

Comparative Analysis of Equalization Methods for SC-FDMA

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Abstract

In this paper we introduce comparative analysis for different types of equalization schemes, based on the minimum mean square error (MMSE) optimization. The following types of equalizers were compared: linear equalization, decision feedback equalization (DFE) and turbo equalization. Performance and complexity of these schemes were tested for Single Carrier Frequency Division Multiple Access (SC-FDMA) system with Single Input Single Output (SISO) antenna configuration. SC-FDMA is a common technique, which is used in the UTRA LTE Uplink, so the results of complexity and performance analysis could be applied to find the appropriate equalization algorithm to be used in the Uplink channel of the LTE – the famous standard in 4G telecommunications. Simulation results in the end in this paper show bit error ratio (BER) and modulation error ratio (MER) for compared schemes.

I. INTRODUCTION

3GPP Long Term Evolution (LTE) is the telecommunication network of the next generation, following 3G. The main advantages of this new technology are high data rate, low latency and packet optimized radio access technology. The LTE specification provides uplink peak rates of at least 50 Mbps in 20 MHz system bandwidth. For the realization of LTE Uplink Single Carrier Frequency Division Multiple Access (SC-FDMA) transmission is used. The reason of choosing this technology is that SC-FDMA has sufficiently low Peak-to-Average Power ratio (PAPR) of signals in comparison with Orthogonal Frequency Division Multiple Access (OFDMA) transmission. It results in significantly lower power consumption in the user equipment (UE).

One of the actual problems in this area is to provide the reliable transmission over the LTE, and for this reason it is necessary to choose the equalization method for the received signal. The main goal of this paper is to analyze and compare the performance for the existing equalization methods for SC-FDMA. We focus mainly on turboequalization as a primary solution, and the structure of investigated turbo equalizer bases on the idea introduced in [1]. In this work we also utilize improved adaptive coefficients solution based on the technique suggested in [2].

The paper is organized as follows. In Section II we provide the mathematical derivation of the equivalent channel impulse response for the SC-FDMA. Section III describes the equalization schemes. In Section IV the simulation results are represented and discussed.

II. SYSTEM MODEL

In this section we derive a mathematical equation of the equivalent channel impulse response for the SC-FDMA.

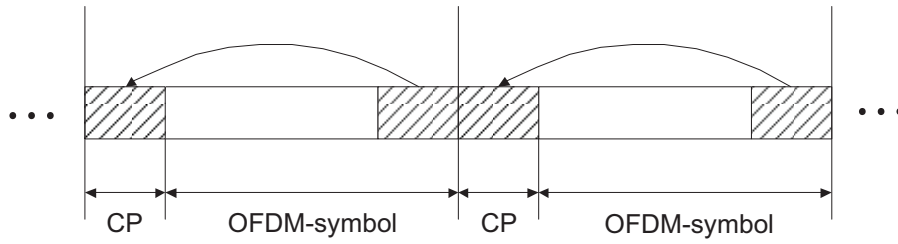


Figure 1. CP usage in SC-FDMA symbol

One slot of information data may consist from one to several Resource Blocks (RB) and each RB has 12 orthogonal subcarriers. The channel resource allocation may be performed dynamically for each subframe of data depending on individual channel features.

SC-FDMA may be considered as OFDMA, which is appended with FFT block for each subcarrier. This conception provides power efficient signal transmission, but, on the other hand, it brings the interference component to the transmitted signal in frequency domain from neighboring subcarriers. Orthogonality between the subcarriers is maintained by use of Cyclic Prefix (CP) (see Fig. 1). It prevents Inter-Symbol Interference (ISI) between SC-FDMA information blocks and transforms the linear convolution of the multipath channel into a circular convolution, enabling the receiver to equalize the channel.

Signal transmission in SC-FDMA can be written as a block diagram in matrix form (see Fig. 2). On the transmitter side the signal is converted to the frequency domain by DFT of the size M . After that frequency domain symbols are mapped on the localized or distributed subcarriers. After conversion to the time domain and CP insertion signal is transmitted through the channel, which is simulated as the Rayleigh fading channel. Inverse process is performed on the receiver side.

