Air Navigation: The Method of Airborne Vehicles' Classification Based on Fuzzy Colored Petri Nets

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Abstract—**Based on mathematical methods of fuzzy colored Petri nets proposed is a method of airborne vehicles' identification in controlled airspace allowing to reduce the dependence of the degree of fuzzy net model of the corresponding software's verification on the degree of dynamic interacting processes in the subject area. Proposed are the data of fuzzy rule-oriented requirements of the fuzzy logical system of airborne vehicles' classification in the process of airspace control. Developed is a unified algorithm of these requirements' implementation. Further, the fuzzy rule-oriented requirements are in structural conformity with the requirements of the Sugeno fuzzy logical system of the first-order. This view of the airborne vehicles' classification process using the fuzzy logical system allows considering both the non-stochastic and subjective nature of operators' decision-making. The developed unified algorithm of the fuzzy rule-oriented requirements' implementation within the Sugeno system of the first-order is the basis for the software of the fuzzy logical system of airborne vehicles' classification.**

I. Introduction

Everyday a lot of industrial facilities such as airports, oil refineries, nuclear power plants, electrical power lines and gas pipelines suffer from attacks with the use of different types of airborne objects, primarily unmanned airborne vehicles. At the same time, unmanned aerial vehicles are used everywhere to monitor and protect these industrial facilities [1]. The specific technical equipment is necessary to facilitate identifying a threat emanating from a certain airborne object in the specified area of responsibility in the automatic or automated modes using the unified data management system [2], [3], [4].

As it may be inferred from the successful completion of similar tasks, one of the most advanced methods of developing the software for such technical equipment controlling airspace is the method of fuzzy Petri nets' interpretation. This method has proved the capability of solving multilevel tasks. Besides, it allows adapting to the peculiarities of the specific subject area's process.

One of the possible methods of developing the software for such technical equipment controlling airspace is the method of fuzzy Petri nets' interpretation. This method has proved the capability of solving multilevel tasks. Besides, it allows adapting to the peculiarities of the specific subject area's process.

As indicated in [5], the process of verifying the software of the fuzzy logical system of airborne vehicles' classification can be generally considered as the research process into the dynamic interacting processes. Moreover, known are the specific theoretical and practical results of using Petri nets for the research into the dynamic interacting processes [6], [7].

On the one hand, although the existing approaches to forming coloured Petri nets and fuzzy colored Petri nets have a wide range of application, their implementation into modern technologies is hardly feasible without further development. On the other hand, there are no methods of direct software verification of the fuzzy logical system of airborne vehicles' classification in the process of airspace control. It became a starting point for using a new type of extended fuzzy colored Petri nets devoid of the shortcomings listed above and allowing for the verification of the software of the fuzzy logical classification system. These Petri nets are characterized by the following features [8], [9]:

- the capability of forming fuzzy net models characterized by natural interpretation, simplicity of description, creation of the dynamic fuzzy interacting processes, presented in numerous «condition-action» relationships taking into account numerous real specifications, features, factors and limits of a certain subject area;
- the adaptation to classes of problems and subject area in addressing the set of considered problems of verifying the software of the fuzzy logical classification's system;
- the solution of the set of problems under consideration as a unified problem of creating models, criteria,

methods and effective tools with the use of modern digital technologies.

II. Methods and Models

Colored Petri nets can be generally described as follows:

$$S = <P,T,F,V,K,C,M_0> \quad (1)$$

where P is a set of positions; T is a set of transition; $F:(P\times T)\cup(T\times P)$ is the function of incidence; C is the function of the colour of the token; V denotes conditions for sequence of transition depending on the colour of the token; K is the capacity of the tokens in regarding to C; M_0 is the vector of initial marking.

The net model of describing interacting processes based on colored Petri nets (1) has a smaller size than the model based on traditional Petri nets. It is due to the fact that each colour $ci \in C$, $\forall i \in I$ contains information about a condition, which is typical only for it, and several tokens of different colours can be located in a certain position. Traditional Petri nets are ordinary Petri nets [10], [11], [12] without any enhancement.

