Phase synchronization and control of clock generators

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Abstract

Investigations of phase synchronization systems, which were held in Saint-Petersburg State University and Jyväskylä University, represented in the work. Aspects of the analysis and application of phase-locked loops and clock generators are represented in the work.

I. Introduction

The group of Prof. G.A. Leonov works in the area of synchronization and control of generators more than 30 years. In 1975-1995 the investigations of PLLs, used in radio communication, were fulfilled. The results obtained can be found in the monographs [1-3]. In 1986 these investigations deserved high evaluation and G.A. Leonov became a laureate of State Prize of USSR. In this period in the frame of the project over 20 PhD dissertations, devoted to the study of different types of phase locked loops, have been defended. The mathematical methods, of theory of phase synchronization, developed by the scholars of group, were successfully used for investigation of electrical machines [4].

In the last ten years the group deals with the application of systems of phase synchronization to computer architectures. It also considered the systems, of phase synchronization, applied to different systems, which are used clock generators. The main particularity of the problems considered consists in the change from analog field to discrete digital high-capacity systems. For the study of such systems a sophistication of previous explorations of as the group of Prof. G.A. Leonov as American scholars A.J. Viterbi and W.C. Lindsey was used. The results obtained in this direction can be found in [5-11]. In 2007 within the framework of agreement between the University of Jyväskylä (Faculty of Information Technology) and Saint-Petersburg State University (Faculty of Mathematics and Mechanics) there was organized a joint research group under the direction of Pekka Neittaanmäki (Dean, Professor) and Gennady A. Leonov (Dean, Director, Professor, Member (corresponding) of Russian Academy of Science). A main object of this research group is to gather the accumulated research work experience in the field of analytical methods of the theory of phase synchronization, numerical procedures and industrial applications, and to use these experiences for rigorous mathematical analysis and synthesis of real applied systems. So research of PLLs received a new impetus and continued within the framework of the joint research group.

The researches, which are carried out in the group, received positive feedback from companies ”Intel” and ”Hewlett-Packard”. Results of research partially included in PhD theses which were defended in University of Jyväskylä (Nikolay Kuznetsov, 2008; Elena Kudryashova, 2009) and were presented at various international conferences [12-14]. Now several patent application for devices of synchronization of processors being prepared in the group.

II. Phase-Locked Loops and Clock Generators Applications

Phase-locked loops (PLLs) are widely used in telecommunication and computer architectures. They were invented in the 1930s-1940s (De Bellescize, 1932; Wendt & Fredentall, 1943) and then intensive studies of the theory and practice of PLLs were carried out [15-18]. One of the first applications of phase-locked loop (PLL) is related to the problems of data transfer by radio signal. In telecommunications PLL is applied to carrier synchronization, carrier recovery, demodulation, and frequency synthesis (see, e.g. [19]).

After the appearance of architectures with chips, operating on different frequencies, the phase-locked loops are used to generate internal frequencies of chips and synchronization of operation of different peripherals and data buses (see, e.g. [20-21] et al.). Nowadays PLL are widely used for the solution of the problems of clock skew and synchronization for the sets of chips of computer architectures (e.g. [22-24] et al.), array processors [25] and chip micro architectures (e.g. the clock skew is very important characteristic of processors [26-28]).

Another actual application of PLL is the problem of saving energy. One of the solutions of this problem for processors is a decreasing of kernel frequency with processor load. The independent phase-locked loops permit one to distribute more uniformly a kernel load to save the energy and to diminish a heat generation on account of that each kernel operates on its own frequency.

According to www.sciencedirect.com this topic is growing up: for ”phase-locked loop” or ”PLL” or ”DPLL” 40,085 articles found, including - 2007 (515), 2008 (596), 2009 (608).

III. Nonlinear Analysis and Synthesis of Phase-Locked Loops and Clock Generators

Various methods for analysis of phase-locked loops are well developed by engineers and are considered in many publications (see, e.g., [29-33]). But recently due to the significant increase of the frequencies used in telecommunications and computer architectures, one should consider adequate nonlinear models and to carry out rigorous mathematical analysis of the models. Such consideration are still far from being resolved and require using special methods of qualitative theory of differential, difference, integral, and integro-differential equations [1-3, 10-11, 34-38].

In the research group has developed a special mathematical theory of phase synchronization based of three applied theories: theory of synchronous and induction electrical motors, theory of auto-synchronization of the unbalanced rotors, theory of phase-locked loops. Its main principle is consideration of the problems of phase synchronization at the three levels:

i) at the level of mechanical, electromechanical, or electronic model,

ii) at the level of phase relations,

iii) at the level of difference, differential, and integro-differential equations.

In this case the difference of oscillation phases is transformed in the control action, realizing synchronization. These general principles gave impetus to creation of universal methods for studying the phase synchronization systems. Modification of the direct Lyapunov method with the construction of periodic Lyapunov-like functions, the method of positive invariant cone grids, and the method of nonlocal reduction turned out to be most effective [2-3, 11]. The last method, which combines the elements of the direct Lyapunov method and the bifurcation theory, allows one to extend the classical results of F. Tricomi and his progenies to the multidimensional dynamical systems. Also were developed special asymptotic analysis methods for high-frequency nonlinear oscillations.

