Efficient Algorithm of Well-Localized Bases Construction for OFTDM Systems

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Abstract

This article presents the research work related to the development of enhanced modulation technique with Orthogonal Time-Frequency Division Multiplexing (OFTDM). Utilization of well–localized bases allows to improve considerably the efficiency and robustness against intercarrier (ICI) and intersymbol interference (ISI) of now existing wideband wireless digital communication systems. Until recently one of the main disadvantages of well-localized bases was the complexity of their construction algorithm. This fact restricted the area of their practical implementation. The algebraic procedure of orthogonal well-localized Weyl-Heisenberg basis construction is considered in the paper. Several basis orthogonality conditions are observed. They allow to formulate computationally efficient orthogonalization procedure on the base of fast Fourier transform. This approach can receive wide implementation in wideband mobile networks (Wi-Max), digital television (DVB-T/H) and other telecommunication systems.

Index Terms: well-localized bases, OFDM, OFTDM, orthogonalization, wideband signals.

I. Introduction

Construction of wireless high-speed digital communication systems often faces with the situation when real radio channels are time-frequency dispersive. Signal coming to the receiving point have multiple reflections form the media inhomogeneitis (city buildings, moving objects, atmospheric layers, etc.). Among the examples of such channels are short and ultra short radio lines, wideband mobile (mobile Wi-Max) and digital TV (DVB-T/H) radio lines.

Because of time-frequency dispersion of the signal on the receiver side such effects as multipath propagation, amplitude/phase fading, Doppler spectrum spreading, frequency offset are observed. Their influence results in intercarrier (ICI) and intersymbol (ISI) interference which worsen considerably receiving characteristics. In addition such interchannel interferences could not be compensated or filtrated by the ordinary digital processing methods.

At present Orthogonal Frequency Division Multiplexing (OFDM) is one of the most efficient and widely spread technologies of data transmission in such highly dispersive channels. The structure and the properties of OFDM signals are determined by the basis used for its construction. Basic functions are taken as the intervals of harmonics with frequencies multiple to some offset frequency v_o. Their amplitude spectrum has narrow main lobe and slowly decay tails, like $\left|\sin(kx)/(kx)\right|$ function. This basis can be constructed by uniform shifts of rectangular impulse within the given frequency range. The signal is generated as the liner combination of real or complex information symbols (determined by the signal constellation: QAM, PSK, ets.). Channel equalization is simplified because OFDM may be

viewed as many slowly-modulated narrowband signals rather than one rapidly-modulated wideband signal.

Physically the appearance of ICI and ISI in time-frequency dispersive channels is caused by the loss of orthogonally of disturbed basic functions. As a result the demodulation procedure in the receiver becomes nonoptimal. The reason of connection breaks when a subscriber enlarges its velocity or when the signal/noise limit is exceed is synchronization upset or inaccurate assessment of channel parameters in the conditions of strong frequency dispersion (strong ICI).

The low symbol rate of OFDM systems makes the use of a guard interval between symbols affordable, making it possible to handle time-spreading and eliminate ISI. The effect of time dispersion can be completely compensated but with the loss of spectral and energy efficiency. Nevertheless the rectangular form of forming function is not optimal from the point of ICI. In addition the level of out-of-band emission in the classical OFDM systems is overrated.

This is why the problem of intercarrier interference rests actual for OFDM systems and in many cases doesn't have satisfactory decision.

This article presents the research work related to the development of enhanced modulation technique with Orthogonal Time-Frequency Division Multiplexing (OFTDM) affording to cope with main disadvantages of classical OFDM mentioned earlier.

In OFTDM systems special orthogonal well-localized bases are used to cope with channel time-frequency dispersion [1]. It can be shown that tightly packed orthogonal bases can be constructed only of several initializing function. So in contrast to OFDM two forming function differing in faze are used for basis generation (generalized Weyl-Heisenberg basis). In addition there are used shifts not only in frequency domain (OFDM) but also in time. These bases minimize the level of channel interference simultaneously in time and frequency domain, but do not spoil system's spectral efficiency.

Until recently one of the main disadvantages of well-localized bases was the complexity of their construction algorithm. This fact didn't allow to use such bases in portable devices. In this article several basis orthogonality conditions are observed. They allow to formulate practical and computationally efficient orthogonalization procedure on the base of fast Fourier transform. Thus the potential spectrum of implementations of OFTDM technology can be considerably broaden.

