Impact of CoM Placement on Quadcopter's Performance and Controller Development

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***Abstract*—The development of Unmanned Aerial Vehicles (UAVs) typically follows an iterative process, where physical design precedes controller design. This study seeks to enhance the development process for quadcopters and drones by elucidating the interplay between physical design parameters and control system effectiveness. Notably, this work could gain attention as its focus is on the relationship between a physical design parameter and its impact on control systems, an aspect that has not been addressed in contemporary research. Initially, we assess two established experimental techniques for determining the Center of Mass (CoM) of a quadrotor, a critical indicator of mass distribution in the assembled or manufactured system. Then, we explore how CoM placement influences control processes and introduce a novel computational methodology for managing mass-imbalanced quadcopters, applicable to both traditional and novel control strategies. Performance evaluations in both balanced and unbalanced conditions had been conducted and compared to demonstrate the effectiveness of the proposed approach.**

I. INTRODUCTION

With the availability of several commercially designed frames, actuators, sensors and controllers in the market, design and development of drones, particularly quadcopters, has seen a tremendous increase in recent times. Even though most of such activities are done on hobby basis, a considerable portion of them are done on academic research and industrial or defense application need basis too. When designed for such critical applications, perfect control of such designed or assembled systems is expected by default. One of the most common problems arising in such situations is the incorrect mass distribution in the system, which deteriorates, disrupts or degrades the control by resulting in drift-away or highly oscillatory attitude responses or in some cases total crash of the system. Hence this paper has attempted to primarily study both sides of this problem, in the form of assessment of mass equilibrium, and correction of it, if found to be offset or incorrect. Considering the scope and complexity, this paper has been limited to the exploration of simple PID controllers with the understanding that the effect of CoM is invariable with the control technique being used. A quadcopter being an underactuated system, needs four fast changing control inputs namely thrust force, and roll, pitch, and yaw torques. For a quadcopter of defined invariable geometry, i.e., a non-transformable or non-reconfigurable drone, a study on its design parameters must include the factors that directly influence these four fast changing control inputs, as well, the two slow changing state controls namely planar control or X-Y position control. As is inferred from the dynamics of quadcopter, the states - roll angle and position along Y are dependent, and pitch angle and position along X are dependent. Since the position of CoM on the body frame defines the fast changing states directly and influences the two slow changing states indirectly, this study gains importance. Here we are presenting a correction technique namely computational thrust correction for CoM-offset in the controller and are observing and comparing the controller results before and after its implementation.

II. BACKGROUND

Bouabdallah et al in [1] threw good light on understanding the dynamics of a standard quadcopter and formulation of classical control laws for the same. Their work also introduced the obstacle avoidance in trajectory planning and flight control modules. However, the limitation of this study was that it explained the dynamics of $'+'$ configuration alone. In [2], the authors had discussed both $'+'$ as well $'\times'$ configurations of quadcopters in detail, including their differences, advantage of each configuration, their aerodynamic as well control stability, etc.. The generalized dynamics of a quadcopter from these two studies, considering both translational and rotational motion, for a quadcopter system as shown, in ***Fig.*** 1, is given as follows:

$$\begin{aligned}\vec{F} &= m\vec{a} + m\vec{g} \\ \vec{\tau} &= I\dot{\vec{\omega}} + \vec{\omega} \times \left(I\vec{\omega}\right)\end{aligned} \tag{1}$$

