Reflective Models of Information Use for System Functioning

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***Abstract*—The article is dedicated to the consideration of the problem of the application of information [1] for system functioning, specifically focusing on the conceptual and formal definitions of researcher's practical problems. The objective is to develop a formal representation and predictive explanation of information utilization, aiming to enable accurate predictions of the impact of information usage in altered circumstances, based on the utilization of mathematical models.**

The author introduces the concept of a complex state as a state that encompasses reflections of information, along with graph-theoretic models of such states in the form of potential sequences of complex states and transitions. These can be leveraged to construct the necessary graph-theoretic and formal algebraic models. These models will facilitate the computation of quantitative success metrics for information application in system functioning.

The article emphasizes various formal research directions in this domain, pertaining to the formulation of mathematical problems concerning information application and the resolution of practical challenges related to information utilization for system functioning, aligning with the corresponding mathematical problems. It presents a novel approach to addressing issues related to information application, as well as the interdisciplinary challenges associated with the formal evaluation of information application across diverse types of system operations, employing mathematical models and methodologies.

I. Introduction

Mathematical modeling is essential for solving many problems related to information application. For instance, in the context of production system operations, it may be necessary to quantitatively estimate the impact of information derived from the system environment at different points in time on the system's performance, depending on the variables of the operational plan being employed. These estimation results can then be utilized to select a more optimal plan based on prevailing environmental conditions.

Such tasks arise, for example, when plans for sustainable system functioning are developed or when agile system functioning is designed to adapt to changing environmental conditions. One of the distinguishing features of such tasks is information obtained through actions designed for information exchange or production (information actions) as part of the system functioning process. This information is acquired and used to modify system functioning based on the information obtained.

Various models are required to simulate the application of such information. Among them is the model of information action, which aims to acquire and/or transmit information. In this article, information is understood by the author as a change in data, description, reflection, or simply as a "reflection of something that makes a difference" (in comparison with well known definition "difference which makes a difference" [1]). The result of an information action is the information transmitted or obtained; therefore, this information or data about it, its reflection, or some kind of model should be part of at least the final state of the information action.

Such complex state reflects information obtained, but may become the cause of changes in other states, including "material" states, i.e., states which are not intended for information transmission/elaboration but for energy and matter transmission in time and/or space.

The mechanism of such information use ("materialization") is through use of actions which are defined, among other things, with information. Examples of such information which alter course of action are instructions, manuals, specifications, route maps and this is information obtained as reflection of other information available to the system. Such information is used during the actions of humans or artificial objects (agents, robots) created by humans for performing activities.

During such activities, according to information reflected, individual actions and their sets (sequences) altered depending on information obtained. Information in such actions is used in cause-and-effect relations, determined by action details, which can be altered depending on human, agent, or robot choice. This choice and, so, cause-and-effect relations realized depends on information reflected and elaborated by the system. In this respect, we may say that reflected information can be "materialized back" through actions. The human mind and artificial objects (agents, robots) created by them, as a result, deemed as two-way bridges between "reflected" and "reflected back".

As a result, the material world "reflected back" through activities and by application of information previously reflected by the humans or objects created by them for that purpose.

These "bridges" can be formalized with possible routes through them and through their possible sequences characteristics (including probabilities structures and material effects characteristics structures). Through this research possibility,

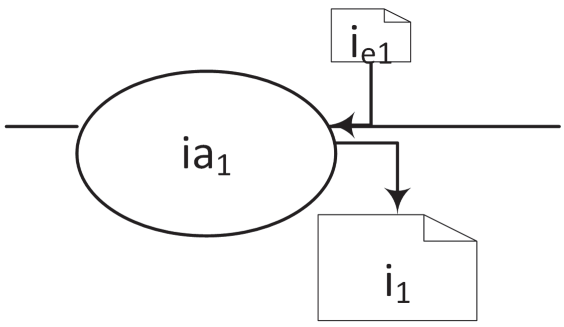

Fig. 1. The result of information obtained as a reflection

one may try to understand and evaluate results of information and information technologies applied for system functioning.

It is necessary to note the eternal analogy of such sequences “through bridges” and the ways matter acts apart from human beings and artificial objects created by humans for activities.

This analogy shall be understood through further cybernetic and system theoretic researches.

The article contains conceptual and structural (graph theoretic) models of information application for system functioning (“bridges” mentioned, formalized as routes in the routes in possible complex state trees).

Further, these routes and trees serve as the basis to develop functional models of information application for action in systems. They, in turn, allow defining various kinds of formal problems of information application for actions in systems definition.

The models and methods obtained allow to solve many practical problems of information application in various kinds of systems, mathematical formulations of problems and to automate such problems decisions using modern digital computer technologies.

II. The Concept of Complex States for Information Receipt, Reflection, and Application

An example schema of obtaining information is shown in Figure 1.

In most cases, as specified in Figure 1, information ie_1 (represented as incoming leaf) is obtained from objects in the environment through the border of the system with the use of some information action ia_1 (represented as the oval), performed by the system on its border (represented as the line). The result of such action can be reflected in some complex state cs_k inside the system, such as $i_1 \in cs_k$ (represented as outcome leaf). Complex state information substate may be represented with various formalisms, depending on reflected features. For example, this can be integral / differential equations and in this case the schema in Figure 1 will represent a kind of hybrid automata [2], acting over the model of the system. Further, “to make a difference” information shall be supplied to one of the other actions in order to change the course of some of them. This can be done by assigning information to substate of some initial state of the “material” action.

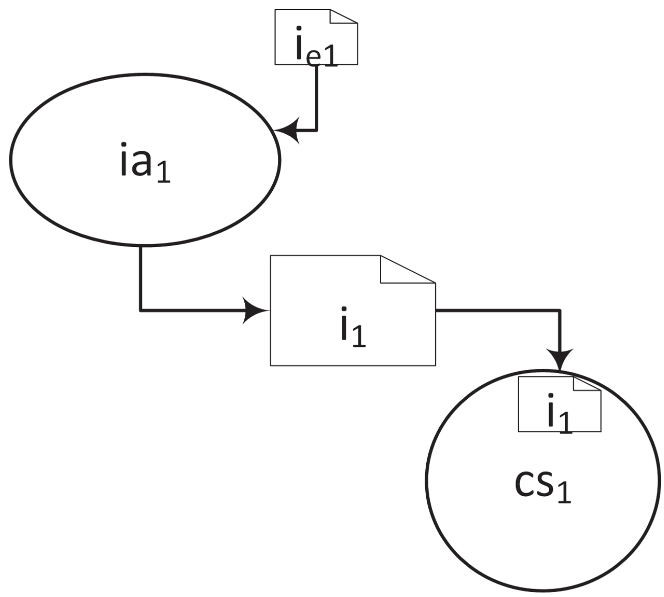

Fig. 2. Complex state creation as a result of information obtained

“Material” action is action intended to produce change in energy or substance or their transmission. Thus, its (beginning or finish or other states do not necessarily contain any information.