By the methods obtained one has made much progress on the problem of study and synthesis of systems PLL and clock generators. New block-diagrams of analog and discrete phase-lock loops which allow eliminating completely clock skew in multi processors systems are developed. At present, in the group the study of floating PLL, which completely eliminates misphasing and clock skew and are globally stable, and permit us to construct new clock frequencies, are been continued.

IV. Asymptotic Analysis and Phase Detectors Characteristics Calculation

Consider the application of the principles outlined above for phase detectors characteristics calculation.

Consider a differentiable 2π-periodic function $g(x)$, having two and only two extremums on $[0, 2\pi] : \; g^- < g^+$, and the following properties.

For any number $\alpha \in (g^-, g^+)$ there exist two and only two roots of the equation $g(x) = -\alpha$:

$$0 < \beta_1(\alpha) < \beta_2(\alpha) < 2\pi.$$

Consider the function

$$F(\alpha) = 1 - \frac{\beta_2(\alpha) - \beta_1(\alpha)}{\pi}$$

if $g(x) < -\alpha$ on $(\beta_1(\alpha), \beta_2(\alpha))$ and the function

$$F(\alpha) = -\left(1 - \frac{\beta_2(\alpha) - \beta_1(\alpha)}{\pi}\right)$$

if $g(x) > -\alpha$ on $(\beta_1(\alpha), \beta_2(\alpha))$ and $a < b$, ω.

Suppose, ω is sufficiently large relative to the numbers a, b, α, π.

Lemma 1. The following relation

$$\int_a^b \text{sign}\,[\alpha + g(\omega t)]dt = F(\alpha)(b-a) + O(\frac{1}{\omega}) \tag{1}$$

is satisfied.

Lemma 1 results from the formula for definitions of $F(\alpha)$.

Consider now the propagation of pulse high-frequency oscillations through linear filter (Fig. 1)

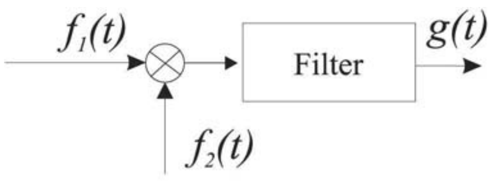

Fig. 1. Multiplicator and filter

Here

$$f_j(t) = A_j \text{sign}\,\sin(\omega_j(t)t + \psi_j), \tag{2}$$

$$g(t) = \alpha(t) + \int_0^t \gamma(t-\tau) f_1(\tau) f_2(\tau) d\tau, \tag{3}$$

$\otimes$ is a multiplicator, $A_j > 0$, ψ_j are certain constants, $j = 1, 2$, $\gamma(t)$ is a pulse transient function of linear filter and $\alpha(t)$ is an exponentially damped function, linearly depending on initial state of filter at moment $t = 0$.

A high-frequency property of generators can be reformulated as the following condition.

Consider a large fixed time interval $[0, T]$, which can be partitioned into small intervals of the form $[\tau, \tau + \delta]$, $(\tau \in [0, T])$, where the following relations

$$\begin{aligned} &|\gamma(t) - \gamma(\tau)| \leq C\delta, \quad |\omega_j(t) - \omega_j(\tau)| \leq C\delta, \\ &\forall\, t \in [\tau, \tau + \delta],\ \forall\, \tau \in [0, T], \end{aligned} \tag{4}$$

$$|\omega_1(\tau) - \omega_2(\tau)| \leq C_1,\ \forall\, \tau \in [0, T], \tag{5}$$

$$\omega_j(\tau) \geq R,\ \forall\, \tau \in [0, T], \tag{6}$$

are satisfied.

We shall assume that δ is small enough relative to the fixed numbers T, C, C_1 and R is sufficiently large relative to the number δ : $R^{-1} = O(\delta^2)$.

The latter means that on small intervals $[\tau, \tau + \delta]$ the functions $\gamma(t)$ and $\omega_j(t)$ are "almost constant" and the functions $f_j(t)$ on them are rapidly oscillating. Obviously, such a condition occurs for high-frequency oscillations.

Consider now 2π-periodic function $\varphi(\theta)$ of the form

$$\varphi(\theta) = \begin{cases} A_1 A_2 \left(1 + \dfrac{2\theta}{\pi}\right) & \text{for } \theta \in [-\pi, 0], \\ A_1 A_2 \left(1 - \dfrac{2\theta}{\pi}\right) & \text{for } \theta \in [0, \pi]. \end{cases} \tag{7}$$

and a block-scheme in Fig. 2

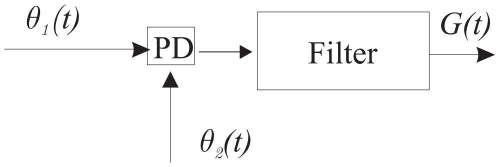

Fig. 2. Phase detector and filter

Here $\theta_j(t) = \omega_j(t)t + \psi_j$ are phases of the oscillations $f_j(t)$, PD is a nonlinear block with the characteristic $\varphi(\theta)$ (being called a phase detector or discriminator) with the output

$$G(t) = \alpha(t) + \int_0^t \gamma(t - \tau)\varphi(\theta_1(\tau) - \theta_2(\tau))d\tau. \tag{8}$$

Theorem 1. *If conditions (4)–(6) are satisfied, then for the same initial states of filter we have*

$$|G(t) - g(t)| \leq D\delta,\ \ \forall\, t \in [0, T]. \tag{9}$$

Here D is a certain not depending on δ number.