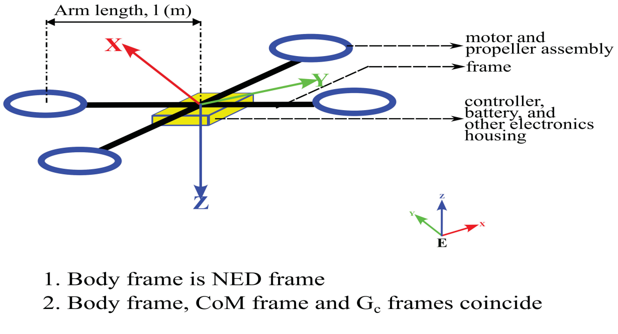


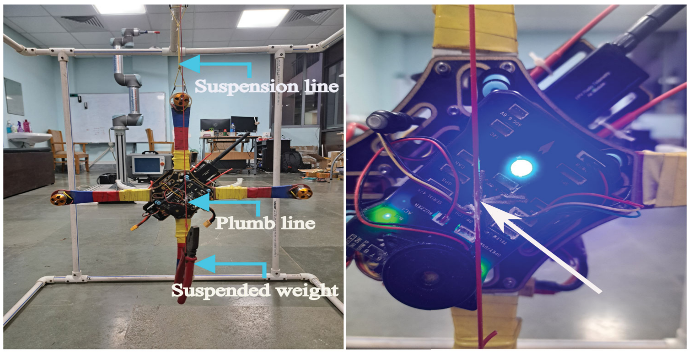
Fig. 1. Frames of reference in a quadcopter and its primary components

As it can be inferred from the system of equations in (1) and ***Fig.*** 1, the critical parameters, from design perspective, could be identified as follows: mass(m), arm length (l), center of mass (CoM), center of Geometry (G_c), Inertia along each axis (J_x, J_y, J_z), and factor of symmetricity (F_s). Here, m and l, can be said to be independent design parameters and CoM, G_c, J_x, J_y, J_z, and F_s are dependent on m and l values. Also, it can be inferred that mass significantly impacts flight dynamics, but it can vary, especially in payload or fuel-based systems. In this context, [3] could gain importance, in which rather than sensing or measuring the mass of a quadcopter preflight or postflight, it had been estimated with the help of input commands to the controller and the IMU (Inertial measurement unit) data during the flight continuously. In confined spaces, the quadcopter's form factor and size are critical. Adjusting arm length, as discussed in [4] and [5], offers practical resizing options and optimized designs. However, fixed arm lengths may not suit all scenarios, prompting research into quadcopters with adaptable geometries, such as [6], [7], and [8]. These adaptable designs are essential for complex aerial robots like those in [9]. These extant literature describe a new class and type of quadcopters that could change their geometry and size factor while flying. To design and control such quadcopters, a deep understanding of their flight dynamics becomes essential. However all of these analyses were limited to the constraint of having constant or solid mass while flying. The studies like [10], [11], [12], and [13] had tried to solve the problem of sloshing while carrying a payload of fluid. The importance of these works, was that they had addressed the control of a time-varying-mass system dynamics. Additionally [14], had discussed the dynamics and control of a slung tank, instead of a fixed tank, another example of variable CoM system. The inference from all such published research is that the correlation between a drone's physical or design parameters and its control mechanisms is crucial for achieving stable flight and facilitating innovative design applications. This presented work addresses these correlations through a case study approach. Previous research, including works by [1], [15], and [10], has explored control techniques for symmetrically designed quadcopters, such as integral backstepping [1] and fuzzy model predictive control [15], etc., often utilizing Computer Aided Design (CAD) to ensure mass symmetry. However, for individuals who assemble quadcopters from readily available components without CAD, there is no current method to estimate the CoM or validate their controller other than through trial and error.

This paper offers a practical technique for estimating the CoM of assembled systems and proposes a method for computationally adjusting thrust generation to mitigate rotational moments caused by mass imbalances. To validate the proposed computational solution, we employed the *Simulink*™ *UAV toolbox* and *Simscape*™ in MATLAB [16] to develop a parameter-defined black-box dynamics model, which enables simultaneous simulation of the controller and plant behavior. An experimental setup was also developed to compare and analyze results from both theoretical and practical approaches. The study is structured into three main sections hereafter: the first details experimental techniques for measuring the CoM; the second examines the effects of CoM variations; and the third proposes a computational methodology for correcting CoM offset impacts on the controller. The research concludes with a summary of findings from both modified and unmodified controllers, highlighting the implications of the results.

