Efficient Finite-State Error Models for Wireless Communications

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July 26, 2007

Outline

1. Introduction
   - Motivation
   - Stochastic Processes
   - Markov Chains
2. Theoretical framework for channel modeling
   - Hidden Markov Models
   - Aggregated Markov Processes
   - Aggregated Renewal Markov Processes
3. Applications
   - Modeling DVB-H laboratory measurements
   - Modeling DVB-H field measurements

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Why use finite-state models for communication channels?

- For speed (=computational simplicity)
- Low memory requirements
- To generate arbitrary amounts of random but statistically coherent channel information (as opposed to using only measurements)

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Discrete-time Stochastic Processes

- Let us denote by $\{X_k\}_{k \in K}$ a stochastic process
- This means a collection of random variables, where for each $k \in K$, X_k is a random variable
- We may interpret k as an expression for time, and consider X_k to be the state of the process at time k
- In the following, we assume that the index set K is countable (more specifically, that $K = \{0, 1, ...\}$) and thus focus on discrete-time stochastic processes

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Markov Chains

- An *m*-th order Markov chain is a discrete-time stochastic process, whose state at a given time is dependent only on the previous *m* states, or the context, of the process
- Formally, for all $i \in K$ and all states x_i:

$$P\{X_i = x_i | X_1 = x_i, ..., X_{i-1} = x_{i-1}\} =$$

$$P\{X_i = x_i | X_{i-m} = x_{i-m}, ..., X_{i-1} = x_{i-1}\}$$

- However, by simply redefining the state space, *m* can be reduced to 1 without loss of generality

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Hidden Markov Processes

- A hidden Markov process (HMP) is a joint stochastic process $\{X_k, Y_k\}_{k=0,1,\ldots}$ where $\{Y_k\}$ is a probabilistic function of $\{X_k\}$ alone
- It is assumed that $\{Y_k\}$ can be observed, but $\{X_k\}$ cannot
- Conceptually, a hidden Markov process can be considered a hidden stochastic process $\{X_k\}$ observed through noise in $\{Y_k\}$

Hidden Markov Models

- An application of HMPs is to use them to model communication channels with memory (thus the term hidden Markov model (HMM) is used)
- A classical example is the Gilbert-Elliott model that consists of two hidden states (good/bad, conceptually) with different probabilities for the output symbols that represent correct and erroneous reception of data
- A typical problem related to HMMs is to determine the most likely model structure and parameters to produce a given output

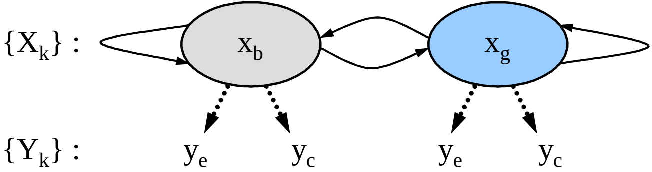

Figure: The Gilbert-Elliott model

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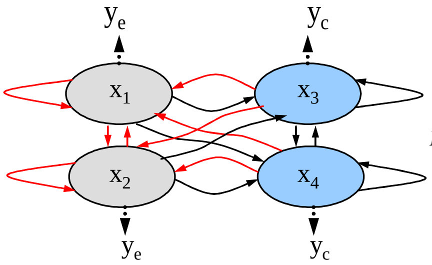
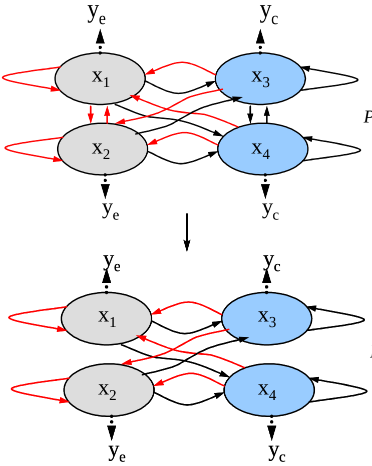
Aggregated Markov Processes

- Aggregated Markov processes (AMP) are a subclass of HMPs, where the output Y_k is a deterministic function of the hidden state X_k
- Any finite-alphabet HMP can be described as an equivalent AMP by augmenting the state space
- For example, the Gilbert-Elliott model is equivalent to a four-state AMP
- Given the state $\mapsto$ output mapping, an AMP is defined by its transition probability matrix P



$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}$$

Figure: State diagram and transition matrix for a four-state aggregated Markov model



$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}$$

$$\downarrow$$

$$P = \begin{pmatrix} p_{11} & 0 & p_{13} & p_{14} \\ 0 & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & 0 \\ p_{41} & p_{42} & 0 & p_{44} \end{pmatrix}$$