Lets define SC-FDMA symbol transmitted to the channel as a column vector $s = [s_{-P}, \dots, s_0, \dots, s_{N-1}]^T$, where P is CP length, N is the size of the IDFT block at the transmitter (or the total amount of available subcarriers). SC-FDMA symbol received from the channel is denoted as $r = [r_{-P}, \dots, r_0, \dots, r_{N-1}]^T$. Lets define multipath channel impulse response by the vector of coefficients $\{h_l\}_{l=0, \dots, L}$, then each component of the vector r is written as

$$r_i = \sum_{l=0}^L h_l s_{i-l} + w_i, \quad i = -P, \dots, N-1, \quad (1)$$

or in the matrix form as

$$r = Hs + w, \quad (2)$$

where w – Gaussian noise vector. In (2) H denotes the $(P+N) \times (P+N)$ matrix of channel response linear convolution:

$$H = \begin{bmatrix} h_0 & 0 & \dots & \dots & \dots & 0 \\ \vdots & h_0 & \ddots & & & \vdots \\ h_L & \vdots & \ddots & \ddots & & \vdots \\ 0 & h_L & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \dots & 0 & h_L & & h_0 \end{bmatrix} \quad (3)$$

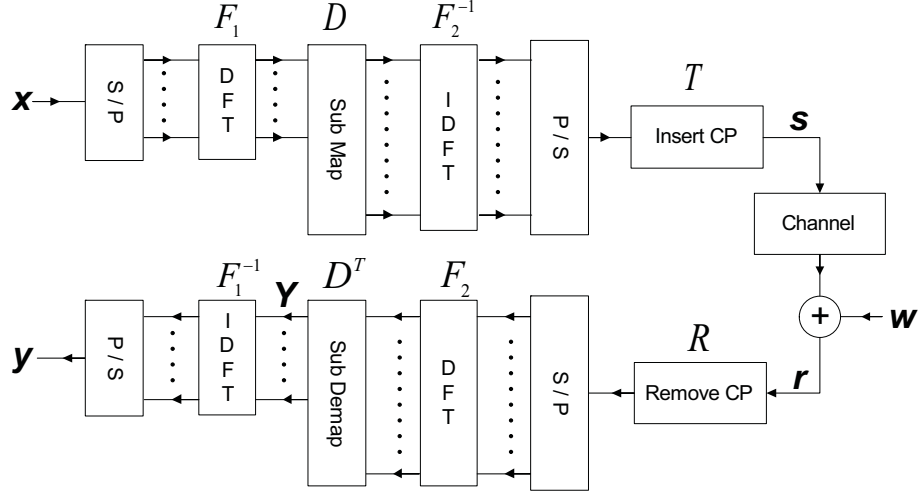


Figure 2. Block diagram of SC-FDMA

Data symbols at the input of the transmitter are denoted as the vector $x = [x_0, \dots, x_{M-1}]^T$, where M is the number of transmitted data symbols in one SC-FDMA block (it is similar to the number of subcarriers allocated for one user and defines the size of the DFT block at the transmitter side). F_1 denotes the $M \times M$ DFT matrix, i.e.

$$F_{1(p+1,q+1)} = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi}{M}pq}, \quad p, q = 0, \dots, M-1 \quad (4)$$

F_2^{-1} denotes $N \times N$ IDFT matrix. It should be mentioned that scaling factor $\frac{1}{\sqrt{M}}$ must be used instead of $\frac{1}{\sqrt{N}}$ to maintain the same output signal power. D is the $N \times M$ matrix that maps m th frequency domain data symbol to the n th available subcarrier, where $m = 0, \dots, M-1$ and $n = 0, \dots, N-1$. The mapping matrix D for localized subcarrier distribution is defined as follows:

$$D_{(n+1,m+1)} = \begin{cases} 1, & n = RAU \cdot M + m \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where $RAU = 0, \dots, \frac{N}{M} - 1$.

