Analysis of Capacity of Picocell with Dominating Video Streaming Traffic

Evgeny Bakin, Anna Borisovskaya, Igor Pastushok

Saint-Petersburg State University of Aerospace Instrumentation

Saint-Petersburg, 2014
Centralized wireless networks may contain a sufficient number of users. If resources are not enough for requiring to quality of service (QoS) for each user, then network is congested. The possible causes of users video playback degradation:

- **Rebuffering** – state of streaming invoked when the playback buffer is emptied.
- **Jitter** – variation of playback speed.
- **Playback smoothness** – frequency of video bit rate switching.

A common problem for a network developer is estimation of number of users, who can simultaneously watch video content without playback degradation.
General description of system model

- Number of active users is constant $N$.
- All the users request videos in the same bit rate:
  $$R_i = R$$
- Maximum channel throughput is calculated by the Shannon formula:
  $$C_i = \Delta F \log_2 (1 + q_i)$$
- User SNR depends on a distance between BS and UE:
  $$q_i = \frac{a}{d_i^\gamma}$$
User disposition can be modeled with an uniform distribution in sphere, which radius corresponds to a maximum distance $d_{max}$.

CDF:

$$F_d(x) = \begin{cases} 
0, & \text{if } x < 0 \\
\frac{x^3}{d_{max}^3}, & \text{if } 0 \leq x \leq d_{max} \\
1, & \text{if } x > d_{max}
\end{cases}$$

PDF:

$$f_d(x) = \begin{cases} 
0, & \text{if } x < 0 \\
\frac{3x^2}{d_{max}^3}, & \text{if } 0 \leq x \leq d_{max} \\
0, & \text{if } x > d_{max}
\end{cases}$$
**Congestion and network capacity**

**Definition**

**Congestion** is an event, when total amount of required resources is greater than one.

\[
Pr\{\text{Congestion}\} = Pr \left\{ \sum_{i=1}^{N} \frac{R_i}{C_i} \geq 1 \right\} = Pr \left\{ \sum_{i=1}^{N} \frac{1}{\log_2(1 + q_i)} \geq \frac{\Delta F}{R} \right\}
\]
# Congestion and network capacity

## Definition

**Congestion** is an event, when total amount of required resources is greater than one.

\[
Pr\{\text{Congestion}\} = Pr\left\{ \sum_{i=1}^{N} \frac{R_i}{C_i} \geq 1 \right\} = Pr\left\{ \sum_{i=1}^{N} \frac{1}{\log_2(1+q_i)} \geq \frac{\Delta F}{R} \right\}
\]

## Definition

**Network capacity** \( N_c \) is a maximum number of users, for which the probability of congestion is less than given level \( p_c \).

\[
N_c = \arg \max_N \left\{ Pr\left\{ \sum_{i=1}^{N} \frac{R_i}{C_i} \geq 1 \right\} \leq p_c \right\}
\]
Auxiliary Inequality

$$\frac{1}{\log_2(1 + x)} \leq \log_2 \left(1 + \frac{1}{x}\right) + \alpha$$

$$Pr\{\text{Congestion}\} \leq Pr \left\{ \sum_{i=1}^{N} \left( \log_2 \left(1 + \frac{1}{q_i}\right) + \alpha \right) \geq \frac{\Delta F}{R} \right\}$$
Approximate Calculation of Congestion Probability

1. Denote: \( \log_2 \left( 1 + \frac{1}{q_i} \right) + \alpha \) as \( X_i \) and \( \sum_{i=1}^{N} X_i \) as \( S_N \).