Simplified AMP

- Given certain (mild) conditions on P, a general AMP can be reduced to a form where transitions between states in a given group are forbidden
- This can be used to simplify the estimation of transition probabilities for a given model
- Still, maximum likelihood estimation of the model parameters given an observed output sequence is computationally relatively demanding (a solution can be obtained using for example the Baum (Baum-Petrie/Baum-Welch) algorithm (equivalent to BCJR), possibly modified to use the simplified form for P)

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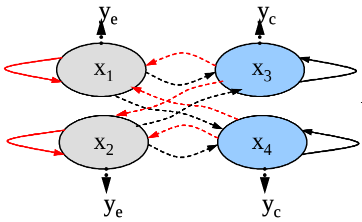
Aggregated Renewal Markov Processes

- Further simplification of finite-state AMPs can be achieved by artificially removing the dependence of Y_k on Y_{k-1}, Y_{k-2},...
- Thus the output of the obtained model is a renewal process
- The abovementioned can be achieved by assigning suitable weights to each state in the model, and using these to determine the transition probabilities between state groups
- This ultimately enables simple parameter estimation such as the method of moments to be used



$$P = \begin{pmatrix} p_{11} & 0 & (1-p_{11})w_3 & (1-p_{11})w_4 \\ 0 & p_{22} & (1-p_{22})w_3 & (1-p_{22})w_4 \\ (1-p_{33})w_1 & (1-p_{33})w_2 & p_{33} & 0 \\ (1-p_{44})w_1 & (1-p_{44})w_2 & 0 & p_{44} \end{pmatrix}$$

Figure: State diagram and transition matrix for a four-state aggregated renewal Markov model

Properties of Aggregated Renewal Markov Processes

- Formally, consider AMPs with $N + M$ states partitioned into groups C and E corresponding to correct and erroneous reception of information passed through *some* channel
- The state transition probability matrix becomes

$$P = \begin{pmatrix} P_{CC} & P_{CE} \\ P_{EC} & P_{EE} \end{pmatrix}$$

where

$$(i) \quad P_{CC} = diag(\alpha_1, ..., \alpha_N),$$

$$(ii) \quad P_{EE} = diag(\alpha_{N+1}, ..., \alpha_{N+M}),$$

$$(iii) \quad P_{CE} = [p_{ij}]_{i \in C, j \in E} = [(1 - \alpha_i) w_j]_{i \in C, j \in E},$$

$$(iv) \quad P_{EC} = [p_{ij}]_{i \in E, j \in C} = [(1 - \alpha_i) w_j]_{i \in E, j \in C}$$

Properties of Aggregated Renewal Markov Processes

- From the previous, the limiting state probabilities are directly obtained as

$$\pi_i = \frac{w_i}{(1-\alpha_i)\left(\sum_{k=1}^{N+M} \frac{w_k}{1-\alpha_k}\right)} \forall i \in \{1, ..., N+M\}$$

- The probability distributions of lengths of sequences of consecutive output symbols (error symbols in the following) are of the form

$$f(n) = \sum_j w_j \alpha_j^{n-1}(1-\alpha_j) \,\forall\, n \in \{1, 2, ...\}.$$

Properties of Aggregated Renewal Markov Processes

- The kth derivatives of the probability generating functions for these distributions are of the form

$$G^{(k)}(z) = \sum_j \frac{k!\alpha_j^{k-1} w_j (1-\alpha_j)}{(1-z\alpha_j)^{k+1}} \ \forall\, k \in \{1, 2, \ldots\}.$$

- Using the results above, expressions for statistics of specific model implementations are obtained, for example for a four-state model, the probability of error is

$$P(Z=e) = \frac{w_3}{(1-\alpha_3)\left(\sum_{k=1}^{4} \frac{w_k}{1-\alpha_k}\right)} + \frac{1-w_3}{(1-\alpha_4)\left(\sum_{k=1}^{4} \frac{w_k}{1-\alpha_k}\right)}$$

Properties of Aggregated Renewal Markov Processes

- The mean dwell time in a given state group is of the form

$$\mu = G^{(1)}(1) = \frac{w_i}{1-\alpha_i} + \frac{(1-w_i)}{1-\alpha_{i+1}}$$

- And the corresponding variance

$$\sigma^2 = G^{(2)}(1) + G^{(1)}(1) - \left[G^{(1)}(1)\right]^2$$

$$= \frac{(1-\alpha_i)^2\alpha_{i+1} - w_i(1-\alpha_{i+1})(\alpha_{i+1}-\alpha_i)(1+\alpha_i) - w_i^2(\alpha_{i+1}-\alpha_i)^2}{(1-\alpha_i)^2(1-\alpha_{i+1})^2}$$