III. Experimental measurement of CoM

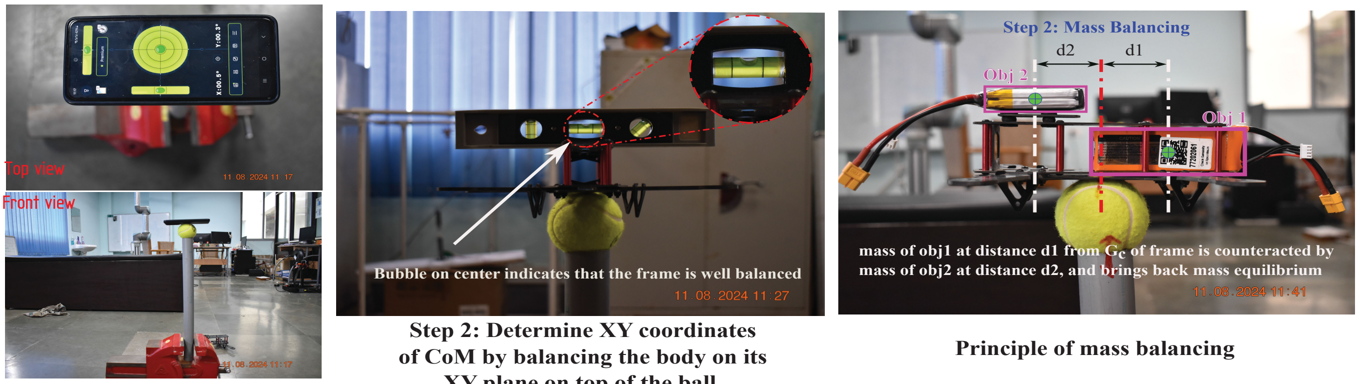
There are two experimental techniques available to measure or calculate the location of CoM in a quadcopter body. The first method known as 'Suspended Balance Method' or 'Parallel Plumb line method', estimates the X and Y coordinates of the CoM effectively. However, the disadvantage of this method is that Z coordinate cannot be effectively determined, as a quadcopter has more cross sectional area in XY plane, than in YZ or XZ planes, which makes it suitable for studies of symmetricity only. However, the second method known as 'Balancing Technique' gives us all three coordinates, even though it allows us to estimate only two coordinates at a time. By repeating this experiment for different orientations of the quadcopter, we can accurately measure or estimate the (x, y, z) coordinates of the CoM of the quadcopter. ***Fig.* 2** illustrates the Suspended Balance Method, wherein the quadcopter is suspended from a fixed height by means of a rope or string, with a plumb line also suspended from the same point. As gravity acts through the CoM of the quadcopter, the line traced by the plumb line on the surface of the quadcopter delineates an axis through which the CoM is aligned. By conducting this experiment with the quadcopter suspended from different points on its body, the intersection of the lines traced by the plumb line will provide the XY coordinates of the CoM. This methodology enables the precise determination of the CoM's location relative to the quadcopter's body in its G_c frame. However, this method is limited in that it only permits estimation of the tilt angle of the Z-axis of the CoM frame, but does not provide information on the CoM's position along the Z-axis.



A. Experimental setup

B. Determined CoM position

Fig. 2. Suspended Balance technique of Measuring X_{CoM} and Y_{CoM}



Bubble on center indicates that the frame is well balanced

mass of obj1 at distance d1 from G_c of frame is counteracted by mass of obj2 at distance d2, and brings back mass equilibrium

Step 1: Determine top center of the ball using spirit level or IMU

Step 2: Determine XY coordinates of CoM by balancing the body on its XY plane on top of the ball.

Principle of mass balancing

* *To determine XZ and YZ coordinates, repeat step-2 by balancing the quadcopter body along its XZ and YZ planes respectively*

Fig. 3. Ball - Balance technique of measuring CoM coordinates in G_C frame

Conversely, the Ball-Balance technique enables the determination of all three coordinates of the CoM. This method involves initially placing the quadcopter on the top of a ball and balancing it to determine the two planar coordinates of the CoM within that plane. The quadcopter is then balanced on another face or plane, which allows for the estimation of the third coordinate. During this process, one of the two measured coordinates will be redundant, while the remaining coordinate will represent the third dimension of the CoM. To validate the measurements, the quadcopter is placed on a third mutually perpendicular face or plane, and the balancing process is repeated. In this case, the two coordinates measured will correspond to two of three obtained previously, thus confirming the accuracy of the measurement process. ***Fig.* 3** shows the setup and measurement process using this technique.

IV. STUDY ON COM VARIATIONS

As in all 3-dimensional systems, CoM of a quadcopter is also denoted in the form of its (x, y, z) coordinates. This is critical, as variation of CoM along each axis has its own impact on the system. For example, the Z_{CoM} is a measure of the effort to be produced by the controller to correct its altitude, while X_{CoM} and Y_{CoM} are the measures of attitude correction efforts.