2. According to Central Limit Theorem (CLT), distribution of \( S_N \) is close to the normal distribution with mean \( E[S_N] \) and variance \( Var[S_N] \).

\[
Pr \{ \text{Congestion} \} \leq Pr \left\{ S_N \geq \frac{\Delta F}{R} \right\} \approx Q \left( \frac{\Delta F - E[S_N] R}{R \sqrt{Var[S_N]}} \right)
\]

3. Here: \( E[S_N] = N \cdot E[X_i] \), \( Var[S_N] = N \cdot Var[X_i] \), since \( X_i \) are independent random variables.
As was mentioned above:

\[ Pr\{\text{Congestion}\} \leq Pr \left\{ S_N \geq \frac{\Delta F}{R} \right\} \]

For finding upper bound for \( Pr \left\{ S_N \geq \frac{\Delta F}{R} \right\} \) Hoeffding inequality can be used. According to it:

\[ Pr \left\{ S_N - E[S_N] \geq t \right\} \leq \begin{cases} e^{-\frac{2t^2}{N(x_{max} - x_{min})^2}}, & t > 0 \\ 1, & t \leq 0 \end{cases}, \]

where \( X_i \in [x_{min}, x_{max}] \), \( x_{min} = \alpha \), \( x_{max} = \log_2 \left( 1 + \frac{d_{max}^3}{a} \right) + \alpha \).

Thus:

\[ Pr\{\text{Congestion}\} \leq \begin{cases} e^{-\frac{2}{N} \left( \frac{\Delta F}{R} - N \cdot E[X_i] \right)^2}, & N < \frac{\Delta F}{R \cdot E[X_i]} \\ 1, & \text{otherwise} \end{cases} \]
Network Capacity Estimation

- **Approximate value** of network capacity, **based on CLT**:

  \[ N_c \approx \left( \frac{-g_2 + \sqrt{g_2^2 - 4g_1g_3}}{2g_1} \right)^2 \]

- **Lower bound** for network capacity, **based on Hoeffding inequality**:

  \[ N_c \geq \left( \frac{-g_4 + \sqrt{g_4^2 - 4g_1g_3}}{2g_1} \right)^2 \]

Where:

\[
\begin{align*}
    g_1 &= E[X_i] \\
    g_2 &= \sqrt{Var[X_i]}Q^{-1}(p_c) \\
    g_3 &= -\Delta FR^{-1} \\
    g_4 &= \log_2 \left( 1 + \frac{d_{max}^3}{a} \right) \sqrt{-\frac{1}{2} \ln p_c}
\end{align*}
\]
Numerical example

- Bandwidth $\Delta F \in \{10\,MHz, 20\,MHz, 40\,MHz\}$
- Carrier frequency $2\,GHz$
- UE noise figure $N_f = 10$
- Video bit rate $R = 500\,kbps$
- Maximum distance $d_{max} = 60\,m$
Possible Use Cases

- Three power picocells
- Five weak picocells
Results:

- Congestion probability for wireless video streaming picocell network was investigated.
- Convenient majorant of function $\frac{1}{\log_2(1+x)}$ was proposed.
- Proposed expressions allows simple estimating of network capacity.
  However the results are applicable only for environments, where path loss factor is close to «3».

Further research:

- Generalization of obtained results for wider conditions may be a direction of further research.
Auxiliary calculations

1. **Calculation of $X_i$ mean:**

\[
E [X_i] = \int_0^{d_{\text{max}}} \left[ \log_2 \left( 1 + \frac{x^3}{a} \right) + \alpha \right] f_d(x) dx
\]

\[
E [X_i] = \frac{1}{d_{\text{max}}^3} \left[ \int_0^{d_{\text{max}}^3} \log_2 \left( 1 + \frac{t}{a} \right) dt + \int_0^{d_{\text{max}}^3} \alpha dt \right] = \frac{k \ln m - 1}{\ln 2} + \alpha,
\]

where $t = x^3$, $m = 1 + \frac{d_{\text{max}}^3}{a}$ and $k = 1 + \frac{a}{d_{\text{max}}^3}$.

2. **Calculation of $X_i$ variance:**

\[
Var [X_i] = E[X_i^2] - E [X_i]^2
\]

\[
E [X_i^2] = \frac{k (\ln m - 1)^2 - k}{(\ln 2)^2} + 2\alpha E [X_i] + \alpha^2
\]