As a result, to model information use, some states of actions of the system functioning shall include information substates obtained due to reflections from another states (substates). An example of such informational substate i_1 is illustrated in Figure 2. It is obtained as a result of information action ia_1 (represented as oval) outcomes i_1 (represented as the outcome leaf) and its further reflection in complex state CS_1 (represented as the circle). We will name such states as complex states (CS_1). Complex states information substates contain reflected data, model, or description. That qualify models created with such complex states use as reflective. Under reflective model, I will define model which parts are reflected from some data / description or information. Complex states (CS_1) and models which are built based on it (transitions between complex states, sequences of complex states, networks of complex states, automata of complex states) are reflective models because they contain reflections of the other informational substates, data, or descriptions.

Such reflection’s objective, among other, is creation of prescriptive information for actions. In Figure 3 action ma_1 (represented as oval) uses prescription information i_1 (represented as incoming leaf) of complex state information substate for performing change (according to reflected information i_1) “material” action.

Information substates of complex states can reflect prescriptive information obtained as a result of information state transmission/transformation/elaboration with some information ac-

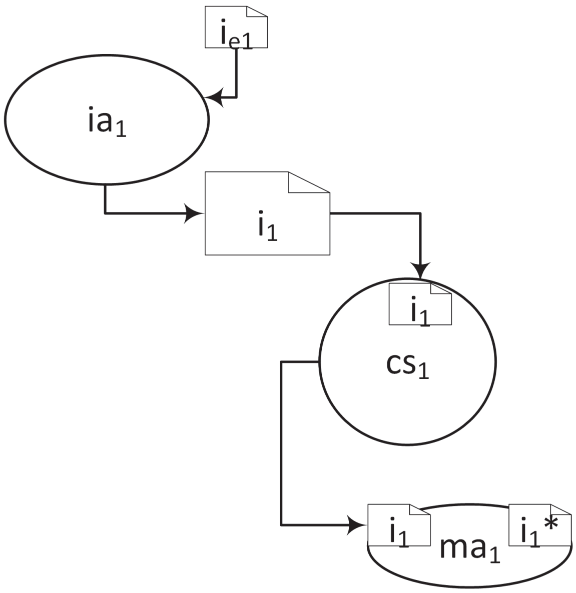

Fig. 3. Starting action based on prescriptive information i_1 in complex state CS_1, associated with action beginning

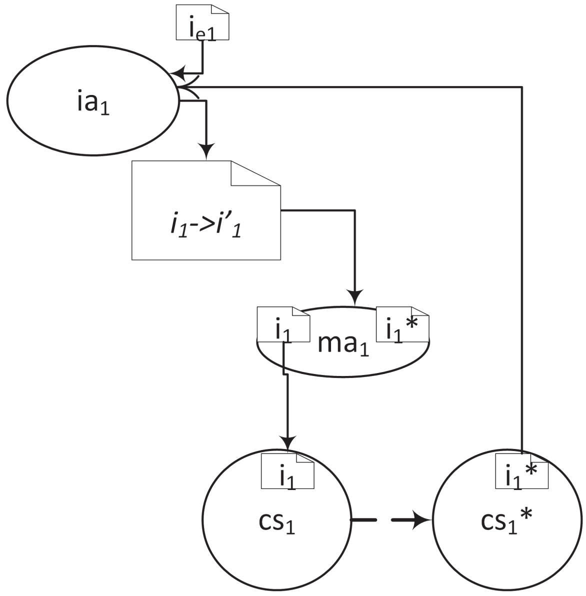

Fig. 4. Sequences of complex states

tion. For example, information substates can be constructed from instruction manuals for action or with other information, which may result in cause-and-effect relationships, realization, and actual change in final "material" states of appropriate actions performed. Then, other states following after the final state of activity, changed by the complex state, can be affected and the effects of the functioning as a whole changed.

As a result, chains of complex states changes sequences formed, which can be modelled with various kinds of reflective models.

Because of this article, these models have been contributed for use by other researchers. These models can be applied to the building of algebraic graph-theoretic models of information use scenarios. Such scenarios' events and transitions enable the creation of mathematical expressions for the effects of information use specific to each scenario. On their basis, additional functional models can be built, such as graph theoretic models' state sequences that serve as summarization indexes and other mathematical operations to account for the overall effects of information application according to the scenario that is modeled.

The one of the simplest model of this kind illustrated in Figure 4. Two complex states CS_1 and CS_1^* shown, represented by circles. The first one is associated with action beginning (the beginning state circle) and the second one — with action end (the final state circle). (CS_1, CS_1^*) pair corresponds to a reflective model of transition between states, caused by appropriate action. Final information substate i_1 reflected back to information action ia_1 for new processing and updated information i_1' reflection.

This process may be compared in some ways to the role information plays in sensory, neural, and reactive actions, as well as the genesis of life in living organisms [3]. A chain of reflections, transmissions, transformations, and uses results in information acquired through the use of certain activities being transferred, possibly using types of reflection, then used to produce useful reflection, and finally used by cause-and-effect relationships and outcome state changes.

In the context of artificial systems, chains of material actions and information function as chains of sensory, neural, and reactive actions, as do chains of (complex) states and (material, information) actions.

Therefore, chains of complex states, information transmissions, and the use of information to change material states can all be represented by reflective models. Their reflective nature stems from the fact that every subsequent complex state is a reflection of a preceding state in the sequence.

Reflective models chains take forms of reflections chains and forms of further cause-and effect relationships and "material" states changes chains as a result of complex states realizations. This process of state changes chains due to reflective models realizations as chains of (complex) states shall be modeled with appropriate models of information application for activity. This application takes dual form of information application and reflective models creation and use as a result of information obtained.

This process can be illustrated with analogies of live organisms, which can produce reactions on information obtained by them and which main function is believed to be information preservation.

Information applicable by reflection and further creation of system complex states results in changes of cause-and-effect relationships and, as a result, in changes of "material" states. This change performed by actions of humans or actions performed under their control but directed by information obtained and/or stored in complex states. This (prescriptive) information can be instructions, prescriptions, plans, and pro-

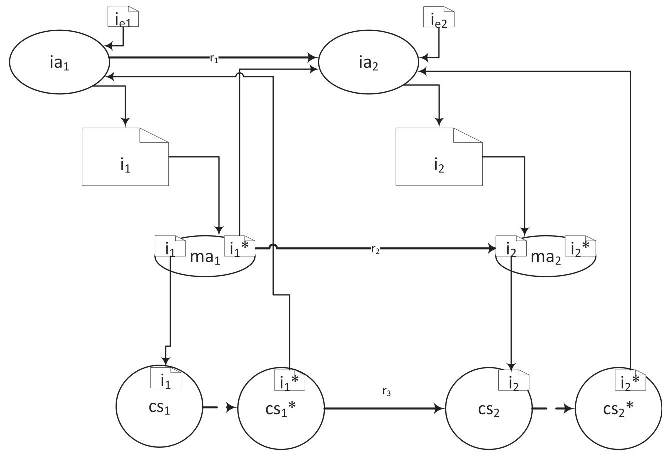

Fig. 5. Network of complex states

grams.