II. WEYL-HEISENBERG BASES

Orthogonal Weyl-Heisenberg basis $\mathsf{B}[J_N]$ and transmitted OFTDM signal $s(t)$ in discrete time can be equivalently presented in the following form

$$s[n] = \sum_{k=0}^{M-1} \left(\sum_{l=0}^{L-1} c_{k,l}^R \, \psi_{k,l}^R[n] + \sum_{l=0}^{L-1} c_{k,l}^I \, \psi_{k,l}^I[n] \right), \; n \in J_N \tag{1}$$

$$\psi_{k,l}^R[n] = g\left[(n-lM)_{\bmod N} \right] \exp\left(j \frac{2\pi}{M} k(n-\alpha/2) \right), \tag{2}$$

$$\psi_{k,l}^I[n] = -jg\left[(n+M/2-lM)_{\bmod N} \right] \exp\left(j \frac{2\pi}{M} k(n-\alpha/2) \right), \tag{3}$$

$$\mathsf{B}[J_N] \triangleq \left\{ \psi_{k,l}^R[n], \psi_{k,l}^I[n] \right\}, \tag{4}$$

where $c_{k,l}^R = \text{Re}(a_{k,l})$ and $c_{k,l}^I = \text{Im}(a_{k,l})$ are real and imaginary parts of complex information QAM symbols $a_{k,l}$; $s[n] = s(nT/M)$; $g[n] = g[nT/M]$ and $g[n+M/2]$ - initializing functions; $J_N = \{0,1,...,N-1\}$, $N = L \cdot M$, $M \geq 2$ - number of subcarriers, L - any natural number unequal zero; α - phase parameter.

The system of basic functions $\mathsf{B}[J_N]$ is orthogonal in terms of real scalar product defined on the Hilbert space of discrete functions on J_N

$$\langle x[n], y[n] \rangle_R = \text{Re} \sum_{n=0}^{N-1} x[n] \cdot y*[n], \tag{5}$$

where $*$ is the sign of complex conjugation.

Orthogonality condition of basis $\mathcal{B}[J_N]$ in matrix form is

$$\text{Re}(\mathbf{U} * \mathbf{U}) = \mathbf{I}_{2N}, \tag{6}$$

where $\mathbf{I}_{2N}$ is $2N \times 2N$ identity matrix; $\mathbf{U} = (\mathbf{U}_R, \mathbf{U}_I)$ is a $N \times 2N$ block matrix with blocks $\mathbf{U}_R$, $\mathbf{U}_I$ - square $N \times N$ matrixes constructed from columns of basic functions $\mathbf{\psi}_{k,l}^R = (\psi_{k,l}^R[0],...,\psi_{k,l}^R[N-1])^T$ and $\mathbf{\psi}_{k,l}^I = (\psi_{k,l}^I[0],...,\psi_{k,l}^I[N-1])^T$ for all indexes $k = 0,...,M-1$, $l = 0,...,L-1$.

Equation (1) describes the modulation algorithm of OFDM/OQAM signal in discrete time. Appropriate demodulation algorithm has the form

$$c_{k,l}^R = \langle s[n], \psi_{k,l}^R[n] \rangle_R, \quad c_{k,l}^I = \langle s[n], \psi_{k,l}^I[n] \rangle_R. \tag{7}$$

III. WELL-LOCALIZED WEYL-HEISENBERG BASES CONSTRUCTION

It is necessary to solve the problem of orthogonal Weyl-Heisenberg basis synthesis to construct a robust OFDM/OQAM system. Ambiguity function for the basis's prototype function

$$\Psi_g(t,f) = Ag(t,f) = \int_{-\infty}^{\infty} g(x+t/2)g*(x-t/2)\exp(-2\pi jfx)dx$$

has to possess maximum localization simultaneously in time and frequency domain. This criteria is analogous to orthogonal system finding which minimizes the left part of inequality responding to the Heisenberg uncertainty principle

$$\left(\int_{-\infty}^{\infty} (t-\tau)^2 |g(t)|^2 \, dt \right) \left(\int_{-\infty}^{\infty} (f-v)^2 |F(g(t))|^2 \, df \right) \geq \frac{\|g(t)\|^4}{16\pi^2}, \tag{8}$$

where $F(g(t))$ is the Fourier transform of function $g(t)$.

It is known [2] that equality in (8) is satisfied for Gaussian function

$$g_0(t) = (2\sigma)^{1/4} \exp(-\pi\sigma t^2). \tag{9}$$

So prototype function $g_0(t)$ is the best one from the point of localization. Unfortunately the basis constructed from Gaussian function as a prototype function (Gabor basis) is not orthogonal.

In the article [1] it was proposed an orthogonalization algorithm. The following problem was solved:

Problem 1. On the subset $\mathfrak{A}$ of complex matrixes of size $N \times 2N$, which satisfy orthogonality condition

$$\text{Re}(\mathbf{U}^*\mathbf{U}) = \mathbf{I}_{2N}, \ \mathbf{U} \in \mathfrak{A}$$

it is necessary to find an optimal matrix $\mathbf{U}_{opt}$ which minimizes the following functional:

$$\mathbf{U}_{opt} : \min_{\mathbf{U} \in \mathfrak{A}} \|\mathbf{G} - \mathbf{U}\|_E^2 , \tag{10}$$

where $\mathbf{G}$ is the Gabor basis matrix and $\|\mathbf{A}\|_E^2 = \text{tr}(\mathbf{A}\mathbf{A}^*)$ is a Frobenius norm.