A fuzzy colored Petri net is as follows:

$$\begin{aligned} S'_C(f) = \langle P', T', F'(f), M'_0(f), M'_C(f), \ldots \\ \ldots L\{x_u\}, u \in U, C', V', K' \rangle, \end{aligned} \quad (2)$$

where P' denotes the numbers of fuzzy positions; T' is a set of fuzzy transition; $L\{xu\}$, $u \in U$ is the predicate referring to the numbers of positions, transition, function of incidence in the condition over space of the fuzzy interacting processes and testifying the additional conditions of hadling the transition; $F'(f) = (P'\times T')\cup(T'\times P')$ is the fuzzy function of incidence; C' is the function of the color of each token $M'(p'i)$ for net positions; V' denotes conditions for the sequence of transition depending on the color of the token; K' is the capacity of the tokens regarding to C'; $M'_0(f)$ is the vector of initial marking; $M'_c(f)$ is the vector of current marking.

The introduction of the predicate $L\{xu\}$, $u \in U$ and features C', V', K' into model (2) significantly increases the model's capabilities in comparison with the existing approaches. The introduction of colour features reduces the cardinality of large sets P', T', matrix sparsity of the function of incidence $F'(f)$ and matrix incidence $H'(f)$ increase.

In turn, the function of incidence $F'(f)$ and matrix incidence $H'(f)$ sparseness can be presented as follows:

$$1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(x''_{ij}(k_0) + y'_{ij}(k_0)\right)}{T'P'} \quad (3)$$

in which

$$x'_{ij}(k_0) = \begin{cases} 1, & if \quad x'_{ij}(k_0) \neq 0, \\ 0, & if \quad x'_{ij}(k_0) = 0, \end{cases}$$

$$y'_{ij}(k_0) = \begin{cases} 1, & if \quad y'_{ij}(k_0) \neq 0, \\ 0, & if \quad y'_{ij}(k_0) = 0. \end{cases}$$

Given the net $S'_c(f)$ and certain vectors $M'_0(f)$ and $M'_c(f)$, then the point $M'_b(p'j)$ of b-th color with $C_bj \in \{C_bj\}$, $b \in B$, $j \in J$ marking the position $p'_j \in P'$ defines the existence of the b-th resource specified on the set of resources $Rb \in \{Rb\}$, $b \in B$ of the stimulated processes in the subject area. In this case, the value $C = \{C\alpha\}$, $\alpha \in A$, in which α is a certain color with A, and the volume of K tokens for a certain position $p'_j \in P'$ are connected as follows: $K_{p'_j} = \left|C_{\alpha p'_j}\right|$.

The rationale of this provision (3) is based on the interpretation of the point $M_\alpha(p'_j) = 1$ of the color α. In this case, in addition to the actual colour, the function $C = \{C_\alpha\}$, $\alpha \in A$ defines the volume of the points of each color in the positions $p'_j \in P'$ of the net $S'_c(f)$.

The presence of a certain set of colored points $|B|$ in the positions of the net $S'_c(f)$ requires the definition of conditions for the permissibility of its transition. A certain transition $t'_i \in T$ of the net $S'_c(f)$ is allowed on condition that:

$$R_t(S'_c(f)) = t'_i \in T' \left| \mu_t(k_0) \geq \mu_t(k_0)^* \right. and$$

$$\forall p'_i \in \{p'_i(i_n)\} \left| \mu_{p'_j}(k_0) \geq \mu_{p'_j}(k_0)^* \right. and$$

$$\left(\forall M'_{c_\alpha}(p'_j) \in M'_c(f) \left| M'(p'_j) > 1 \; and \; z_{p'_j}(k_0) \geq z_{p'_j}(k_0)^* \right.\right) and$$

$$x_{ij}(k_0) \geq x_{ij}(k_0)^* \; and \; L = true \; and \; V' = true$$

in which $M'_{c_\alpha}(p'_j)$ is the token of the color α in the position p'_j; $V = V\left(K_{p'_j \in \{p'_i(i_n)\}}, C_{\alpha p'_j \in \{p'_i(i_n)\}}\right)$ is the condition for the sequence of the transition depending on the color and volume of the tokens.

The suggested model based on the fuzzy colored Petri net $S'_c(f)$ can solve a set of practical tasks quite effectively, including the verification of the software of the fuzzy logical system for airborne vehicles' classification in the process of airspace control.