Reflective models of complex states and following them material states, resulted due to actions from complex states, can be viewed as models of information application. The network of all possible complex states during system functioning represent the model of system functioning under possible changes of conditions and appropriate information processing and use. The example of such network is shown in Figure 5.

III. Approach to Formulation of Problems of Information Application for System Functioning

The network of scenarios suggested in Figure 5 should serve as contribution for use by other researchers in their research of information application. These models can be applied to the building of algebraic graph-theoretic models of information use.

There is analogy of this process with organism's genesis due to actions performed by organism according to genetic information. After information materialized, new information generated about results of information application, and it is combined in complex states of the new sequences of states and actions.

Modelled sequences of complex states and transitions, due to actions (information, material) form probabilistic partial ordered sets (or networks of actions) with information traces embedded into sequences of actions and states.

These sequences can be modelled with various means, and suggested graph theoretic models are one of such means. Other ones may include theoretic algebra means, linguistic means, partial ordered sets theoretic means.

Based on graph theoretic or other models of sequences, functional mathematical models can be built, which shall represent information application effects formation mathematical dependencies based on graph theoretic models created. In case of graph theoretic models used numbers of vertices in parse order may serve as indexes of functional expressions, which represent quantitative characteristic's of effects computation in sequences.

As a result, researchers obtain the ability to predict effects characteristics of information application based on mathematical models constructed in the article.

Mathematical models reflect changes which happens due to information obtained during functioning. Such changes reflected as changed complex states and, next, to changed material states through performed action, This process can be referred as reflection of information in practice.

The author investigated the effectiveness and efficiency of information application, as well as their interaction with information quality measures. The study delved into the peculiarities and existing methods for constructing information application measures. A literature review on information quality (IQ) and the assessment of decision quality was conducted. Furthermore, the paper presented a survey of methods for estimating the value of information, focusing on fundamental and mathematical approaches, as referenced in [4] and by various researchers adopting an empirical approach. Y. Lee, R. Wang, and D. Strong [5] critiqued this approach, noting that "the correctness or completeness of the results cannot be proven via fundamental principles." The concept of 'fitness for use' was explored by Lee [5]. L. Floridi and P. Illari emphasized that qualitative descriptions of terms like 'information quality' or 'timeliness' differ from the formal metrics required for their measurement, which are essential for practical implementation, as discussed in [6], [7].

In this article, an approach based on the mathematical modeling of sequences of reflective complex states proposed in the context of information utilization for systemic activity. This approach draws upon concepts and formal models introduced by the author earlier [8]. The novel measures presented in the article are refined using probabilistic and entropy measures previously suggested. These measures are calculated using mathematical models that describe information application and the corresponding measures of system potential. Such measures, along with the mathematical models that develop them, could facilitate the resolution of various practical issues associated with information application and digital transformation—treating them as mathematical problems, such as those found in operational research and mathematical programming. The foundation of the proposed models is graph-theoretic models, which are constructed based on the schematics of information application for actions within systems. From these graph-theoretic models, probabilistic functional models have been developed.

This approach bears similarity to the information process modeling approach proposed by C. Batini and M. Scannapieco, yet it exhibits certain limitations as identified by the authors themselves. They critique that it "does not distinguish between or provide specific formalisms for operational processes that use elementary data and decision processes that use aggregated data" [9]. A fundamental aspect of information processing is its inevitable influence on purposeful action modification and environmental interaction [10]. However, the mathematical models required to articulate these shifts in human activities have not yet been adequately addressed in the literature. This

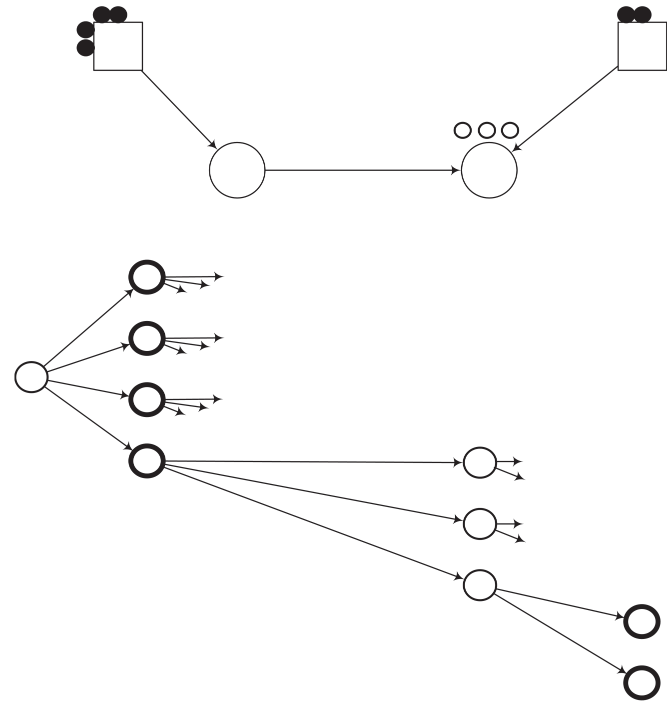

Fig. 6. Parsing of complex states chains

gap can be bridged by employing diverse methodologies to characterize dynamic activity, such as the theory of functional systems, provided it is mathematically articulated with suitable formal tools [11]. My research is committed to the development and utilization of such tools.

The work presented encompasses both conceptual and formal models for the application of information to activities within systems. These formal models are instrumental in defining a set of pressing practical problems. As a result, researchers can now utilize the material provided to tackle practical issues related to information application, conceptualized as mathematical problems, by employing mathematical techniques.

IV. Structural Modelling of Information Application for System Functioning

As it was shown earlier, the formalisms proposed in the article are predicated on sequences of complex states and their transitions. These sequences are constructed using the models previously suggested, through the execution of parsing algorithms. As a result of these parsing algorithms, trees representing potential reflections, complex states, and the responses of "material" states to complex states are generated.

Parsing of complex states chains is illustrated in Figure 6.

The methods and models proposed are limited by their discrete and probabilistic nature, the complexity of the modeling process due to the vast number of possible sequences of changes and reactions to represent, and the constraints on data availability.

V. Formulations of Typical Problems of Information Application for System Functioning Functional Modelling

Let's examine the types of formal problem definitions and solving, intended for resolution using the formalisms of information application in system activities. Consequently, a classification of these formal problem definitions and solving methods will be established. Researchers can now articulate their specific formal definitions and methods of practical problems solving, based on their resemblance to already classified formal problem definitions and methods to solve such problems.

This may lead to fewer efforts and time required for researchers to create appropriate formal problem definitions, as well as defining appropriate methods to solve problems stated.