It means that the equation of transmitted to the channel data block may be written as follows:

$$s = TF_2^{-1}DF_1x \quad (6)$$

On the receiver side inverse process is used. So, the equation of the output signal is

$$y = F_1^{-1}D^TF_2Rr, \quad (7)$$

where T and R are the matrices of insertion and removing of CP, which may be defined as

$$T = \begin{bmatrix} I_{CP} \\ I_N \end{bmatrix}, \quad (8)$$

$$R = \begin{bmatrix} O_{N \times P} & I_N \end{bmatrix}, \quad (9)$$

where I_N is $N \times N$ identity matrix, I_{CP} is $P \times N$ matrix that copies the last P rows of I_N , $O_{N \times P}$ is the $N \times P$ null matrix.

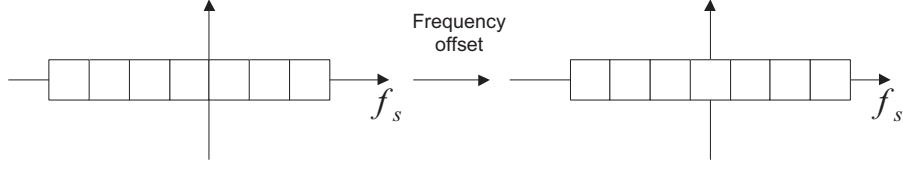


Figure 3. Frequency offset of SC-FDMA data blocks to the half of subcarrier band gap

After substituting the (2), (6) into (7), we will get the equation of the SC-FDMA equivalent channel:

$$y = F_1^{-1} D^T F_2 (RHT) F_2^{-1} D F_1 x + w', \quad (10)$$

where w' is the effective noise at the output of the receiver:

$$w' = F_1^{-1} D^T F_2 R w \quad (11)$$

In practice the spectrum of the SC-FDMA data block should be zero symmetrical. For this purpose the frequency offset (see Fig. 3) to the half of subcarrier band gap Δf is used. When the sampling rate is f_s the frequency offset is:

$$\phi^n = e^{-j2\pi \frac{\Delta f}{2} t} = e^{-j\pi \Delta f \frac{n}{f_s}} = e^{-j\pi \frac{f_s}{N} \frac{n}{f_s}} = e^{-j\pi \frac{n}{N}} \quad (12)$$

The use of CP in the SC-FDMA data blocks transforms the matrix H of linear convolution to circular one. Lets define matrix H_0 as

$$H_0 = RHT = \begin{bmatrix} h_0 & 0 & \dots & 0 & h_L & \dots & h_1 \\ \vdots & h_0 & \ddots & & \ddots & \ddots & \vdots \\ h_{L-1} & \vdots & \ddots & \ddots & & \ddots & h_L \\ h_L & h_{L-1} & & \ddots & \ddots & & 0 \\ 0 & h_L & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \ddots & 0 \\ 0 & \dots & 0 & h_L & h_{L-1} & \dots & h_0 \end{bmatrix} \quad (13)$$

H_0 is the circulant matrix and may be diagonalized by the following property:

$$\tilde{H}_0 = F_2 H_0 F_2^{-1} = \text{diag} \{ \tilde{h}_0, \tilde{h}_1, \dots, \tilde{h}_{N-1} \} \quad (14)$$

Matrix \tilde{H}_0 corresponds to the equivalent channel impulse response in the frequency domain, i.e.

$$\tilde{h}_k = \sum_{l=0}^L h_l e^{-j \frac{2\pi}{N} k l} \quad (15)$$

After subcarrier mapping/demapping procedure the channel impulse response may be written in the following form:

$$\tilde{H}' = D^T \tilde{H}_0 D \quad (16)$$

and \tilde{H}' keeps to be diagonal and is set by the elements from \tilde{H}_0 as:

$$\tilde{H}'_{(m+1, m+1)} = \tilde{H}_0_{(n+1, n+1)}, \quad (17)$$

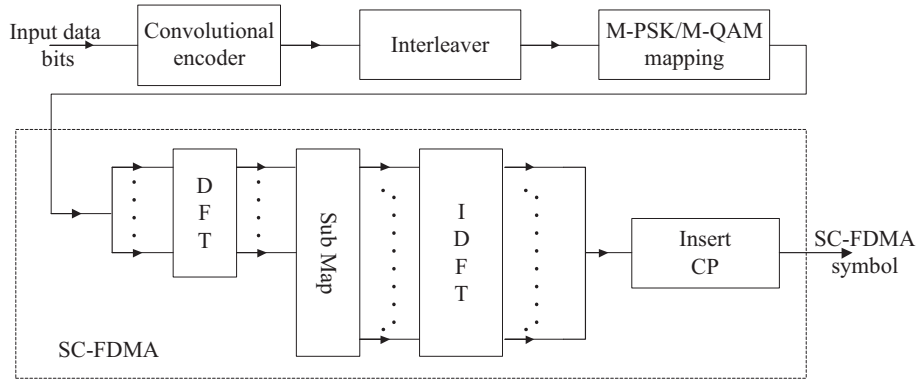


Figure 4. SC-FDMA transmitter with convolutional coding and interleaving

where the values of m and n corresponds to (5). Matrix \tilde{H}' defines the equivalent impulse channel response for each user subcarrier. As \tilde{H}' is diagonal matrix, it could be written as composition of circulant matrix and DFT/IDFT matrices:

$$\tilde{H}' = F_1 H' F_1^{-1} \quad (18)$$

From this follows that (10) may be rewritten as

$$y = F_1^{-1} (F_1 H' F_1^{-1}) F_1 x + w' = H' x + w', \quad (19)$$

where H' is the matrix of the equivalent channel response in the UTRA LTE Uplink with the following coefficients:

$$h'_l = \frac{1}{M} \sum_{k=0}^{M-1} \tilde{h}'_k e^{j \frac{2\pi}{N} kl}, \quad l = 0, \dots, L', \quad (20)$$

where L' is the length of the equivalent channel response in the Uplink and may be estimated, according to [4], as:

$$L' = \left\lceil (L + 1) \times \frac{M}{N} \right\rceil - 1 \quad (21)$$

III. DESCRIPTION OF THE USED EQUALIZATION SCHEMES

To provide the correct comparison of equalization schemes, we need to use the same coding in all of them. In the turbo equalization it is necessary to use error-correcting code. In this work convolutional coder with rate $\frac{1}{2}$ from [5] is used. In this case the structure of the SC-FDMA transmitter should be changed according to Fig. 4.

As SC-FDMA may be interpreted as OFDMA proceeded by a DFT block, the primitive equalization schemes may be deployed there without significant difficulties. For example, in this paper we use linear (LE) [6] frequency domain MMSE [6] equalization (see Fig. 5), that is used as a starting point to more advanced schemes. MMSE criteria gives the following coefficients for LE:

$$P_{LE,k} = \frac{H_k^*}{|H_k|^2 + SNR^{-1}}, \quad (22)$$

where H is the estimation of the channel impulse response in frequency domain (it is calculated according to [3]), SNR is Signal-to-Noise ratio.