The algorithm consists of several successive steps:

1. Gabor basis matrix $\mathbf{G} = [\mathbf{G}_R, \mathbf{G}_I]$ is constructed, where

2. $\quad \mathbf{G_R}(n, lM+k) = g_0\left[(n-lM)_{\text{mod} N}\right]\exp\left(j\frac{2\pi}{M}k(n-\alpha/2)\right),$

3. $\quad \mathbf{G}_I(n, lM+k) = -jg_0\left[(n+M/2-lM)_{\text{mod} N}\right]\exp\left(j\frac{2\pi}{M}k(n-\alpha/2)\right).$

4. Extended real Gabor matrix is constructed

5. $\quad \mathbf{G_B} = \begin{bmatrix} \text{Re}\,\mathbf{G} \\ \text{Im}\,\mathbf{G} \end{bmatrix}.$

6. Spectral decomposition of matrix $\mathbf{G_B G_B}^*$ is performed: $\mathbf{G_B G_B}^* = \mathbf{S\Lambda S}^*$

7. The following matrixes are calculated $\mathbf{\Sigma} = \mathbf{\Lambda}^{1/2}$, $\mathbf{W} = \mathbf{G_B}^*\mathbf{S\Sigma}^{-1}$

8. The optimal real matrix is calculated $\mathbf{V}_{opt} = \mathbf{SW}^*$

9. Matrix $\mathbf{V}_{opt}$ is decomposed in two parts

10. $\quad \mathbf{V}_{opt} = \begin{bmatrix} \mathbf{V}_{1opt} \\ \mathbf{V}_{2opt} \end{bmatrix}$

11. The optimal matrix of orthogonal Weyl-Heisenberg basis is constructed $\mathbf{U}_{opt} = \mathbf{V}_{1opt} + j\mathbf{V}_{2opt}$. Optimal prototype function $g[n]$ is specified be the first column of optimal matrix: $g[n] = \mathbf{U}_{opt}(n,1)$.

This algorithm allows to solve *Problem 1* and thus to find orthogonal Weyl-Heisenberg basis which has the best localization characteristics, because it is the closest to Gabor basis under the Frobenius norm.

Unfortunately this algorithm is based on the matrix orthogonal decomposition. This operation is computationally inefficient, so we need some other algorithm to use it in real systems.

In fact, the structure of Weyl-Heizenberg basis is determined by its forming function $g[n]$. So basis construction algorithm can be simplified by using $g[n]$ function's orthogonality criteria in instead of basis orthogonality conditions.

IV. BASIS ORTHOGONALITY CONDITIONS

Consider a particular case, when $g[n]$ is a real function and has a property of conjugate N-symmetry $g[n] = g*\left[(-n)_N\right]$. Optimal value if faze parameter corresponding to this type of symmetry is $\alpha = (M/2)_{\mathrm{mod}\,M}$ [2]. Under these conditions it can be shown that the basis [3]

$$\mathsf{E}[J_N] = \{g_{l,m}[n]\}, \quad \langle g_{l,m}[n], g_{l',m'}[n] \rangle = \delta_{l,l'}\,\delta_{m,m'}$$

$$g_{l,m}[n] = g\left[(n-lM)_N\right]\exp\left(j\frac{4\pi}{M}mn\right),\ m \in J_{M/2};\ l \in J_L \tag{11}$$

is orthogonal in term of ordinary scalar production, if and only if the the Weyl-Heizenberg basis, constracted on the base of the same function $g[n]$ is also orthogonal but in terms of real scalar production. The following theorem is valid:

Theorem 1. Necessary and sufficient condition of $\mathsf{E}[J_N]$ (and also $\mathsf{B}[J_N]$) basis orthogonality in time domain is the following equality

$$\sum_{r=0}^{2L-1} g\left[\left(n-r\,M/2\right)_N\right]g\left[\left(n-r\,M/2 - lM\right)_N\right] = \frac{2}{M}\delta_{l,0},\ \forall n \in J_N,\ \forall l \in J_L. \tag{12}$$

The next step is to come from the orthogonality of basis $\mathsf{E}[J_N]$ to the orthogonality of Wiener basis.

Definition 1. The set of functions $f_0, f_1, ..., f_{N-1}$ of linear space $V \subset \tilde{\mathbb{C}}^N$ ($\tilde{\mathbb{C}}^N$ is the space of N-periodical complex discrete functions) is called Wiener basis if discrete-periodical analog of Wiener theorem is valid for them: functions $\left\{g[\cdot - kN/J]\right\}_{k=0}^{J-1}$ form the basis of space V if and only if $g \in \tilde{\mathbb{C}}^N$ can be represented in the following form $g = \sum_{k=0}^{J-1} a_k f_k$ and all a_k are nonzero.