Typical formal problems definition and solving consist of a few parts, main of them are:

1) Learning structure of the model of information application for action.
2) Learning functional models (parameters values, variables constraints, functional dependencies kinds) of information application for action.
3) Using models constructed for solving problems (such as prediction of effects).

Given:

1) Apriori structural models of information application for activity $\mathbf{M}_i^I(I_i, D)$, which were built as a result of previous problem decision. Such models can be built algorithmically, based on structural considerations of metamodels and data.
2) Current data $\mathfrak{D}^{\mathfrak{I}\mathfrak{H}}$ of one of the kinds or modes $i \in I^u$, which describes application of information $\mathfrak{I}^{\mathfrak{H}}$, obtained in connection to activity observed.
3) Hyperparameters. That is, the values, which determine parameters of models constructed and model learning process. For example, intervals of possible values, initial values, algorithms steps values, algorithms thresholds.

Construct:

1) Mathematical, functional model $\mathbf{M}^F(I_i^H, D^{IH})$ learned according to actual information I_i^H obtained. Learning process includes determining such parameters and variables of the model as probabilities of model parts actualization, parameters of stochastic values distributions, functional dependencies parameters.
2) Predictions $Pred$ with use a model $\mathbf{M}^F(I_i^H, D^{IH})$ constructed to get estimation of vector of parameters of information application for activity effect's probability distributions, $\mathfrak{F}(I_i^H)$ – depending on information I_i^H used. I_i^H considered as obtained or processed during action. Such use can be expressed in the form of a function, which maps data obtained to prediction of the vector of projected parameters of effects probability distributions $\mathbf{M}^{\mathbf{F}}(I_i^H, D^{IH})$. This model further used to

calculate effects $Y^p(M^F(I_i^H))$ of information application for action based on function $Y(\mathfrak{M}^{\mathfrak{F}}(I_i^H))$:

$$Pred : \mathfrak{D}^{\mathfrak{I}\mathfrak{H}} ... \xrightarrow{\mathfrak{F}(D^{IH})} ... \mathfrak{M}^{\mathfrak{F}}(I^H) ... \xrightarrow{\mathfrak{F}(D^{IH})} ... Y^p(M^F(I^H)) \quad (1)$$

Where

1) $Pred = \mathfrak{F}(D^{IH}) \diamond \mathfrak{Y}(\mathbf{M^F}(I^H)$, where $\diamond-$ functions composition.
2) $\mathfrak{F}(D^{IH})-$ function, which construct functional model $\mathbf{M^F}(I^H)$. $IH = /left/I_i^H, i \in I^u/right/$.
3) $Y(\mathfrak{Y}(\mathbf{M^F}(I^H))$ – function, which provide effects Y^p probabilistic values construction based on functional model $\mathbf{M^F}(I^H)$.
4) $Y^p(\mathbf{M^F}(I^Hs))$– function, which defines effects of activity with information application (effects defined as results, which have defined requirements to their possible values, according to the current objective of functioning).
5) $\pi_g(\mathbf{M}_i^{\mathfrak{I}}, \mathfrak{D}, \mathfrak{I}_i)-$ function which generates information I_i application functional models and functional models of effects probability distribution based on (structural and other) models $\mathbf{M}$ of information application for activity, built during previous problem decision;
6) $\mathfrak{M}_i^{\mathfrak{I}}$ - metamodel of information application of the kind i;
7) $\mathfrak{D} = \left\{D_d, d \in \bar{(}1, K)\right\}$ - data of various kinds d, existing for analysis and models of information application building.
8) I_i — information obtained during action.
9) Functions π_g may take form, for example, category theoretic generators of sequences of complex states, according to category schemas, or graph automata generators of marked (hyper)graphs. The result of such generation is one of a kind of structural model $\mathfrak{M}_i^{\mathfrak{I}}$v, which describe possible sequences structure of information use to alter activities and appropriate resulting states flow.

Effects Y^p, obtained as a result of $\mathfrak{I}$ application, described by the data $\mathfrak{D}$, $\mathfrak{I} = \left\{I_i, i \in \overline{1, I^p}\right\}$, with use of the functional model $\mathbf{M^F}(I^H))$, constructed earlier as a result of appropriate problem decision.

$$\begin{aligned} Y^p = \mathbf{F}(I^H)(\mathfrak{D}) = \\ = \mathfrak{F}(D^{IH}) \diamond \mathfrak{Y}(\mathbf{M^F}(I^H) = \\ = \{\pi_g(\mathbf{M}_i^I), \mathfrak{D}, \mathfrak{I}_i), i \in \overline{1, I^p}\} = \\ = \left\{\mathbf{F}I^H)_i^I(\mathfrak{D}), \mathfrak{I}_i) \diamond \mathfrak{Y}(\mathbf{M^F}_i^{\mathfrak{I}}(I^H, I_i), i \in \overline{1, I^p}\right\}. \quad (2)\end{aligned}$$

VI. Problem of Information Application for activities in Systems Results Quality Assessment

Such problem can be considered as the system of subproblems related to possible activity cases and its results quality assessment in the relation to information application for such activity cases, performed in the change conditions:

1) Action quality assessment regarding information use in changed conditions.
2) Action results (effects) quality assessment regarding information use in changed conditions.
3) Effects characteristics assessment regarding information use in changed conditions.

These three subproblems are interconnected and necessitate a robust quality assessment technique. To address this need, we propose a probabilistic approach for estimating the realization measure of probabilistic values. Specifically, this measure can be derived using stochastic indicators based on the theory of stochastic processes. By employing techniques such as stochastic relationship indicators, we can assess the correspondence between probabilistic values. This measure represents a probabilistic value distributed within the range of probability values, i.e. $[0, 1]$. The system of such measures, estimated as a result of solving the subproblems, can be represented as a vector of probabilistic values. Additionally, stochastic indicators provide valuable insights, including probabilities, stochastic graphs, and stochastic indicator graphs. This representation enables us to handle complex quality measures as vector probability values. Consequently, we can leverage various techniques, including modern data science and machine learning (such as generative machine learning for constructing vector probability distributions) as well as game-theoretic approaches. Scalar indicators are essential for constructing the objective function, denoted as Obj. These indicators quantify the quality of information application to action. For instance, they can be computed as various moments of the distributions of probabilistic values.