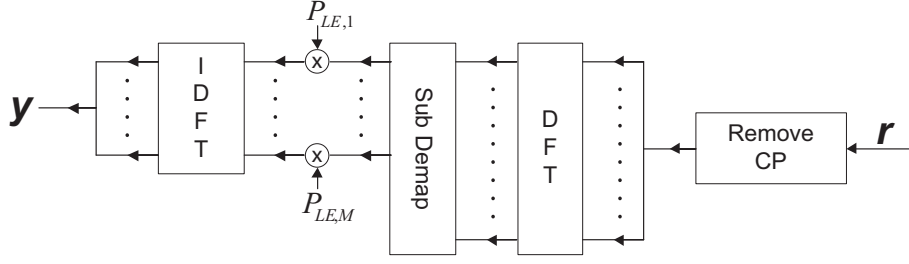


Figure 5. Linear equalization for SC-FDMA system

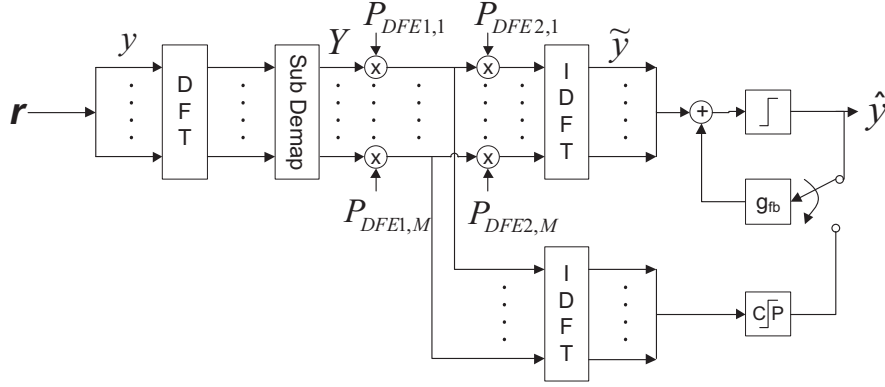


Figure 6. DFE structure for SC-FDMA system

Decision feedback equalization (DFE) [7] is another solution that performs better than LE due to its ability to cancel Inter-Symbol Interference (ISI) component of the received signal with the help of previously received data symbols [6]. In this paper we use approach from [4], which introduces DFE scheme for SC-FDMA (see fig. 6) with frequency-domain feedforward (FF) filter and a time-domain feedback (FB) filter. The derivation of the coefficients is done in [7] and here we provide only the results for reading simplicity. The FB coefficients may be obtained by solving the equation $A_{MMSE} \cdot g_{FB} = b_{MMSE}$, where matrix A_{MMSE} and vector b_{MMSE} are given by

$$[A_{MMSE}]_{i,l} = \sum_{k=0}^{M-1} \frac{e^{-j\frac{2\pi}{M}k(l-i)}}{|H_k|^2 + SNR^{-1}}, \quad 1 \leq i, l \leq L' \quad (23)$$

$$[b_{MMSE}]_i = - \sum_{k=0}^{M-1} \frac{e^{j\frac{2\pi}{M}ki}}{|H_k|^2 + SNR^{-1}}, \quad 1 \leq i \leq L' \quad (24)$$

The FF coefficients are

$$P_{DFE,k} = \frac{H_k^*}{|H_k|^2 + SNR^{-1}} \times (1 + G_{FB,k}), \quad (25)$$

where $G_{FB} = DFT(g_{FB})$ and $P_{DFE1,k}$ is the first term in the (25), $P_{DFE2,k}$ is the second one.

Based on advanced iterative equalization and error-correcting decoding technique turbo equalization [8], [9] allows to significantly increase performance of the data transmission