Thus the existence of Wiener basis is necessary and sufficient for the existence of function g whose vector of shifts

$$\vec{g}[n] \triangleq \left(g[n], g\left[(n-N/J)_N\right], ..., g\left[(n-(J-1)N/J)_N\right]\right)^T$$

is the basis of space V.

Definition 2. Transform which allows to pass from the vector of shifts to the orthogonal Wiener basis is called Wiener transform and can be written in the following form

$$\eta_k^{M/2}[n] = \sum_{r=0}^{2L-1} g\left[\left(n - r\frac{N}{2L}\right)_N\right]\exp\left(\frac{2\pi j}{2L}rk\right).$$

The inverse Wiener transform is

$$g[n] = \frac{1}{2L}\sum_{k=0}^{2L-1}\eta_k[n].$$

Theorem 2. Necessary and sufficient condition of $\mathsf{E}[J_N]$ (and also $\mathsf{B}[J_N]$) basis orthogonality in time domain is the following equality

$$\left|\eta_k^{M/2}[n]\right|^2 + \left|\eta_{k+L}^{M/2}[n]\right|^2 = \frac{4}{M}. \tag{13}$$

V. Efficient Orthogonalization Algorithm

Orthogonality condition (13) allows to formulate an effective orthogonalization procedure. Firstly, a real conjugate N-symmetrical function $g_o[n], n \in J_N$ should be chosen. To receive an orthogonal well-localized basis $g_o[n]$ also have to possess a good localization. For example, Gaussian function has the best time-frequency localization. Secondly, the Wiener basis is constructed in the following from:

$$\eta_k^{M/2}[n] = \frac{2\tilde{\eta}_k^{M/2}[n]}{\sqrt{M\left|\tilde{\eta}_k^{M/2}[n]\right|^2 + M\left|\tilde{\eta}_{k-L}^{M/2}[n]\right|^2}}, \tag{14}$$

where $\tilde{\eta}_k^{M/2}[n] = \sum_{r=0}^{2L-1} g_o\left[\left(n - r\frac{N}{2L}\right)_N\right]\exp\left(\frac{2\pi j}{2L}rk\right)$. It is obvious that the calculation of $\tilde{\eta}_k^{M/2}[n]$ in every point $n \in J_N$ comes to a discrete Fourier transform. Thus computationally efficient fast Fourier algorithm can be implemented N times.

It is easy to test directly that Wiener basis (14) satisfies orthogonality condition(13). So inverse Wiener transform can be used to receive the required forming function $g[n]$ of Weil-Heisenberg basis.

VI. Modeling Results

Two algorithms (on base on orthogonal decomposition and on base on FFT) of orthogonal well-localized bases were compared. It was discovered that these methods have the same output, i.e. forming function constructed from the same $g_o[n]$ (Gaussian function) match perfectly.

Graphs of ideally localized Gabor basic function $g_o[n]$ and optimal Weyl-Heisenberg basic function $g[n]$ are shown in Fig.1. For clearness basic function are transmitted in the centre of the interval J_N. It is apparent that basic functions $g[n-N/2]$ and $g_0[n-N/2]$ are close enough.
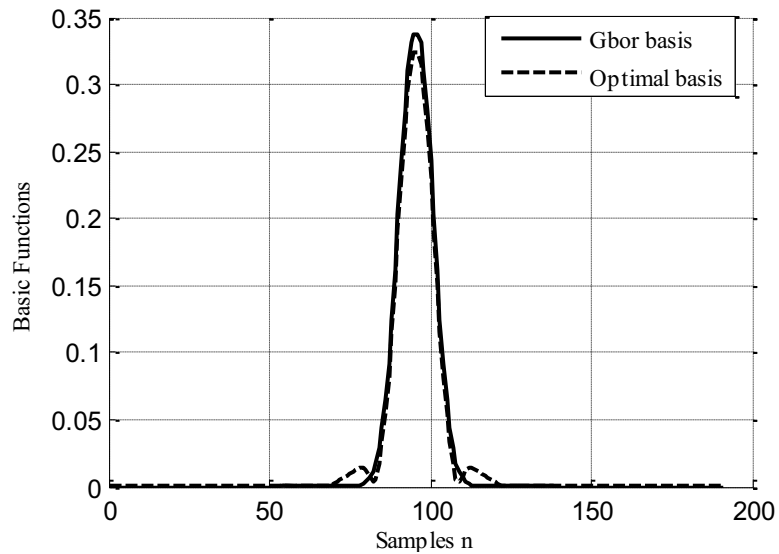
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Figure 1. Excellently localized Gaussian function and received optimal forming function

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