Now, let's delve into a specific sub-problem—one that is particularly significant: assessing the quality of information application effects on actions. We evaluate this quality based on the model $\mathbf{M}^F(I_i^H, D^{IH})$ learned according to actual information I_i^H obtained. Its construction was discussed as solution of previous problem. To estimate quality, technique of probabilistic measure of stochastic values correspondence estimation applied. Given:

1) Apriori (structural and other kinds) models $\mathbf{M}_i^I(I_i, D)$, which were built as a result of previous problems decision. Such model's distinguishing feature is they can be generated algorithmically, based on graph-theoretic, structural considerations of metamodels and using data obtained. Other models can be built with use of machine learning methods and machine learning algorithms, and by combining these methods.
2) Current data $\mathfrak{D}^{\mathfrak{I}\mathfrak{H}}$ of one of the kinds or modes i according to SC^mI^n schema, $i \in I^u$, which describes application of information $\mathfrak{I}^{\mathfrak{H}}$, obtained in connection to activity observed. This data may be historical as well as real time.
3) Hyperparameters of (functional) model and its creation process. That is, the values, which determine parameters

of models constructed and of model learning process. For example, intervals of possible values, initial values, algorithms steps values, algorithms thresholds.

Construct:

1) Predictions with use a model $\mathbf{M}^F(I_i^H, D^{IH})$ constructed to get estimation of the probabilistic measure of activity effect's correspondence to projected requirements, $W(I_i^H)$ – depending on information I_i^H used. The value $W(I_i^H)$ can be used as objective function value $ObjF(I_i^H)$. I_i^H considered as obtained or processed during action. Such use is expressed in the form of a function (objective function) which maps information obtained to predicted values of the measure of correspondence of projected effects to projected requirements:

$$\mathfrak{D}^{IH} \xrightarrow{ObjF(\mathbf{M}^F(D^{IH}))} W(I^H) = Prob(Y^p \leq \setminus > Y^d) \quad (3)$$

Here, $W(I^H) = Prob(Y^p \leq \setminus > Y^d)$ can be computed with use of theory of stochastic indication. In this case, the probability function of the stochastic indicator can be computed, as a mapping from Y^d.

Effects Y^p, obtained as a result of $\mathfrak{I}$ application, described by the data $\mathfrak{D}$, $\mathfrak{I} = \left\{I_i, i \in \overline{1, I^p}\right\}$ using the functional model $\mathbf{M^F}$.

Data according $SC^m I^n$ used to obtain models described.

A. Problems of Information Application to Activity Digital Technologies Support

The Information Application for Activity and Decision (IAAD) problems constitute a system of sub-problems aimed at systematically supporting and enhancing information application for activities, particularly in the context of using digital technologies amidst changes. These IAAD sub-problems share a close relationship with the Information Application Synthesis (IAS) problems related to decision technologies, which I will describe further below.

The connection between IAAD and IAS is twofold. On one hand, while not all IAAD sub-problems are directly tied to synthesis (optimization) or operations research problems, many of them exhibit such connections. On the other hand, although not all IAS problems are explicitly linked to the use of digital technologies, a significant subset of them does involve digital technology applications.

1) Action digital twin imitation and experimentation regarding information use in changed conditions.
Such digital twins systematically use data according to $SC^m I^n$ schema to build and update a model of action regarding information use. This process of model and data synchronization shall be such that digital twin after updates behaves very similar to action. That shall be true for information use for action as well.
2) Artificial activity digital technologies.
Such technologies shall allow, based on the data according to $SC^m I^n$ schema use, to simulate human's action (both material and informational).
This shall allow building, learning, updating and using of models of information use for action in digital form. Such models could be used as the basis for developing a lot of new digital technologies. Some of them are thoroughly described below:
3) Digital representation of cause – and-effect relationships and actions should be developed, analogously to representation of texts with use of word-2-Vec and doc-2Vec. Such representations could further allow storing of technologies in specialized databases (like vector ones), analyzing and generation of new technologies, especially technologies, concerned with information application for activity. Such technologies' representation requires extensive use of formalism, which describes causes of changes, effects obtained as changes results, peculiarities of effects formation as well change, and quality of actions performed in changing environments.
4) Description, research, and application of technological knowledge formation and application of it to activity, digital representation of these cognition processes.
This can be implemented based on models to be created based on $SC^m I^n$ schema data. Such models and knowledge generate die to their use shall allow predictions of various activities results, representations of dependencies between action modes, information use, their structure, parameters, and organization and quality of action results regarding information use;
5) Further, such models shall allow generation of better technologies, better design of new technologies, better use of information for activity, better changes and cause-and-effect-relationships realization, which allows achieving better quality of activity results. Such generation could be referred as generative technology models;
6) Digital experimentation technologies.
Such technologies can be performed with, for example, Digital Twin of activity. They can allow generating more data, to conduct experiments automatically and to imitate human activity, including technological knowledge creation and its approval in practice;
7) Activity generation, generative research of the new activity. Such new digital techniques could allow using generative machine learning models to create new activities descriptions, possibly performed with new technologies. Such activity shall be created in such a way that activity results quality will the best among possible ones;
8) Reinforcement learning, simulation-based optimization using model of information application for activity;
9) Intelligent Artificial Agents technologies based on information application and resulting system behavior, including modelling for distributed activity;
10) New digital technologies design created regarding information use and resulting system behavior modelling in changed conditions.
11) Predictions with use a model $\mathbf{M}^F$ constructed to get estimation of parameters of activity effect's probability distributions, $\mathfrak{F}(I^H, D^{IH})$ – depending on information

I^H used and data collected according to schema SC^mI^n. I^H is obtained (processed) during action;

12) Such use can be expressed in the form of a function, for example, of prediction $\mathfrak{Pred}$ kind, which maps data obtained to prediction of the vector of projected parameters of effects probability distributions $\mathfrak{F}(I^H, D^{IH})$.

Based on data given, let us formulate the problem discussed. Such problem is viewed as a system of subproblems. Let us explain it on examples, in a unified manner, as the set of example subproblems formulations:

1) Given:
   a) $IH-$ structured set of possible information obtained, intended for application to action in various circumstances;
   b) Data D^{I^H}, collected and structured according to described schema SC^mI^n.
   c) Functional model $\mathbf{M}^F(D^{IH})$ constructed.

   Construct:
   a) Vector $\mathfrak{F}(I^H, D^{IH})$ of parameters of effects distribution functions. Constructed on the base of algorithm and model estimation problem decision $\mathfrak{Pred}$ based on model of information I^H application for action:

$$\mathfrak{Pred} : \mathfrak{D}^{IH}, IH : \xrightarrow{Pred(\mathfrak{M}^{\mathfrak{F}}(D^{IH}), IH)} \mathbf{F}(IH, D^{IH}) \quad (4)$$

   b) Digital twin of activity DTA : use problem: Model $\mathbf{M}^F$ is the base of digital twin of activity DTA use. It uses latest data D^{IH} to update model $\mathbf{M}^F(IH, D^{IH})$ of functioning and predicted effects $\mathbf{F}(IH, D^{IH})$:

$$(IH, D^{IH})\xrightarrow{\mathbf{M}^F(IH, D^{IH})}\mathbf{F}(IH, D^{IH}) \quad (5)$$

   c) Technological knowledge $\mathfrak{TK}$ digital representation:

$$\mathfrak{TK} \quad : \quad \mathbf{M}^F(IH, D^{IH})\xrightarrow{\mathbf{C}^F(IH, D^{IH})}ET \quad (6)$$

   where:
   a) $\mathbf{C}^F-$ Encoding function;
   b) $ET-$ Encoded Technology Results Model $\mathbf{M}^F$ (Model, represented with $\mathbf{C}^F$ use).