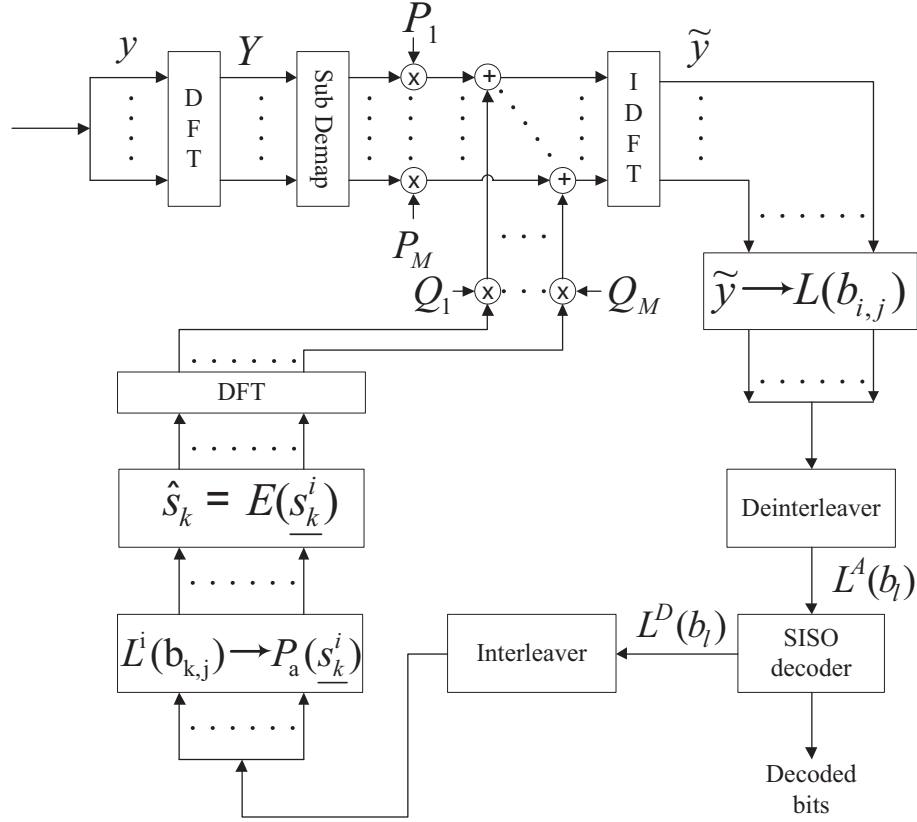


Figure 7. Turbo equalization for SC-FDMA system

over a frequency selective fading channel. The approach (see Fig. 7), described in this paper, is based on the turbo equalizer for SC-FDMA for Single Input Multiple Output (SIMO) antenna configuration, which is introduced in [1] and uses adaptive coefficients. For testing purposes SISO antenna configuration is used.

On the each iteration of the scheme, algorithm calculates log-likelihood ratios (LLR) of all the coded bits as follows:

$$L(b_{k,j}) \approx \frac{\min_{\epsilon_k^i: b_{i,j}=0} \epsilon_k^i - \min_{\epsilon_k^i: b_{i,j}=1} \epsilon_k^i}{2\sigma_w^2}, \quad (26)$$

where σ_w^2 is the power of the noise in channel, $b_{i,j}$ is the j -th bit of the i -th constellation point, k is the index of the transmitted symbol and ϵ_k^i denotes the squared Euclidean distance of the equalizer output to constellation point.

Then algorithm calculates improved LLRs $L^D(b_l)$ with the help of SISO decoder, based on the BCJR algorithm [5]. With the help of those improved LLRs $L^D(b_l)$ is possible to calculate the $(N \times M)$ apriori symbol symbol probability matrix $[P_a(s_k^i)]$ [1] and compute the estimation of the transmitted symbol [1]:

$$\hat{s}_k = E\left(\underline{s}_k^i\right) = \sum_{i=1}^M s^i P_a(s_k^i) \quad (27)$$

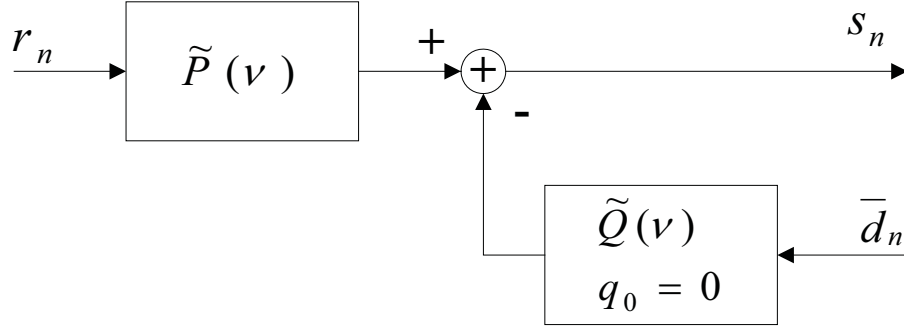


Figure 8. Structure of infinite length equalizer

The theoretical transfer function of the infinite length linear MMSE equalizer with a priori information could be used to derive the adaptive coefficients for turbo equalizer. The calculation is made in [2] and here we introduce only the results.

