,45 dB by level $FER = 10^{-2}$ compared to the known MMSE algorithm (4).

VI. The Analysis of Computational Complexity of Iterative Detector

Table II shows the computational operations of the iterative algorithm for detection of Alg.1 at each step of the algorithm

TABLE II. Computational complexity of iterative detector Algorithm 1

Step number	Operation	Computational complexity	Number of repetitions
Step 1.	$\mathbf{Y} = \mathbf{H}^H \mathbf{y}$	$Z_{3MMATVEC}(N) = 7N^2 + N$	1
Step 6.	$\mathbf{g} = \mathbf{H}\mathbf{p}^{(l)}$; $\mathbf{u} = \mathbf{H}^H \mathbf{g}$;	$Z_{3MMATVEC}(N) = 7N^2 + N$	L

	$\mathbf{f}=\mathbf{H}\mathbf{s}^{(l)}$; $\mathbf{w}=\mathbf{H}^H\mathbf{f}$		
	$2\sigma^2\left(\mathbf{I}\mathbf{s}^{(l)}\right)=2\sigma^2\mathbf{s}^{(l)}$; $2\sigma^2\left(\mathbf{I}\mathbf{p}^{(l)}\right)=2\sigma^2\mathbf{p}^{(l)}$; $\alpha^{(l)}\mathbf{q}^{(l)}$; $\alpha^{(l)}\mathbf{p}^{(l)}$; $\omega^{(l)}\mathbf{s}^{(l)}$	$Z_{NUMVEC}(N)=2N$	L
	$\left(\mathbf{r}^{(l)},\bar{\mathbf{r}}^{(0)}\right)$; $\left(\mathbf{q}^{(l)},\bar{\mathbf{r}}^{(0)}\right)$; $\left(\mathbf{v}^{(l)},\mathbf{s}^{(l)}\right)$; $\left(\mathbf{v}^{(l)},\mathbf{v}^{(l)}\right)$	$Z_{SKAL}(N)=8N-2$	L
	$\mathbf{u}+2\sigma^2\mathbf{p}^{(l)}$; $\mathbf{w}+2\sigma^2\mathbf{s}^{(l)}$; $\mathbf{r}^{(l)}-\alpha^{(l)}\mathbf{q}^{(l)}$; $\alpha^{(l)}\mathbf{p}^{(l)}+\omega^{(l)}\mathbf{s}^{(l)}$; $\hat{\mathbf{x}}^{(l)}+\left[\alpha^{(l)}\mathbf{p}^{(l)}+\omega^{(l)}\mathbf{s}^{(l)}\right]$	$Z_{VECSUM}(N)=2N$	L
	$\alpha^{(l)}=\frac{\left(\mathbf{r}^{(l)},\bar{\mathbf{r}}^{(0)}\right)}{\left(\mathbf{q}^{(l)},\bar{\mathbf{r}}^{(0)}\right)}$; $\omega^{(l)}=\frac{\left(\mathbf{v}^{(l)},\mathbf{s}^{(l)}\right)}{\left(\mathbf{v}^{(l)},\mathbf{v}^{(l)}\right)}$	$Z_{DIV}(N)=N$	L
Step 7.	$\omega^{(l)}\mathbf{v}^{(l)}$; $\beta^{(l)}\mathbf{p}^{(l)}$; $\omega^{(l)}\mathbf{q}^{(l)}$	$Z_{NUMVEC}(N)=2N$	L-1
	$\left(\mathbf{r}^{(l+1)},\bar{\mathbf{r}}^{(0)}\right)$; $\left(\mathbf{r}^{(l)},\bar{\mathbf{r}}^{(0)}\right)$	$Z_{SKAL}(N)=8N-2$	L-1
	$\frac{\alpha^{(l)}}{\omega^{(l)}}$; $\frac{\left(\mathbf{r}^{(l+1)},\bar{\mathbf{r}}^{(0)}\right)}{\left(\mathbf{r}^{(l)},\bar{\mathbf{r}}^{(0)}\right)}$	$Z_{DIV}(N)=N$	L-1
	$\beta^{(l)}\left[\omega^{(l)}\mathbf{q}^{(l)}\right]$; $\frac{\left(\mathbf{r}^{(l+1)},\bar{\mathbf{r}}^{(0)}\right)}{\left(\mathbf{r}^{(l)},\bar{\mathbf{r}}^{(0)}\right)}\times\frac{\alpha^{(l)}}{\omega^{(l)}}$	$Z_{MULT}(N)=N$	L-1
	$\mathbf{s}^{(l)}-\omega^{(l)}\mathbf{v}^{(l)}$; $\mathbf{r}^{(l+1)}+\beta^{(l)}\mathbf{p}^{(l)}$; $\beta^{(l)}\mathbf{p}^{(l)}-\beta^{(l)}\omega^{(l)}\mathbf{q}^{(l)}$	$Z_{VECSUM}(N)=2N$	L-1

As Table II shows the total complexity of the proposed detection algorithm Alg.1 can be calculated by the following equation:

$$Z_{ITERMIMO}(N)=Z_{3MMATVEC}(N)+ \\ +L*\begin{bmatrix}4*Z_{3MMATVEC}(N)+5*Z_{NUMVEC}(N)+\\+4*Z_{SKAL}(N)+5*Z_{VECSUM}(N)+2*Z_{DIV}(N)\end{bmatrix}+ \\ +(L-1)*\begin{bmatrix}3*Z_{NUMVEC}(N)+2*Z_{SKAL}(N)+\\+3*Z_{VECSUM}(N)+2*Z_{DIV}(N)+2*Z_{MULT}(N)\end{bmatrix} \tag{6}$$

By simplifying the equation (6), we get the following expression:

$$Z_{ITERMIMO}(N)=7N^2+N+ \\ +L*\begin{bmatrix}28N^2+4N+10N+\\+32N-8+10N+2N\end{bmatrix}+ \\ +(L-1)*\begin{bmatrix}6N+16N-4+\\+6N+2N+2N\end{bmatrix}= \\ =7N^2+N+L*\left[28N^2+58N-8\right]+(L-1)*\left[32N-4\right] \tag{7}$$

Table III gives the comparison of the number of operations required to calculate the estimate of the application of the known MMSE algorithm (4) and the proposed iterative detector Alg .1 for antennas configurations and 256×256.

TABLE III. COMPUTATIONAL COMPLEXITY OF KNOWN AND PROPOSED DETECTOR FOR COMMUNICATION SYSTEMS WITHFEC

Complexity of calculating estimate of the vector of the transmitted symbols				The ratio between number of operations proposed detector Algorithm 1 and MMSE (4)
Detector	MMSE (4)	Proposed detector Algorithm 1		
Antennas configuration	Number of additions and multiplications		Number of iterations	
	16 924 160 [2]	4 664 844	10	0,28
256×256	134 806 784 [2]	22 390 900	12	0,17

VII. CONCLUSION

The developed iterative detection algorithm Alg .1 allows to perform the procedure of calculating the vector estimation of transmitted characters on the receiving side with less computational complexity and acceptable losses in resistance to noise compared to the known MMSE algorithm (4), which is essential for wireless communication systems built on the basis of massive MIMO technology.

Losses in resistance to noise during the application of the developed detection algorithm Algorithm 1 in massive MIMO systems with 16-QAM modulation and turbo-coding (rate ½) is 0,9 dB and 0,45 dB by $FER = 10^{-2}$ while reducing the computational complexity by 3,6 and 6 times compared to the known MMSE algorithm (4) for massive MIMO antennas configurations and 256×256, respectively.

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