A. Theory of Z_{CoM} variation

There are two scenarios to be considered while studying Z_{CoM}. If the mass is added uniformly across the quadcopter, the Z-coordinate of the CoM will not change significantly. The CoM will remain approximately at the same position since the added mass is distributed evenly. Nevertheless, if the mass is added in specific areas, especially above or below the original CoM, the Z-coordinate of the CoM will shift towards the new mass. For example: Adding mass below the original CoM (e.g., a battery under the frame) will lower the CoM, whereas adding mass above the original CoM (e.g., a payload on top) will raise the CoM. Thus, the total mass of a quadcopter can be said to be the sum of three main systems of components, namely the frame, the electrical or controller-actuator system of components, and the payload. Since the payload is an optional component, its inclusion depends on the specific use case and requirements of the quadcopter. The inclusion of a solid payload mass, like for delivery missions and other such, the CoM gets lowered along Z-axis and the altitude response does have a similar effect as in the case of frame mass variation. On the other hand, for a fluid mass, as in the cases of fertiliser spraying for agricultural missions or paint spraying etc., the CoM of the payload itself is variable. Hence, in such cases, the effect of sloshing is to be considered. Compartmentalization or installation of baffles to slow down the fluid movement will greatly help to prevent sloshing and in further treating the receding fluid mass like a variable solid mass.

The vertical position (Z coordinate) of the CoM relative to the plane of the propellers significantly influences the performance of the altitude controller in a quadcopter.

1) **CoM aligned with plane of propellers:** If the CoM lies within or near the plane of the propellers, the thrust generated by each motor is evenly distributed. This ensures that the altitude controller can maintain a stable hover or controlled ascent/descent with minimal effort.
2) **CoM above or below plane of propellers:** When the CoM is significantly above or below the plane of propellers, the thrust required to maintain altitude may become uneven. The motors may need to produce additional thrust to counteract any tilting moment generated by the offset CoM. This can introduce complexity into the control dynamics, making altitude maintenance more challenging.
    - **Higher Z_{CoM}:** If the CoM is kept above the plane of propellers, the quadcopter may become more prone to pitching or rolling during altitude changes as the system behaves like an inverted pendulum. This can result in a more oscillatory response in altitude control, as the altitude controller must also counteract the induced moments.

- **Lower Z_{CoM}:** If the CoM is found to be below the plane of propellers, the quadcopter tends to be more stable in terms of pitch and roll, as the system behaves similarly to a pendulum. However, this can also make the altitude controller more sensitive to changes in thrust, potentially causing rapid altitude changes or overcorrections.

B. Impact of Z_{CoM} on Control of Flight

The position of the CoM along the Z-axis necessitates different gain settings for the altitude controller. A higher CoM will require lower gains to avoid overcompensation and oscillations, while a lower CoM will allow for higher gains for a quicker response without overshoot. Also, a higher CoM will make the quadcopter less stable in presence of external disturbances, while a lower CoM will make it more resistant to tipping, as well potentially more responsive to thrust changes. If the Z coordinate of the CoM is significantly off the propeller plane, the motors may need to work harder to maintain a stable altitude, leading to increased power consumption. The altitude controller must compensate for the additional energy required to counteract any moment introduced by the offset CoM. Such offsets can introduce a coupling between altitude and attitude dynamics, leading to potential oscillatory behavior. This can manifest as small altitude oscillations, if the controller struggles to balance the thrust and induced moments. The altitude controller will need to be more tightly coupled with the attitude controller, especially when the CoM is not in the propeller plane. This ensures that any induced pitch or roll due to the CoM offset is corrected promptly, maintaining stable altitude control.