   This encoding is similar by its objective to one used in machine learning applications to represent (encode) complex objects.

2) Function and algorithm $\mathfrak{T}$ creation of the new technology creation in encoded representation form:

$$\mathbf{ET}^* \xRightarrow[ET_m \in \mathfrak{ET}]{\mathbf{ArgMax} W(IH, D^{IH}, ET_m \in \mathfrak{ET}, A_n \in ET_m)} . \quad (7)$$

   where
   a) $\mathfrak{ET}$ is a set of possible technologies, represented in encoded form of ET_m. $ET_m \in \mathfrak{ET}, m \in \overline{1, M}$.
   b) $\mathbf{ET}^*$ is the best possible in current circumstances (optimal) technology, represented in encoded form. Best technology is one which allow best plan of actions A_n in planned circumstances.
   c) $W(IH, D^{IH}, A_n, ET_m)-$ quality indicator of the information application for activity, performed according to the one of the plans A_n and technology ET_m.
   d) Computation of $W(ET_m)$ for $A_n \in ET_m$ is a separate problem of nested optimization. Among techniques used are approximating function $W(IH, ET_m)$ for each chosen ET_m based on the construction of the best possible action plan A^*. For example, by solving nested optimization problem: $A^*(ET_m)$:

$$(W(A^*) \xRightarrow{A_n \in \mathbf{A}(\mathbf{ET_m})} W(IH, D^{IH}, A_n) \quad (8)$$

   e) $A^* \in ET_m$;
   f) $\mathbf{A}(ET_m)-$ set of A_n, which corresponds to possible cases of technology ET_m use for A_n construction (according to technology descriptions, rules, and constraints);
   g) In turn, each A_n is a set of actions (of various kinds) and order relations between them, as well as parameters of these actions – such as optional mode of operation, time to start, resources assigned. Further, A_n- n-th possible action plan.
   h) $\mathbf{A}(ET_m) = \left\{A_n \in \mathbf{A}(ET_m), n = \overline{1, M_n}\right\}$ is the set of a number of M_n action plans, created according to technology ET_m.
   i) $W(A^*)-$ optimal point(s) of the function $W(A_n)$ of information use for action quality indicator W depending on action plan A_n.
   j) Optimal points can be found by solving the optimization problem on $A_n \in ET_m$, described above. Other methods to build or approximate such points can be approximating and learning function points.

   Examples of values $(W(A^*, ET_m, IH, D, A_n)$ are:
   a) Approximation of $W(A^*)$ values in $\mathfrak{ET}$ space. It can be done in $mathfrakET$ space by using Data Science, Process Science and Machine Learning methods.
   b) Generation of the new activity with the best possible estimation of quality of information application for action in the given circumstances (for example. with genetic programming methods).
   c) Given activity according plan $A_n(ET_m)$ transformation $\mathfrak{Tran}$: $A_n \to A_{n+1}$ to the activity according plan A_{n+1} for the same technology ET_m with better estimation of quality of information application for transformed action A'_{n+1} : $W(A'_{n+1}) > W(A'_n)$ in the same technology.
   d) Transformation $\mathfrak{Tran}$: $A^*_n \to A^*_{(n,m+1)}$ of the given activity $A^*_{n]}$ to the activity according to the plan $A^*_{(n,m+1)}(ET_{m+1})$ with optimal (the best possible) estimation of quality of information application for transformed action $A^*_{(n,m+1)}$ according to technology $ET_{m+1})$.
   e) Approximation of the functional dependence between activity according plan A_n and the value $W((A_n)$ of

estimation of quality of information application for activity according A_n.

B. Information Application Synthesis Problems

Such (short, IAS) problem can be considered as the system of synthesis sub problems. Many of them are related to digital technologies use, and thus, to problems of Information Application to Activity Digital Technologies Support (IIAD), considered above. The difference is in that, IAS is not necessarily related to use of digital technologies (such as Digital Twin of Activity or Coders and Decoders of technological, information use for activity, cause-and-effect relations data. But, a number of digital technologies, such as generative models, are closely related to IAS. Their relation as well as relation between cybernetics, digital technologies use for activity, optimization, operation research models and new digital technologies, such as generative techniques, transformers techniques, encoders/decoders of technological information and information use in activity shall be subject of future research.

Formal definitions of main IAS problems are listed below as a system of optimization sub problems. Many practical problems of information use for actions formulated as optimization sub problems, classified below:

a) Interruption and optimal update of activity plan $A*$ according to new information obtained about environment;
b) Interruption and optimal update of activity plan $A*$ according to new information obtained about updated objectives;
c) Synthesis of optimal plans of new activities to be performed and their plan according to information obtained;
d) Synthesis of the sets of activities performed and projected optimal plans of new activities to be performed and their plan according to possible information obtained;
e) Synthesis of the technologies used and projected optimal plans of new activities to be performed according these technologies as well as their plans according to possible information obtained;
f) Synthesis of the information technologies used and projected optimal plans of new activities to be performed according to possible information obtained or processed as a result of synthesized technologies use;
g) Synthesis of the new information technology based on current information technology use results and feasible changes of current information technology – such that activity will be enhanced in the best way possible in current circumstances and constraints given by information processed;
h) Synthesis of the information technology changes based on current information technology use results and feasible changes of current information technology - such that activity will be enhanced in the best way possible in current situation (circumstances and constraints) given by information processed;
i) Synthesis of the technology changes. Such changes shall lead to activity being enhanced in the best way possible.

Other sub problems may be concerning complexes, which include sub problems described above and relations between them, for example:

3) Synthesis $\mathfrak{Syn}ET_{(n,m)}, (A^*_{(n,m)}(ET_{(n,m)}))$ of system of optimal technologies $ET_{(n,m)}$ as well as corresponding to them sets of (information and "material") activities (optimal plans). Such plans shall be performed according to possible predefined information IH_i, which projected to be obtained before action start;
4) Synthesis of system of optimal information technologies organization, modernization, and update policies as well as corresponding them policies of optimal plans changes, and changes of corresponding to plans changes information use cases (to be performed according to possible predefined information);
5) System of actions $Ap(ET(IH_i))$ - tree structured set of A_n sequences with probability distribution defined on the set of A_n sequences (elements of $Ap(ET(IH_i))$) under condition of information IH_i obtained;
6) $\mathfrak{ET}-$ set of possible technologies representations: $\mathfrak{ET} = \{ET_m m \in \overline{1,M}\}$.
7) $ET_m - m-$th possible technology representation.
8) $ET*$ = best technology determined, and represented in standard encoded form.
9) $A_n(ET_m)$ is the action set and partial order relations as well as their parameters (action plan), representing information application for activity, computed for the case of technology ET_m use, element of $Ap(ET_m; IH_i, D^{IH_i})$;
10) $\mathfrak{ET}^*(IH_i)$ best possible – in current constraints (i. e. optimal) – incremental $i-$th change of technology.
11) $i-$th change performed in condition of IH_i obtained before change.