The presentation of interacting processes is considered for the following cases:

1) The process is performed given only one input condition and only one output condition

$$\exists t'_i \in T' \left\| \{p'_i(in)\} \right| = \left|\{p'_i(out)\}\right| = 1 \quad (4)$$

2) The process is performed given several unequal input conditions and only one output condition

$$\exists t'_i \in T' \left\| \{p'_i(in)\} \right| > 1 \; and \; \left|\{p'_i(out)\}\right| = 1 \quad (5)$$

3) The process is performed given only one input condition and several unequal output conditions

$$\exists t_i' \in T' \left\| \{p_i'(in)\} \right| = 1 \; and \; \left| \{p_i'(out)\} \right| > 1 \quad (6)$$

4) A certain condition for the execution of a process has several unequal input processes and only one output process

$$\exists p_i' \in P' \left\| \{t_i'(in)\} \right| > 1 \; and \; \left| \{t_i'(out)\} \right| = 1 \quad (7)$$

5) A certain condition for the execution of a process has

$$\exists p_i' \in P' \left\| \{t_i'(in)\} \right| = 1 \; and \; \left| \{t_i'(out)\} \right| > 1 \quad (8)$$

only one input process and several unequal output processes

6) A certain condition has only one output process

$$\exists p_i' \in P' \left\| \{t_i'(in)\} \right| = 0 \; and \; \left| \{t_i'(out)\} \right| = 1 \quad (9)$$

7) A certain condition has only one input process

$$\exists p_i' \in P' \left\| \{t_i'(in)\} \right| = 1 \; and \; \left| \{t_i'(out)\} \right| = 0 \quad (10)$$

8) The process is performed in the presence of several unequal input conditions and several unequal output conditions

$$\exists t_i' \in T' \left\| \{p_i'(in)\} \right| > 1 \; and \; \left| \{p_i'(out)\} \right| > 1 \quad (11)$$

9) A certain condition for the execution of a process has several unequal input processes and several unequal output processes

$$\exists p_i' \in P' \left\| \{t_i'(in)\} \right| > 1 \; and \; \left| \{t_i'(out)\} \right| > 1 \quad (12)$$

in which $\{t'_j(in)\}$ denotes a set of input processes of the position p'_j; $\{t'_j(out)\}$ denotes a set of output processes of the position p'_j.

Formula (11) can be represented by sequential connection of formulas (5) and (6); formula (12) can be represented by sequential connection of formulas (7) and (8).

For each of formulas (4) – (8), the conditions for the allowed transitions and the conditions for marking the positions of the models are determined.

Taking into consideration that formulas (4) – (6) differ only in the number of input |{p'i(in)}| and / or initial positions |{p'i(out)}| of the given transition t'j, then the corresponding conditions are valid for them

$$\begin{gathered} t_i' \in T' : \mu_{t_i'}(k_0) \geq \mu_{t_i'}(k_0)^* \; and \; \forall p_j' \in \{p_i'(in)\} \\ \left| p_j' : \mu_{p_i'}(k_0) \geq \mu_{p_i'}(k_0)^* and \right. \\ \forall M'(p_j') \in M'(f) \left| M'(p_j') \geq 1 \; and \right. \\ z_{p_j'}(k_0) \geq z_{p_j'}(k_0)^* and \\ x_{ij}(k_0) \geq x_{ij}(k_0)^* \; and \; L = true \end{gathered} \quad (13)$$

in which $\mu_{t_i'}(k_0)^*$, $\mu_{p_i'}(k_0)^*$, $z_{p_i'}(k_0)^*$, $x_{ij}(k_0)^*$ denote the limits of the corresponding functions' value, k_0 is a certain meaning of the variable k, which determines the specific meaning of the corresponding function based on expert assessments of the subject area.

Taking into consideration that formulas (7) – (10) differ in the number of input $|\{t'_j(in)\}|$and (or) output $|\{t'_j(out)\}|$ transitions of the given position p'_j, then for the transitions $\{t'_i\}$, which are included in the corresponding formula with (7) – (10), the above-mentioned conditions of their resolution are also valid.

The interacting dynamic fuzzy processes corresponding to the software verification of the fuzzy logical system for airborne vehicles' classification in the process of airspace control can be formally presented as analytical representations and predicate logic containing logical, int. al., fuzzy operations: AND, OR , NOT, their derivatives and logical functions; graphical representation, e.g. in the form of graph-schemes of algorithms, including fuzzy production rules based on *if then* relations, which generally contain fuzzy representations.

Thus, describing interacting dynamic fuzzy processes represented by predicate logic, it is essential, at least, to represent logical operations AND, OR, NOT and membership functions, $\mu_{I'i}$, $i \in I$ components of the software verification model using a fuzzy colored Petri net fuzzy logical system of airborne vehicles' classification.