The equalizer consists of two filters, which are used for signal r_n received from the Uplink channel and its soft estimation \bar{d}_n (see Fig. 8). The theoretical transfer function for these filters are derived as follows:

$$\tilde{Q}(\nu) = \tilde{P}(\nu)\tilde{H}(\nu) - \lambda\beta \quad (28)$$

$$\tilde{P}(\nu) = \frac{1}{1 + \gamma \frac{\sigma_d^2}{\sigma_w^2} \left(1 - \frac{\sigma_d^2}{\sigma_w^2}\right) \left|\tilde{H}(\nu)\right|^2 + \frac{\sigma_w^2}{\sigma_d^2}} \tilde{H}^*(\nu), \quad (29)$$

where

$$\beta = T \int \frac{\left|\tilde{H}(\nu)\right|^2}{\left(\sigma_d^2 - \sigma_w^2\right) \left|\tilde{H}(\nu)\right|^2 + \sigma_w^2} d\nu \quad (30)$$

$$\lambda = \frac{\sigma_d^2}{1 + \beta\sigma_d^2} \quad (31)$$

$$\gamma = T \int \frac{\left|\tilde{H}(\nu)\right|^2}{\left(1 - \frac{\sigma_d^2}{\sigma_w^2}\right) \left|\tilde{H}(\nu)\right|^2 + \frac{\sigma_w^2}{\sigma_d^2}} d\nu, \quad (32)$$

where σ_d^2 is the power of the signal estimation on the each iteration of the turbo equalizer, σ_w^2 is the power of the transmitted signal in the frequency domain, σ_w^2 is the power of the Gaussian noise in the channel and T is the sampling time.

IV. SIMULATION RESULTS

For equalization process simulation in the UTRA Uplink LTE channel the typical parameters (Table I) were chosen according to [3].

For performance evaluation of the equalization schemes, that were described in this paper, BER and MER values were calculated for different values of SNR. Also, for the purpose of the experiment's accuracy, different lengths of the channel impulse response values were used.

Table I
PARAMETERS USED IN THE SIMULATION PROCESS.

Carrier bandwidth	5 MHz
FFT size	512
Sampling rate	7,68 MHz
Number of subcarriers	300
Number of RBs	25
Bandwidth efficiency	90%
Modulation	QAM-16

Lets define the SNR formula:

$$SNR = 10 \log_{10} \frac{\overline{E}_s}{2\sigma_w^2}, \quad (33)$$

where \overline{E}_s is the average power of the signal, which is transmitted in the channel, σ_w^2 is the power of the Gaussian noise.

MER may be defined as follows:

$$MER = 10 \log_{10} \frac{\sum_i |x_i|^2}{\sum_j |x_j - y_j|^2}, \quad (34)$$

where x_i is the transmitted constellation point, y_i is its estimation at the output of equalizer.

For the sake of making the correct comparison of the simulation results of turbo equalizer with linear and DFE schemes, convolutional encoder [5] and maximum a posteriori (MAP) decoding [5] were added to the last ones.

From the results of MER and BER performance comparison (see Fig. 9) could be seen, that DFE scheme provides better performance than linear equalization, when the length of the channel impulse response grows up. Moreover, the length of the feedback (FB) filter N_{FB} may be set less than the length of the channel impulse response without significant losses in the efficiency of the DFE scheme.

Results from the Fig. 9 shows that deployment of the turbo equalization scheme in the SC-FDMA allows to increase the efficiency of the transmission up to 3 dB for the error probability 10^{-3} in comparison with DFE and linear schemes. From the series of the simulations that were done during the experiment we conclude that the gain from turbo equalization does not increase significantly after 3 iterations. Moreover, it may decrease due to the the disadvantage of error propagation existence in turbo equalization.

V. CONCLUSION

This paper demonstrates the comparison of linear, DFE and turbo equalizers. For these purposes the simulation model of the SC-FDMA transmission was developed and tested for presented equalization schemes. The coefficients were estimated based on the MMSE technique. A DFE gives better performance due to its ability to remove inter symbol interference (ISI) component from the output signal, but suffers from error propagation as well as turbo equalizer does.