Parameter estimation

- Equating the previous (error probability, means and variances) to (measured) sample statistics results in a group of equations with error model parameters as the unknown variables

$$\begin{cases} P(Z = e) = P(q_E) \\ \mu_C = \overline{L}_C \\ \mu_E = \overline{L}_E \\ \sigma_C^2 = S_{L_C}^2 \\ \sigma_E^2 = S_{L_E}^2 \end{cases}$$

- Unfortunately this is an underdetermined group of nonlinear equations with no explicit solution (numerical methods may be used, but this is not always fast or accurate)

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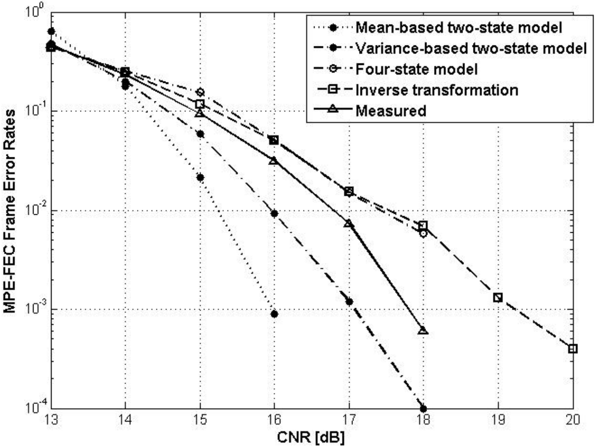
Some results for modeling laboratory error measurements

- The models considered above were applied in simulating DVB-H TS packet error behavior in a typical urban setting (DVB-T/H operated over a TU-6 hardware channel simulator)
- The necessary sample statistics were observed and used to obtain the model parameters to match a given measurement
- Error traces were generated also using the inverse transformation method which may be considered to provide an upper limit for the accuracy of renewal models such as the finite-state models used


Mean-based two-state model
Variance-based two-state model
Four-state model
Inverse transformation
Measured
MPE-FEC Frame Error Rates
CNR [dB]
10^0
10^-1
10^-2
10^-3
10^-4
13
14
15
16
17
18
19
20

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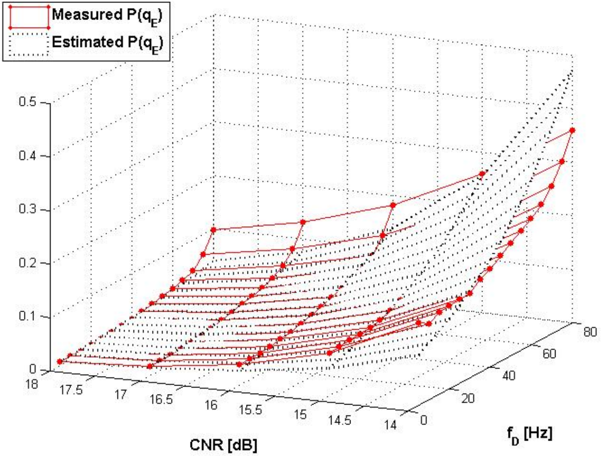
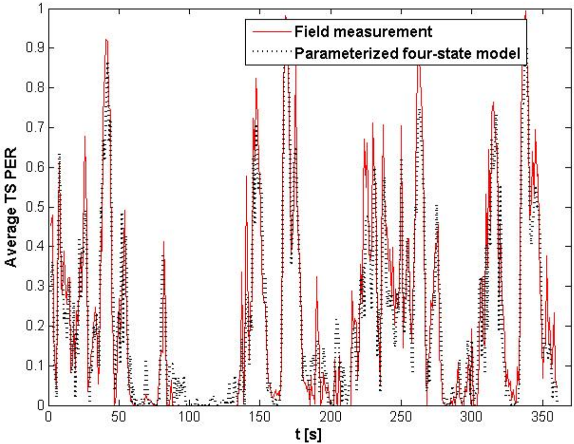
Some results for modeling field measurements

- By approximating the sample statistics of laboratory packet error measurements as functions of the CNR and Doppler frequency, a functional model for mobile reception in field conditions can be constructed
- Using the approximated statistics with measured time-dependent vehicle speeds and received signal strengths, separate finite-state models are calculated to correspond to each sampling interval for the speed and RSSI


Measured P(q_E)
Estimated P(q_E)
0.5
0.4
0.3
0.2
0.1
0
18
17.5
17
16.5
16
15.5
15
14.5
14
CNR [dB]
0
20
40
60
80
f_D [Hz]


Field measurement
Parameterized four-state model
Average TS PER
t [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50
100
150
200
250
300
350

Summary

- In the work presented here, a very limited subset of finite-state stochastic processes is considered for error modeling
- It seems that the proposed simplifications in model structure do not severely degrade simulation performance compared to measurements (in the cases studied)
- Potential applications are for example high protocol level system simulations, and block error simulations with large-scale fading