The operation < *and* a_i >, $i \in I$ can be represented by formula (5). This position is based on the property of resolving the transition t'_i of formula (5) marking all the input positions $p'_j \in \{p'_i(in)\}$ of the segment. The operation < *or* a_i>, $i \in I$ can be represented by the segment of model (7). This position is based on the property of marking the position p'j of segment (7) performing at least one transition $t'_i \in \{t'_j(in)\}$ of the position p'j of the segment.

In this interpretation, security violation and the occurrence of a conflict are possible in segment (7), which must be taken into account in the practical implementation of the software verification process for a fuzzy logical system of airborne vehicles' classification.

The operation NOT can be represented by introducing an inhibitory arc into formula (4) and modifying the sparsity of its transition so that it will be allowed if the formula is true

$$\begin{gathered} \exists t_i' \in T' \left\| p_i'(in) \right| = \left| p_i'(out) \right| = 1 \; and \\ \left(M'(p_j') = 0 \; or \; z_{p_j'}(k_0) < z_{p_j'}(k_0)^* \right) \end{gathered} \quad (14)$$

This position is based on the property of inhibitory arcs, when the sparsity of the transition t'_i of the modified segment (4) is possible if there is no marking of the input position $p'_j \in \{p'_i(in)\}$ of the segment and / or formula (13) and (14) is true.

In the case of the problem of interacting dynamic fuzzy processes, complex procedures in the form of a graphical representation of algorithms, it is essential to select and describe such elements of a fuzzy logical system of airborne vehicles' classification as computational process, control process, decision-making; developing processes according to a logical condition; developing processes for the implementation of at least one of the previous processes; parallelization of processes; developing processes upon the completion of all the previous ones; initiating processes' development; achieving the desired result.

Separate segments of algorithms, e.g., pertaining to the «conditioned process» type, the output of results to an external device, etc., can be represented by the above-mentioned formulas without any loss of data and adequacy of their display.

The main statements determining the interpretation of the model's segments are formulated as follows:

- the computational process, control process, decision making I'i can be represented in the model by formula (4);
- the fuzzy processes' development for the fulfillment of a logical condition in the model can be represented by formula (9);
- the development of processes for the implementation of at least one of the previous processes in the model can be represented by formula (10);
- the procedure of parallelizing the processes {I'i} can be represented by formula (6) of the model;
- the procedure of developing the processes {I'i} upon the completion of all the previous processes can be represented by formula (5);
- the procedures for starting the development of processes and obtaining the desired result can be represented, respectively, by formulas (9) and (10).

The procedures for presenting production rules of the form if/then are determined by formulas (4) – (10) of the model. If the segments of the basic task are specified by the production rules of the form if/then, then it is obvious that the production rules can be represented by formulas of the model S'(f), S'c(f), similarly to the representation of interacting processes by predicates, logical functions based on logical operations.

Consider the structure of the production rules in a clear representation of knowledge

$$\textit{if } A \textit{ and } B \textit{ and } C \textit{ then } D \tag{15}$$

In a verbal expression, (18) can be represented as follows: if A and B and C are true, then perform action D, or, in the language of Boolean logic:

$$D = true \,|\, A \textit{ and } B \textit{ and } C = true \tag{16}$$

For formulas similar to (16), solutions have already been found regarding the representation of logical operations by the corresponding formulas with (4) – (8) of fuzzy net models $S'(f)$, $S'_c(f)$. It is possible to show the truth of the corresponding solutions for the production rules, which contain the operations OR, NOT and their derivatives.

Fuzzy properties of both predicate components and graphical representations of algorithms and production rules are completely determined by the corresponding membership functions. Similarly to (15) and (16), in a fuzzy representation, it can be defined as

$$\begin{gathered} \textit{if } A' \textit{ is } \mu_{A'}(k) \textit{ and } B' \textit{ is } \mu_{B'}(k) \textit{ and} \\ C' \textit{ is } \mu_{C'}(k) \textit{ then } D' \textit{ is } \mu_{D'}(k) \end{gathered} \tag{17}$$

$$\begin{gathered} D' = true \big| \big((A' \textit{ and } B' \textit{ and } C') = true\big) \textit{ and} \\ \big(\mu_{A'}(k_0) \ge \mu_{A'}(k_0)^*\big) \textit{ and} \\ \big(\mu_{B'}(k_0) \ge \mu_{B'}(k_0)^*\big) \textit{ and } \big(\mu_{C'}(k_0) \ge \mu_{C'}(k_0)^*\big) \textit{ and}, \\ \big(\mu_{D'}(k_0) \ge \mu_{D'}(k_0)^*\big) \end{gathered} \tag{18}$$

in which $\mu_{A'}(k_0)^*, \mu_{B'}(k_0)^*, \mu_{C'}(k_0)^*, \mu_{D'}(k_0)^*$ are the valid values of the corresponding membership functions.