C. Symmetricity of mass and geometry

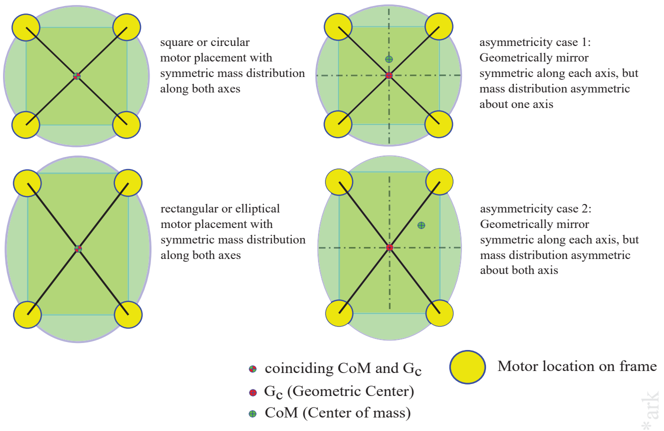


Fig. 4. Cases of symmetricity and asymmetricity based on location of CoM and G_C

Ideally, the $CoM_{(x,y)}$ should align with the geometric center to avoid additional unwanted torques. If X_{CoM} or Y_{CoM} are offset from the geometric center, it can cause an imbalance in the quadcopter's moment distribution. This imbalance can create undesired rotational moments around the pitch and roll axes, leading to instability in flight. For example, when CoM and G_c are not aligned with each other, i.e., their frames of references are not coincident, then it implies that $J_x \neq J_y$ and that the design is asymmetric. However, there is a special case of symmetricity, where the design is symmetrical around X and Y axes individually but not mutually and hence, $J_x \neq J_y$, but origins of CoM and G_c frames will coincide. This is called as mirror symmetricity about certain axis. When the design is mirror symmetric about both X and Y axis, but not mutually, it is called as bilateral symmetricity. This type of situation will happen, when the designer chooses to have an elliptical frame over circular, or rectangular motor placement over square. Thus, bilaterally symmetric designs are still stable systems that are to be considered as symmetric quadcopters. In short, symmetricity of the quadcopter depends on both, geometry as well its mass distribution.

It is critical for any flying machine, not just quadcopters to have at least one axis of geometric symmetricity and perfect symmetry of mass distribution along all axes. This is because geometric symmetricity helps in maintaining a predictable and stable flight path while perfect symmetry of mass distribution along all axes is essential to avoid imbalances that can lead to unwanted rotations or oscillations and hence uncontrollable dynamics. This is particularly important for the control systems to function effectively, as they rely on predictable responses to control inputs. For quadcopters, this symmetry helps in achieving stable hover and precise manoeuvres. In more complex flying machines, like fixed-wing aircraft or drones with morphing capabilities, maintaining symmetry becomes even more critical to ensure safe and efficient flight. In case of asymmetricity of mass, as portrayed in ***Fig.*** 4, case 1, the controller design process will remain same. However, the motor mixing algorithm will have to be changed slightly.

V. CORRECTIVE MEASURES FOR STABLE CONTROL

A. Proposed computational methodology

In perfectly mass-symmetric quadcopters, the thrust force per motor is estimated as $F_i = F_{z,total}/4$. However, in case of mass asymmetricity along one axis, the following approach must be followed.

Let location of motor m_i be (x_i, y_i) from CoM in G_c frame. The ratios of their distances from CoM along X and Y axes are found as shown in ***Fig.*** 5. Now, the torques around CoM, are resolved by using the following equations:

$$F_1 + F_2 + F_3 + F_4 = F_{Z,total} = T \tag{2.a}$$

$$F_1 - F_2 + F_3 - F_4 = 0 \tag{2.b}$$

$$-F_1 - F_2 + (k_1 \times F_3) + (k_1 \times F_4) = 0 \tag{2.c}$$

$$-(k_2 \times F_1) + F_2 + F_3 - (k_2 \times F_4) = 0 \tag{2.d}$$

Here, equations (2.a), (2.b), (2.c), and (2.d) represent the equations for thrust, yaw, pitch and roll respectively.
Note that (2.b) is valid, assuming the motor torque and force constants are the same for all motors, and are proportional. By