Proof. It is readily seen that

$$g(t)-\alpha(t)=\int\limits_0^t \gamma(t-s)A_1A_2\mathbf{sign}\,[\cos((\omega_1(s)-$$
$$-\omega_2(s))s+\psi_1-\psi_2)-\cos((\omega_1(s)+\omega_2(s))s+$$
$$+\psi_1+\psi_2)]ds=$$
$$=A_1A_2\sum_{k=0}^{m}\gamma(t-k\delta)\left[\int\limits_{k\delta}^{(k+1)\delta}\mathbf{sign}\,[\cos((\omega_1(k\delta)-\right.$$
$$-\omega_2(k\delta))k\delta+\psi_1-\psi_2)-cos((\omega_1(k\delta)+\omega_2(k\delta))s+$$
$$\left.+\psi_1+\psi_2)]ds+O(\delta^2)\right],\ t\in[0,T].$$

Here the number m is such that

$$t\in[m\delta,(m+1)\delta].$$

By Lemma 1 this implies the estimate

$$g(t)=\alpha(t)+A_1A_2\left(\sum_{k=0}^{m}\gamma(t-k\delta)\varphi(\theta_1(k\delta)-\right.$$
$$\left.-\theta_2(k\delta))\delta\right)+O(\delta)=G(t)+O(\delta).$$

This relation proves the assertion of Theorem 1.

Consider now a block-scheme of typical phase-locked loop [1–6] (Fig. 3)

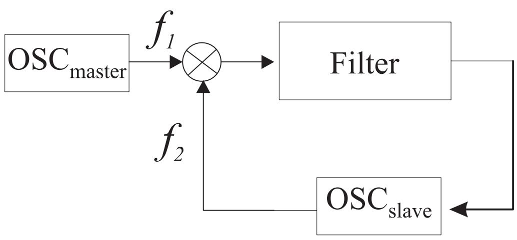

Fig. 3. Phase-locked loop with multiplicator

Here $\text{OSC}_{\text{master}}$ is a master oscillator, $\text{OSC}_{\text{slave}}$ is a slave (tunable) oscillator and block $\otimes$ is a multiplier of oscillations of $f_1(t)$ and $f_2(t)$.

From Theorem 1 it follows that for pulse generators, at the outputs of which there are produced signals (2), this block-scheme can be asymptotically changed (for high-frequency generators) to a block-scheme on the level of frequency and phase relations (Fig. 4)

Here PD is a phase detector with characteristic (7).

Thus, here on basis of asymptotical analysis of high-frequency pulse oscillations (Lemma 1 and Theorem 1) a characteristic of phase detector (7) is computed.

We give now a scheme for computing characteristics of phase detector for PLL with squarer. Consider a block-scheme in Fig. 1 with

$$f_1(t)=A_1^2(1+\mathbf{sign}\sin(\omega_1(t)t+\psi_1))^2$$
$$f_2(t)=A_2\mathbf{sign}\sin(\omega_2(t)t+\psi_2)$$

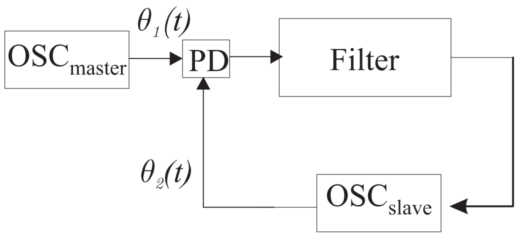

Fig. 4. Phase-locked loop with phase detector

Consider then a block-scheme in Fig. 2, where PD is a block with characteristic $F(\theta) = 2A_1\varphi(\theta)$.

Theorem 2. *If conditions (4)–(6) are satisfied, then for the same initial states of filter the relation*

$$|G(t) - g(t)| \le D\delta, \quad t \in [0, T]$$

holds true. Here D is a certain independent of δ number.

Proof of Theorem 2 is similar to that of Theorem 1.

Finally it may be remarked that for modern processors a transient process time in PLL is less than or equal to 10 sec. and a frequency of clock oscillators attains 10Ghz . Given $\delta = 10^{-4}$ (i.e. partitioning each second into thousand time intervals), we obtain an expedient condition for the proposed here asymptotical computation of phase detectors characteristics:

$$\omega^{-1} = 10^{-10} = 10^{-2}(\delta^2) = O(\delta^2).$$

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