The membership functions $\{\mu_{I'_i}(k)\}$ of the set of processes $\{I'_i\}$ determining the conditions and actions in the subject area are displayed on the set of fuzzy positions $\{p'_j\}$ and fuzzy transition $\{t'_i\}$ into the spatial states of the model $S'(f)$, $S'_c(f)$. This is due to the fact that the fuzzy model $S'(f)$, $S'_c(f)$ reflects the fuzzy processes in the subject area. Therefore, to represent a segment of certain fuzzy knowledge as a production rule containing fuzzy conditions and fuzzy actions it can be written as in (17).

The process of airborne vehicles' classification is considered as establishing the attribution of a given airborne object $x \in X$ to a predetermined class of airborne objects according to the relation:

$$F : X \to \{K_i^{\varepsilon}\}, \tag{19}$$

in which X is the set of all detected airborne objects; x is the airborne object N_i^{ε}, $i = \overline{1, L}$, L is the number of detected airborne objects; K_i^{ε} is the class of the airborne object, $i = \overline{1,10}$.

Formally, the predetermined classes of airborne objects are specified as a set $\{K_i^{AO}\}$, the components of which are:

- class K_1^{ε} «non-identified» is determined automatically or set automatically;
- class K_2^{ε} «flight request» is determined automatically or set automatically;
- class K_3^{ε} «intruder» is determined automatically or set automatically;
- class K_4^{ε} «not signalling» is determined automatically or set automatically;
- class K_5^{ε} «signalling» is determined automatically or set automatically;

- class K_6^ε «jammer» is determined automatically or set automatically;
- class K_7^ε «friend» is set automatically;
- class K_8^ε «control object» is set automatically;
- class K_9^ε «border violator» is set automatically;
- class K_{10}^ε «foe» is set automatically.

The value of the set of airborne objects' features and the classes of airborne objects of the set $\{K_i^\varepsilon\}$ are in the binary interrelation (19), which is specified by the corresponding matrix of relations

$$\rho = [d_{ji}]_{m \times n}, \qquad (20)$$

in which m is the number of values of airborne object's features; n is the number of airborne objects' classes.

For the elements of the set $\{C_i^\varepsilon\}$ for airborne vehicles' classification, the following features are considered:

- the sign of determining state affiliation $\{C_i^\varepsilon\}$;
- the sign of flight plan correlation C_2^ε;
- the sign of a violation of the flight mode C_3^ε;
- the sign of trajectory tracking according to the triangulation method C_4^ε.

The values of the features C_1^ε, C_2^ε, C_4^ε are determined by the results of generalization of the trace information about the airborne object and are linguistic variables (LV) in terms of fuzzy sets.

The sign value C_3^ε is set by default to 0 (no violation) and changes automatically to 1 (there are violations), if the entered type of violation of the flight mode is applicable for the corresponding airborne object, and there are linguistic rules in terms of fuzzy sets.

In general, the meaning of features C_1^ε, C_2^ε, C_4^ε and C_3^ε are indistinct numbers describing the terms of the corresponding linguistic rules.

III. Results and Discussion

The classification of airborne vehicles is performed in the automatic or automated modes. The classification in the automated mode ensures that an airborne vehicle belongs to the following classes:

- non-identified;
- flight request;
- intruder;
- not signalling;
- signalling;
- jammer.

The automatic classification process is carried out according to the following linguistically described rules:

- the «non-identified» class is attributed automatically to those airborne vehicles which feature of state affiliation is not specified;
- the «flight request» class is attributed automatically to those airborne vehicles which do not disturb the set mode of flight and for which there is complementarity of «flightplan»;
- the «intruder» class is attributed automatically to those airborne vehicles which disturb the set mode of flight and for which there is complementarity of «flightplan». The previous documented violations (side trip, altitude deviation, excess of the quantitative composition) is determined automatically by introducing this violation from the data entry console;
- the «not signalling» class is attributed automatically to those airborne vehicles, for which there is no complementarity of «flightplan», while the feature of state affiliation is «foe»;
- the «signalling» class is attributed automatically to those airborne vehicles, for which there is no complementarity of «flightplan», while the feature of state affiliation is «friend» or «neutral»;
- the «jammer» class is attributed automatically to those airborne vehicles, for which there is no complementarity of «flightplan», while the feature of state affiliation is «foe» and the trajectory is processed using the triangulation method.