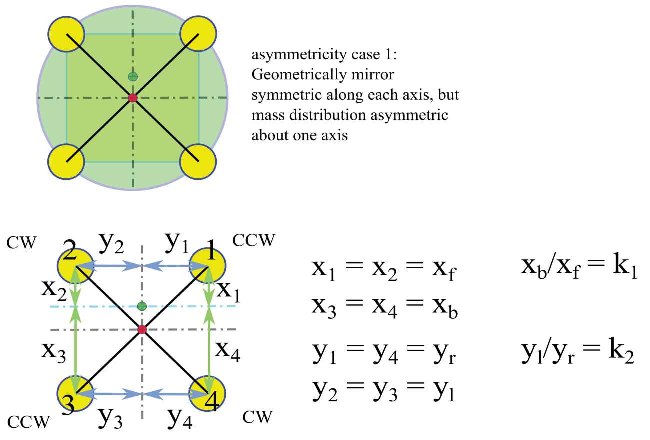


Fig. 5. Deduction of ratios of symmetricity around axes of CoM

solving this system of equations (2), we get, the thrust forces for altitude control as follows:

$$F_1 = \left(T \times \frac{((3 \times k_1) - k_2 + (k_1 \times k_2) + 1))}{(4 \times (k_1 + 1) \times (k_2 + 1))} \right) \tag{3.a}$$

$$F_2 = \left(T \times \frac{(k_1 + k_2 + (3 \times k_1 \times k_2) - 1))}{(4 \times (k_1 + 1) \times (k_2 + 1))} \right) \tag{3.b}$$

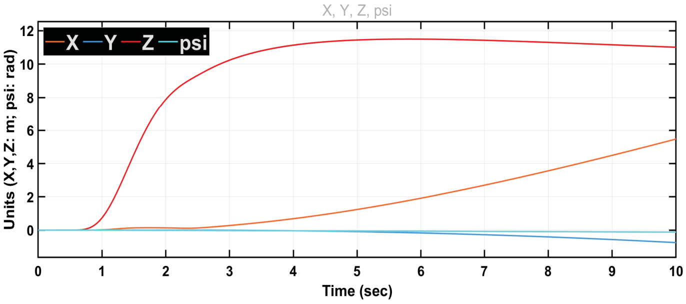
$$F_3 = \left(T \times \frac{((3 \times k_2) - k_1 + (k_1 \times k_2) + 1))}{(4 \times (k_1 + 1) \times (k_2 + 1))} \right) \tag{3.c}$$

$$F_4 = \left(T \times \frac{(k_1 + k_2 - (k_1 \times k_2) + 3))}{(4 \times (k_1 + 1) \times (k_2 + 1))} \right) \tag{3.d}$$

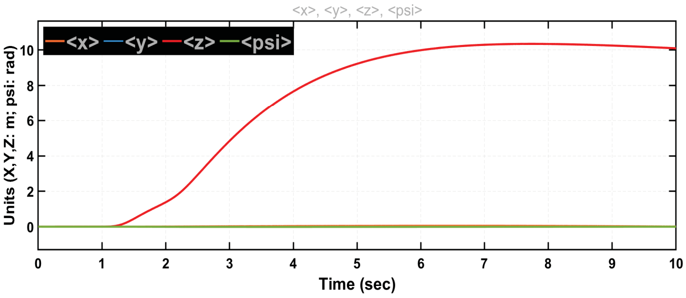
Once the roll and pitch effects due to mass offset are corrected and nullified using this method, we can control roll, pitch, and yaw moments around geometric centre as usual. There will be no other difference in controller design process. The plant may be of critically damped, under damped or overdamped in nature. Hence, linearization in such cases is to be avoided and it is better to treat the plant as a nonlinear system in such and similar cases while designing controller. By following this method, asymmetricity in case-2 (shown in ***Fig.*** 4) can also be solved, but the nature of the plant identified under such circumstances cannot be guaranteed to be stable. Further investigation is required in such scenarios and conventional control techniques may prove to be of inadequacy in such situations. So, in asymmetric mass distribution cases as illustrated in ***Fig.*** 4 case 2, intelligent adaptive techniques may be used, provided that the designer has good understanding of its dynamics and controller design requirements. This proposed methodology is limited to the use of quadcopters and octacopters alone, however a modified version of this can be applied to hexacopters, after careful consideration.

