Analytical model of WiMAX cell in AMC environment

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Abstract

Adaptive modulation and coding (AMC) is one of the key features in IEEE 802.16e standard. Different modulation and coding schemes allow for dynamic adaptation of the transmission of data to radio conditions. According to the value of the signal to interference plus noise ratio (SINR) the appropriate AMC scheme can be chosen in order to maximize transmission rate. In this paper, we develop and solve a new analytical model, considering a WiMAX cell in AMC environment. The balance equations are derived and the main performance characteristics are calculated.

Index Terms: Adaptive Modulation and Coding, Analytical Model, Performance Evaluation.

I. Introduction

Mobile WiMAX has become highly deployed in more than 150 countries recently, and its deployment is rapidly growing. In order to stay competitive in mobile data networks, continuous innovations of the standard are of great importance. Moreover, there is a need for manufactures and operators to have efficient tools for network performance evaluation [1-3]. Adaptive modulation and coding (AMC) is one of the important features in mobile WiMAX [1, 2]. In order to achieve the highest spectral efficiency the selection of appropriate AMC scheme is done according to the value of the signal to interference plus noise ratio (SINR).

Several models investigated performance of a WiMAX cell in AMC environment [1-2]. The performance of call admission control algorithms for two types of traffic, data sessions and voice calls of higher priority, was studied in [2]. The role of elastic traffic on the system performance was demonstrated. However, only two AMC schemes were considered. Here, we develop a new analytical model for a single WiMAX cell, obtain the balance equations and derive the performance metrics of interest.

II. Description of the Analytical Model

A. Basic assumptions

Assume that the total number of AMC schemes is K. The selection of appropriate AMC scheme is carried out according to the value of SINR. For the sake of simplicity, we consider basic channels (Bch) to be the smallest unit of resource. The total number of Bchs available for data transmission is M.

The arriving call requires the AMC scheme k (k-AMC) with the probability p_k, $p. = 1$. Here and further, dot instead of the index implies the full sum of the variable. Assuming uniform distribution of the mobile stations (MS) within a WiMAX cell, the probability p_k corresponds to the area s_k of the cell in which k-AMC is used. Thus, the probability of choosing k-AMC is obtained as $p_k = s_k/s.$. Note that the areas are not necessarily circles.

We assume that calls arrive according to the Poisson process with intensity λ. Hence, $\lambda_k = p_k\lambda$ is the arrival rate of a call with k-AMC. The call service time associated with k-AMC is exponentially distributed with the average value μ_k^{-1}.

Let x_i be the number of the AMC scheme required by a call, occupying the corresponding Bch i, $x_i = \overline{1,K}$, $i = \overline{1,M}$. When Bch is in outage we have $x_i = 0$. Here and further $i = \overline{1,M}$. Let us also denote $\vec{x}^{\mathrm{T}} = (x_1, x_2, \dots, x_M)$, $\vec{\lambda}^{\mathrm{T}} = (\lambda_1, \lambda_2, \dots, \lambda_K)$, $\vec{\mu}^{\mathrm{T}} = (\mu_1, \mu_2, \dots, \mu_K)$, $\vec{e}_i^{\mathrm{T}} = (0,0, \dots, 1_i, \dots, 0)$.

The presented system can be described by Markov process $\vec{\xi}(t) = \vec{x}(t)$, $t \geq 0$, with the state space $X = \{\vec{x} | x_i = \overline{0,K}, i = \overline{1,M}\}$. Notice that the state space is not trivial. In models investigated in [1, 2], the state space describes the number of active users in the system. Here we represent the state space $(x_1, \dots, x_M)$ as the number of the AMC scheme $x_i = \overline{0,K}$ used at the corresponding Bch i.

B. The solution of the model

In what follows, we will use square brackets for probability notation. According to the assumptions there is a unique [4] steady-state probability distribution $\{[\vec{x}]\}$, that can be described by solving the following balance equations:

$$[\vec{x}]\left(\sum_{i=1}^{M} u(1-x_i)\lambda + u(x_i)\mu_{x_i}\right) = \sum_{i=1}^{M}\left\{u(x_i)\lambda_{x_i}[\vec{x} - \vec{e}_i x_i] + u(1-x_i)\sum_{k=1}^{K}\mu_k[\vec{x} + \vec{e}_i k]\right\},$$

$\vec{x} \in X$, with the normalizing equation $\sum_{\vec{x}\in X}[\vec{x}] = 1$ and Heaviside function $u(x) = \begin{cases} 1, x > 0 \\ 0, x \leq 0 \end{cases}$.

The steady-state probability distribution can be obtained by the numerical iterative algorithms. Notice that the state space grows exponentially fast and is high even for small values of M and K.

III. Performance measures

The steady state probability distribution allows us to obtain the main performance characteristics of the system.

Let π be the probability that there is no idle Bchs in the system, then

$\pi = P\{\vec{x}: \prod_{i=1}^{M} x_i \neq 0\} = \sum_{x_1=1}^{K} \dots \sum_{x_M=1}^{K}[x_1, \dots, x_M]$.

The probability that the call with k-AMC will be blocked is $\pi^k = p_k\pi$.

The mean number of calls with k-AMC is

$\bar{N}^k = \sum_{\vec{x}\in X}[\vec{x}]\sum_{i=1}^{M}\delta(x_i, k)$,

where $\delta(x_i, k)$ is a Kronecker symbol, $\delta(i,j) = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$

According to Little's law the mean transmission time of the call with k-AMC is calculated

$W^k = \frac{\bar{N}^k}{\lambda_k(1-\pi^k)}$.

Utility can be found

$U^{\mathrm{k}} = \frac{\sum_{\vec{x}\in X}[\vec{x}]\sum_{i=1}^{M}\delta(x_i,k)}{M}$.

The probability of the outage is $p_0 = [\vec{0}]$. The probability that M_0 out of M Bchs are in the outage state is given by

$p_0(M_0) = \sum_{\vec{x}\in X}[\vec{x}]\sum_{i=1}^{M}\delta(x_i, 0)$.

IV. CONCLUSION

A WiMAX cell in AMC environment was analyzed. Markov model for the case of K AMC was described. The main performance characteristics were obtained. However, the dimension of a state space is very high, which makes the model intractable for realistic values of K and M. Hence topics of future research are the effective ways of transition the multi-dimensional Markov chain to a simplified univariate distribution in compliance with the model described above.

REFERENCES

[1] S. Doirieux, B. Baynat, T. Begin. On finding the right balance between fairness and efficiency in WiMAX scheduling through analytical modeling // Proceedings of the 17th IEEE/ACM International Symposium on Modeling, Analysis and Simulation of Computer and Telecommunication Systems, MASCOTS 2009 – September London (England).

[2] T. Efimushkina, D. Moltchanov, Y. Koucheryavy. Analysis of WiMAX Cell with two AMC modes and Elastic Data Traffic // April 26-30, 2010. –Proceedings of 7th Finnish-Russian University Cooperation in Telecommunications (FRUCT) Conference, Saint-Petersburg.

[3] Basharin G.P. Lectures on mathematical teletraffic theory// Moscow, PFUR Publ., 2009. – 342 p.

[4] Feller W. An Introduction to Probability Theory and Its Applications // John Wiley & Sons, 1968, vol.1.