The automated mode allows the operator to change the automatically classified attribution of airborne vehicles to the classes from the above-mentioned list, as well as to set or change the attribution of airborne vehicles to the following classes:

- friend;
- control object;
- border violator;
- foe.

The automated classification is performed according to the conditions of the above described rules, as well as the following rules:

- the «friend» class is attributed automatically after an operator determined the fact of an airborne vehicle's takeoff, identifying and taking it for escort; it is based on the results of the air situation's assessment, which is shown on the means of display for individual and collective use;
- the «control object» class is attributed automatically if an operator decides that the airborne vehicle is «friendly» and flew out for drilling, training, etc.;
- the «border violator» class is attributed automatically or at an operator's decision to those air vehicles, for which there is no complementarity of «flightplan», which are foreign and have crossed the state border illegally;

- the «enemy» class is attributed automatically or at an operator's decision based on the air situation developing in a certain airspace control zone.

Formally, the rules for airborne vehicle's classification determining the matrix of relations (20) are determined in the form of a set of the following fuzzy production rules:

$$\textit{if } C_1^{\varepsilon} = 0 \textit{ then } x = K_1^{\varepsilon}, \tag{21}$$

$$\textit{if } C_3^{\varepsilon} = 0 \textit{ and } C_2^{\varepsilon} = 1 \textit{ then } x = K_2^{\varepsilon} \tag{22}$$

$$\textit{if } C_3^{\varepsilon} = 1 \textit{ and } C_2^{\varepsilon} = 1 \textit{ then } x = K_3^{\varepsilon} \tag{23}$$

$$\textit{if } C_2^{\varepsilon} = 0 \textit{ and } C_1^{\varepsilon} = 01 \textit{ then } x = K_4^{\varepsilon} \tag{24}$$

$$\begin{gathered}\textit{if } C_2^{\varepsilon} = 0 \textit{ and } \left(C_1^{\varepsilon} = 10 \textit{ or } C_1^{\varepsilon} = 11\right) \\ \textit{then } x = K_5^{\varepsilon}\end{gathered} \tag{25}$$

$$\begin{gathered}\textit{if } C_2^{\varepsilon} = 0 \textit{ and } C_1^{\varepsilon} = 01 \textit{ and } C_4^{\varepsilon} = 1 \\ \textit{then } x = K_6^{\varepsilon}\end{gathered} \tag{26}$$

$$\textit{if } C_1^{AO} = 10 \textit{ and } G^d = G_1^d \textit{ then } x = K_7^{AO} \tag{27}$$

$$\textit{if } C_1^{\varepsilon} = 10 \textit{ and } G^d = G_2^d \textit{ then } x = K_8^{\varepsilon} \tag{28}$$

$$\begin{gathered}\textit{if } C_1^{\varepsilon} = 10 \textit{ and } C_2^{\varepsilon} = 0 \textit{ and } G^d = G_3^d \\ \textit{then } x = K_9^{\varepsilon}\end{gathered} \tag{29}$$

$$\textit{if } C_1^{\varepsilon} = 01 \textit{ and } G^d = G_4^d \textit{ then } x = K_{10}^{\varepsilon} \tag{30}$$

in which:

G^d is the sign of an operator's action on the analysis of the data necessary for an airborne vehicle's classification, and an action on the implementation of his decision on an airborne vehicle's class, which is formally considered as an AS (air-strip) in terms of fuzzy sets;

G_1^d determines the fact of a «friendly» airborne vehicle's take-off, identification and taking it for escort, making a decision by the operator about the class of the airborne vehicle as «friend»;

G_2^d determines the fact of take-off, identification and taking an airborne vehicle for escort for the purpose of drilling, training, etc. and making a decision by the operator about the class of the airborne vehicle as «control object»;

G_3^d determines the fact of crossing the state border by a foreign airborne vehicle and making a decision by the operator about the class of the airborne vehicle as «enemy»;

G_4^d making a decision by the operator about the class of an airborne vehicle as a «foe», based on the air situation developing in the area of his responsibility.