Best possible means its implementation allows maximum (in current constraints and information IH_i obtained) potential indicator $\Psi(Ap(ET_m); IH_i, D^{IH_i,A_n})$ of the information application for activity, performed incrementally according $Ap(ET_m)$ and IH_i used.

C. Problems of Mixture Kind

Mixture problems are problems which can be defined as the system of sub-problems of various kinds, described above. The system of sub-problems is the set of sub-problems with relations defined between them. Such relations may define constraints, sequences of actions in decision problems, cause-and-effect relationships between models construction. Many real life problems can be formulated as the mixture of sub-problems of various kinds.

1) Model encoding problem of optimal activities according encoded technology ET_m and activity plan A^* (for example, as graph encoding) - $\mathfrak{TR}$. Then, Synthesis

problem of analogical activity in technology ET_{m+1} : $\mathfrak{Syn}(A^*_{(n,m+1)})$ based on technologies ET_{m+1}, ET_m and plan A^*. Then - digital technologies problem to approximate $\mathfrak{Apr}$ points of $W(A*)$ recurrently, for all other technologies ET_k through transforming $\mathfrak{Tran}A^*(n,k-1) \to A^*_{(n,k)}$ plans for such technologies based on transformers developed. Here, we defined sequence of subproblems, described their use and cause-and-effect relationships which determine system of subproblems decision.

2) Problem of digital transformation of business processes. This problem is based on previous one and concern to research of $\mathfrak{TR}$ properties in relation to the company current technology used ET_m in varying environments and possible technologies set $\mathfrak{ET}$ which can be used by the company due to possible business process modernization on the short-to-medium planning range.
3) Problem of organization digital transformation strategy planning.
   This problem is based on previous one. It concerns $\mathfrak{TR}$ properties in relation to company strategy in varying environments and possible technologies set $\mathfrak{ET}$ used by the company in such conditions on the long-term horizon of the planning.
4) Problem of public services digital transformation.
   This problem is based on previous one as its part. For example, previous problem can be considered as one of the similar parts of transformation problem. Transformation $\mathfrak{TR}^{\mathfrak{o}}$ of public services $\mathfrak{TR}^{\mathfrak{p}}$ can be considered as a mean to transform organizations of concerns (organizations, public services are intended for) $\mathfrak{Org}$ and their business processes. Thus, this problem can be formulated as research of $\mathfrak{TR}^{\mathfrak{p}}$ properties in relation to $\mathfrak{TR}^{\mathfrak{o}}$ application to organizations of concern $\mathfrak{Org}$ results (depending on $\mathfrak{TR}^{\mathfrak{p}}$) in varying environments. This mean dependencies $\mathfrak{TR}^{\mathfrak{p}}(\mathfrak{TR}^{\mathfrak{p}})$ shall be investigated. Next, resulting due to transformation possible technologies sets $\mathfrak{ET}(\mathfrak{TR}^{\mathfrak{p}})$ shall be modelled (for example, in encoded form) and further, system of optimal technologies shall be synthesized by solving $ET_*k : \mathfrak{Syn}(A^*_k)$. As a result, a system of technologies $ET_*k(\mathfrak{TR}^{\mathfrak{o}})$ and appropriate transformations $\mathfrak{TR}^{\mathfrak{p}})$ can be built. Further, problem of $\mathfrak{TR}^{\mathfrak{o}}$ synthesis can be considered, such as resulting $ET_*k(\mathfrak{TR}^{\mathfrak{o}})$ shall allow achieving the best possible results of organizations $\mathfrak{Org}$ of concerns functioning in changed environments.
   Optimal public service is one, which provide the best possible measure of potential $\psi^*(Ap(ET_d(IH_i)$ due to organizations of interest transformations.
5) Problem of Information Quality Measurement, Management, and Assessment.
   Problems of information Quality (IQ) Measurement, Management, and Assessment (MMA) is the set of subproblems which are aware of information application by humans. These problems are closely related to problems considered by information systems theory, management information systems, continuous information quality improvement, information foraging [12] software engineering, action theory, situated action, context awareness [13], human computer interaction, cause-effect relationships, information application mining and others. The common feature of these theories and problems is their focus on information application process and its results modelling. For example, model driven approach is considered Thus, such problems decision is based on models of information application [14]$\mathbf{M}^*(IH, D(IH))$, where $*$- kind of model ($S-$structural, $F-$functional),IH- structured set of information obtained (situations), D^{IH}- data available for situations.
   Thus, solving problems described can be represented as a twofold process:
6) Building models $\mathfrak{M}^y$ of various kinds y. Models $\mathfrak{M}^y$ can be built using available data, information, expert knowledge. Model's function is to allow predicting characteristics of vector of probability distributions functions (generally, conditional) of activity results values $\mathfrak{F}(t)$ over time. Such models shall allow estimation of probability distributions functions $\mathfrak{F}(t, IH, D(IH))$ characteristics $\mathfrak{Ch}_{\mathfrak{F}}(\mathfrak{t})$, depending on information obtained IH and data available $D(IH)$ as a result of information. $\mathfrak{F}(t, IH, D(IH))$ can be regarded as a system of probabilistic mappings, random processes of various kinds; As an example, y can represent various kinds of such dependencies. For example, graphs probabilistic mappings, marked graphs probabilistic mappings, vector random processes, probabilistic martingales.
   Thus, by its nature, the model building problem is similar to generative machine learning for multidimensional vector probabilistic dependencies.
7) Using models created to predict action results. Results are predicted in the form of joint probability distribution $\mathfrak{Y} =< \mathbf{P}^y(t.IH, D^{IH}) >$.
8) Using models for predicting action results. Results are predicted in the form of joint probability distribution $\mathfrak{Y} =< \mathbf{P}^y(t.IH, D^{IH}) >$.
9) Using joint probability distribution predicted to estimate probabilistic measures of information use qualities indicators in the space of possible measures.
10) Defining the problem to be solved as the search problem in the space of possible measures of information use quality indicators.
11) Solving the practical problem as mathematical problem of search in the space of possible information use quality indicators.
12) Decision interpretation and use for performing activity in changing conditions Models and methods required can be built with a variety of existing and developed concepts, methods, models, and their combinations. Among models and methods available are:
    a) Meta-models [15] of information application can be used to build models and interpret data available about information application;
    b) Linguistic technologies for building models of infor-

mation application for activity;
c) Data and Process mining methods [16], [17] to build models of activity regarding information use;
d) Change and Information use cases mining methods to build models of information application for activity [18], [19];
e) Change mining methods [20]
f) Machine learning methods for models of information application for activity;
g) Knowledge Engineering model building, model management methods
h) Abstraction management software [21], [22], for example, STRIPS or ALPINE [23].
i) Expert based model construction methods

Among information technologies used to build appropriate software are open source as well as other technologies, among them:

a) VizzAnalyzer from arisa.se [14]
b) X-Develop IDE
c) Process and Change mining technologies [24], such as PROM [16], [25] PM4PY [26], [27] and other similar technologies used to solve process mining type problems [28]
d) NTools
e) Analyst4j [29] CCCC
f) AALA, SOFM, RUM [23].
g) Software metrics used are CBO, DIT, LCOM-CK, LCOM-HS, LOC, NOC, NOM, TCC, WMC [30], [31]
h) CodeGuide [32]
i) EclipseJDT [33]
j) information quality framework [34]
k) Prometheus, Logit.io, Thanos [35],

These technologies may be the base for creating new information technologies and appropriate software to build and use models of various kinds of information application for activity.