The best performance is introduced by turbo equalization due to the improvement of the equalized signal estimation on the each iteration and better cancelation of the ISI component. The results of experiments show that usage of 2-3 iterations is enough to get significant increase in the performance of the SC-FDMA system. The use of turbo equalization in the

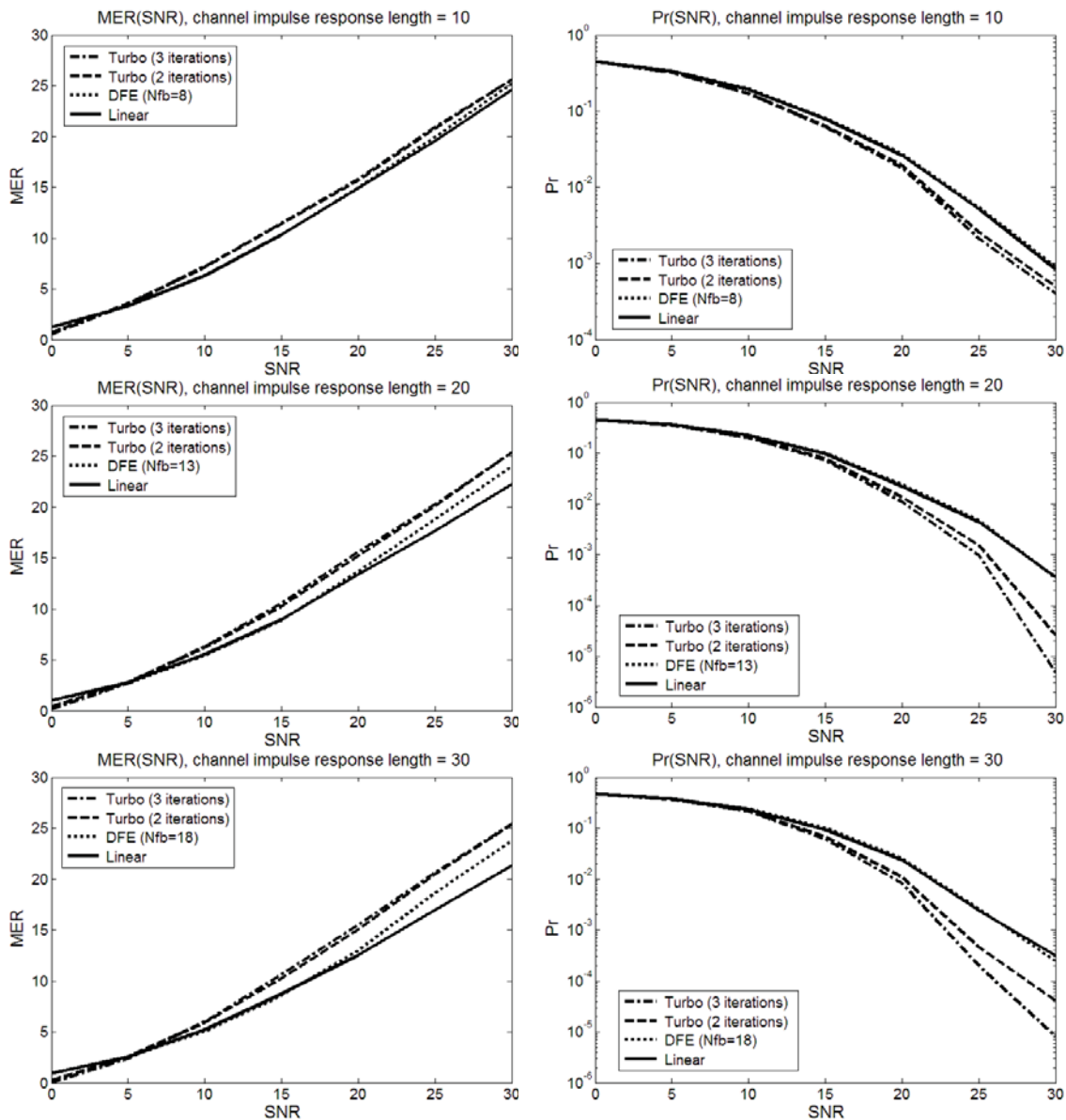


Figure 9. BER and MER comparison for linear, DFE and turbo equalizers in SC-FDMA

Uplink channel of the UTRA LTE network will result in extra receiver complexity, but isolated from the base station, which does not have strong power constraints. Since this is viable, SC-FDMA with turbo equalization can be considered for use in the Uplink channel of the UTRA LTE network to increase system performance and efficiency.

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