Generally, the values of the attribute of the operator Gd are fuzzy numbers describing the terms of the corresponding linguistic rules.

The structure of the directly generalized algorithm for the implementation of fuzzy production rules for airborne vehicles' classification (the results of fuzzy logical inference based on the corresponding rule base) includes the following algorithms for solving particular problems based on the use of zero-order Sugeno fuzzy inference mechanisms:

- the algorithm for solving a particular problem of automatic airborne vehicles' classification according to their characteristics;
- the algorithm for solving a particular problem of automated airborne vehicles' classification according to their characteristics.

The execution of the algorithm implementing fuzzy production rules for airborne vehicles' classification (21) – (26) includes the following operations and actions:

- the data on the airborne vehicle are received according to the results of route information's generalization;
- for a given airborne vehicle, the sign of violation of the flight mode is set to «0» by default;
- the significance of the signs of the state border's violation by an airborne vehicle is analyzed;
- if the sign of violation of the state border is not determined, then the airborne vehicle is classified as «non-identified»;
- if the sign of violation of the state border is determined, then the value of the sign of the «flightplan» correlation is analyzed;
- if there is no «flightplan» correlation, then the operator's introduction of a violation of the flight mode for the given airborne vehicle is checked;
- if a violation is entered by the operator, then the sign of violation of the flight mode is set to «1» by default for this airborne vehicle, so it is classified as a violator of the flight mode;
- if the violation is not entered by the operator, then the airborne vehicle is classified as a flight request;
- if there is a correlation of the «flightplan», then the sign of violation of the state border is analyzed;
- if the sign of violation of the state border has the value «foe» («01»), then the sign of tracking the trajectory of its flight is analyzed according to the triangulation method;
- if there is no tracking of the trajectory of an airborne vehicle by the triangulation method, then it is classified as not signalling;
- if there is a tracking of the trajectory of an airborne vehicle by the triangulation method, then it is classified as a jammer;

- if the sign of violation of the state border does not have the value «01», then the airborne vehicle is classified as signalling;
- the classification results are shown on operators' visual display units.

The execution of the algorithm implementing the fuzzy production rules (21) – (30) for airborne vehicles' classification includes the following operations and actions:

- the automatic airborne vehicles' classification is performed and the data is simultaneously received on non-identified airborne vehicles according to the results of the route information's generalization;
- operators makes decisions on the classification of airborne vehicles which have not been automatically classified, or upon changing the results of automatic classification;
- the selection of the class of an airborne vehicle when the appropriate conditions are met in accordance with the rules for classifying airborne vehicles;
- show the classification results on operators' visual display units.

The above-mentioned fuzzy production rules for the classification of airborne objects in the process of airspace monitoring correspond to a fuzzy colored Petri net (2). This fuzzy colored Petri net is a fuzzy network model for software verification of a fuzzy logical system for classifying airborne objects and is considered a feasible model within the framework of formal software verification methods for further research in the field of creating an airspace control system.

IV. Conclusion

The extended fuzzy colored Petri nets have been chosen as the basic mathematical tool for verifying the software [13–20] of the fuzzy logical system for classifying airborne vehicles in the process of airspace control. This class of Petri nets provides a decrease in the dependence of the dimension of the fuzzy net verification model on the dimension of the dynamic interacting processes in the subject area.

Moreover, a set of fuzzy production rules of the fuzzy logical system for the classification of airborne vehicles in the process of monitoring the use of airspace and a generalized algorithm for the implementation of these rules has been developed. Structurally, fuzzy production rules correspond to the rules Sugeno fuzzy logical system of the first order. The formal representation of the process of airborne vehicles' classification using the fuzzy logical model allows taking into account the non-stochastic and subjective nature of the decision-making process by operators controlling the airspace. The developed generalized algorithm for the implementation of fuzzy production rules within the Sugeno system of the first-order is the basis for creating software for the fuzzy logical system for airborne vehicles' classification.

The considered approach can be used as the basis for the software verification method for the fuzzy logical system for airborne vehicles' classification, which ensures the transformation of the set of fuzzy production rules into a workable model based on the use of fuzzy colored Petri nets within the model of formal software verification methods. As a result of the method's implementation, there should be an analytical report on the possible strategy of the model and the identification of all possible classes of airborne vehicles, the data on which are received by the air traffic control system.

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