VI. VALIDATION, RESULTS AND DISCUSSION

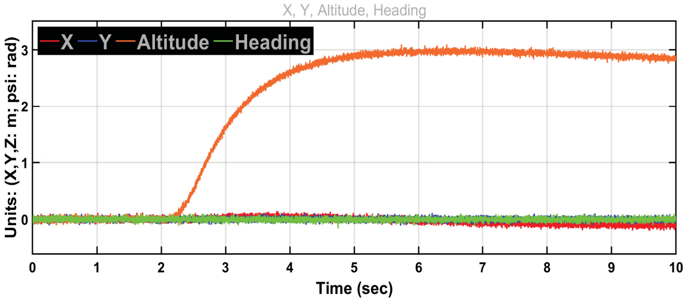
To validate the proposed methodology, the authors had applied the same to an F450 type quadrotor assembled by them (as shown in ***Fig.* 2**), experimentally, and also, in the form of a *Simulink*™ simulation. In this quadcopter, due to the offset axis of the battery placement, the origins of CoM frame and G_c frame did not coincide and the coordinates of the CoM in G_c frame were measured experimentally to be $(1.2, 0.1, -1.6)cm$. Hence, the altitude control command to reach $(0, 0, 1)m$ always resulted in drifting off of the drone along $+Xaxis$. Sometimes, when the attitude control was intentionally coupled with altitude control, it was observed that the quadcopter exhibited multiple flips and would eventually crash. The drift-away behavior can be observed in ***Fig.* 6a**.



(a) Captured drift behavior of the assembled quadcopter in response solely to altitude control in simulation; step input cmd: $(0, 0, 10)$, time delay:$0.5s$



(b) Stable hover response after implementation of proposed methodol-ogy in simulation; input step cmd: $(0, 0, 10)m$, time delay:$1.0s$



(c) Datalog plot from experiment: fairly Stable hover response after implementation of proposed methodology in px4 autopilot; input step cmd: $(0, 0, 3)m$, time delay:$2.0s$

Fig. 6. Various responses captured from simulation and exper-iment, before and after implementation of proposed algorithm to a mass-imbalanced quadcopter.

To implement the proposed algorithm, the motor mixing section of the *px4* firmware code was modified and uploaded into the flight controller hardware using *QGroundControl*™

software as a custom firmware package. It can be observed from ***Fig.*** 6c, during experimentation, the assembled quadcopter with mass offset did not crash, but had held its $X = 0m$ & $Y = 0m$ position fairly stable during hover condition, after integration of the proposed methodology, into the autopilot firmware. Similarly, in the *Simulink*™ simulation data shown in ***Fig.*** 6b also, we could see that stable hover at $(0, 0, 10)m$ had been achieved. The results shown in ***Fig.* 6b, 6c** and ***Table*-I** validate that the proposed computational thrust adjustment methodology could perform as expected.

TABLE I. COMPARISON OF THE RESULTS BEFORE AND AFTER IMPLE-MENTATION OF PROPOSED METHODOLOGY IN SIMULATION STUDY

	Before implementation of proposed thrust correction		**After implementation of proposed thrust correction**	
	ITAE	***ISE***	***ITAE***	***ISE***
Z	*70.65*	*96.5*	*60*	*86.9*
Y	*13.74*	*0.6701*	*0.007877*	*3.134e-07*
X	*100.2*	*31.63*	*3.939*	*0.06762*

VII. CONCLUSION

This study had seeked to inform design engineers, who are engaged in assembling and operating their own quadcopter systems, about the impact of CoM location on design outcomes. Additionally, it had addressed the considerations and decisions necessary for achieving an effective controller design for the same model, with a focus on the implications of mass distribution. A novel thrust force distribution algorithm had been contributed to counteract the effects of mass asymmetricity and the practical techniques of estimating mass distribution using ball-balance method and suspended-balance method had also been illustrated. Simulation study and experimental study on the flight performance of an assembled quadcopter, before and after implementation of the proposed strategy had been given and compared. In short, it was observed that,

- CoM position is crucial in finding the nature of the plant in a control system design point of view.
- Even if the CoM is offset from G_c, the resulting moments had been nullified using proposed approach.
- The ability to modify an existing quadcopter design, due to the unavailability of spare parts, could be achieved by carefully measuring the modified CoM using the techniques discussed. Thus, the proposed approach had enabled not only effective assembly but also efficient repair work.

Thus overall, this study had made an attempt on being a helpful tool for design engineers in understanding the criticality of quadcopter CoM coordinates and their impact on controller design process.

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