Further, to solve practical problems of information application for action in systems, it is suggested to use models build according to description above for calculations in the space of probabilistic measures of quality indicators. Such calculations are performed for search of decisions of mathematical problems of information application for activity. Such mathematical problems formulated using structured space of probabilistic and entropy measures constructed.

Such structured space shall be constructed in such a way it allows conducting various search and other similar algorithms, intended for solving kind of problems, which can be solved by performing algorithms in structured space of probabilistic and entropy measures of information application for action in systems. It is suggested to structure space with use of graphs and partially ordered sets. Such structures shall represent organization of action chains of changes in changing conditions, from one hand, and chains of reflections, abstractions used to predict future outcomes – from another hand. As a result, space for problem decision search can be represented as trajectory in four dimensional space, formed by quality measures, entropy measures, action organization measures and level of reflection (abstraction).

For such measures creation following theories can be used:

a) information theory
b) stochastic indication theory
c) stochastic processes theory
d) graph abstraction and graph encoding theories [22], [23]
e) probabilistic relations theory

Some technologies and libraries are possible to use for such measures computation and application:

a) DOCplex programming in Python [36], as well as PuLp, DipPy, PYOmo, Gurobi libraries
b) Optimization problems decision libraries [37]
c) pyentropy Optimization problems decision libraries [37]library in Python
d) STRIPS, ALPINE, STAR abstraction techniques [23]
e) StochPy Python library [38]
f) PYmc3 in Python

To use models build for calculations in the space of probabilistic measures for deciding search problem following theories can be used:

a) optimization theory
b) mathematical programming theory
c) probabilistic graph theoretic structures theory
d) knowledge graphs and abstraction theory [23], [39]

To use decisions obtained, some technologies a library are possible to use:

a) DOCplex programming in Python
b) SCIPy-optimize, SCIKIT-Learn-optimize optimization library
c) Pulp, PYomo libraries
d) entropy computation libraries [40]
e) Poodle - AI planning in Python
f) Dynamic capabilities research libraries [41].

Further, decisions found due to search in structured space shall be interpreted and used.

Interpretation consists in forming thorough description of decisions (plans), corresponding to requirement To describe decisions obtained as a result of deciding search problem following theories can be used:

a) operations research theory
b) probabilistic planning and scheduling theories
c) human-computer interaction theory

Finally, decisions (plans) interpreted must be implemented and used in practice. To use decisions obtained, some technologies an dlibraries are possible to use:

a) Poodle - AI planning in Python
b) GOAPy planning in Python library
c) Poodle - AI planning in Python

$< \mathbf{M}^y(IH, D^{IH}) >$

13) Dynamic Capabilities Measurement, Management, and Assessment.
Dynamic capabilities are abilities to achieve goals which can be actualized for achievement in the case of certain changes chain $Cc_k \in CmathbfCc$ realized (as a result of conditions changes).
DC MMA Problem schema:

$$\mathbf{M}^f(CC_k) \to \psi^f(t, Cc, IH, D^{IH}) \to \pi(t, Cc, IH, D^{IH}) \quad (9)$$

where
  a) $\mathbf{M}^f(CC_k)$ - activity model in conditions of chains of changes realization $\mathbf{Cc}$;
  b) $\mathbf{Cc}$- the set of structured sets of changes and probability distributions defined on it;
14) Problem of choosing strategy for organizational change. Let us define strategy St as a set of plans π_j mappings $\delta : \pi_j \to \pi_m$
15) General Problem of Information Application Statement. Based on particular problem definitions, we can inductively formulate common problem definition and decision parts. It consists of a few typical subproblems:
  a) Database backend (storing log files data, cause-and-effects storage)
  b) Models building and updating
  c) Models use: effects probabilistic vectors and processes distribution prediction based on models
  d) Problem formulation: functional definition of problem and its decision
  e) Problem decisions representation (information system front-end, static)
  f) Problem decisions application (information system front-end, dynamic)

Formal problem definition may take form:
Perform mappings:

$$\begin{aligned}(IH, D^{IH}, t, \mathfrak{M}) &\xrightarrow{M} < \mathbf{M}^y(t, IH, D^{IH}) >; \\ &\xrightarrow{P} < \mathbf{P}^y(t.IH, D^{IH}) >; \\ &\xrightarrow{Pf} < \mathbf{Pf}^y(t, IH, D^{IH}) >; \\ &\xrightarrow{Pd} < \mathbf{Pd}^y(t, IH, D^{IH}) >; \\ &\xrightarrow{A} < \mathbf{Y}^y(t, IH, D^{IH}) >. \end{aligned} \quad (10)$$

VII. Conclusions

The paper elaborates on reflective models of information use for system functioning. These models serve as a foundation for understanding how information is applied in the context of system operations, particularly when digital technologies are involved.

Conceptual and formal models are presented, and they play a crucial role in formulating a set of typical problems related to information use for system functioning. These models and formal definitions pave the way for future research, potentially leading to the creation of a contemporary theory of information application for activity. Such a theory would be well-suited for formal, mathematical descriptions of how information use results are generated and how outcomes can be predicted using formal methods and models.

The models and methods obtained allows to solve many practical problems of information application in various kinds of systems, as well as to make mathematical formulations of problems and to automate such problems decisions using modern digital computer technologies.

Classification of these formal problem definitions and solving methods was suggested. Researchers can now articulate their specific formal definitions and methods of practical problems solving, based on their resemblance to already classified formal problem definitions and methods to solve such problems.

This may lead to fewer efforts and time required for researchers to create appropriate formal problem definitions, as well as defining appropriate methods to solve problems stated.

Looking ahead, the scope includes further developing the suggested formalisms. If these formalisms are established, they could be seamlessly integrated into system science, complexity theory, cybernetics, and various activity theories. By constructing models of information application and techniques for predicting the results of information use, researchers will be better equipped to address a wide range of issues related to improving information utilization and strategically adapting systems and their operations in response to changing conditions.

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