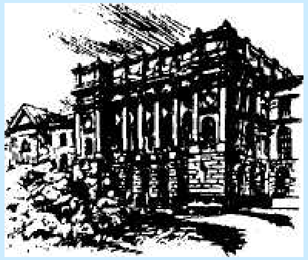
Saint-Petersburg State Polytechnical University



Spectrally efficient OFDM signals

Post-graduate student: A. Rashich
Scientific Advisor: Prof. S. Makarov

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Outline

- Basis sets for constructing spectrally efficient orthogonal frequency division multiplexing (OFDM) signals

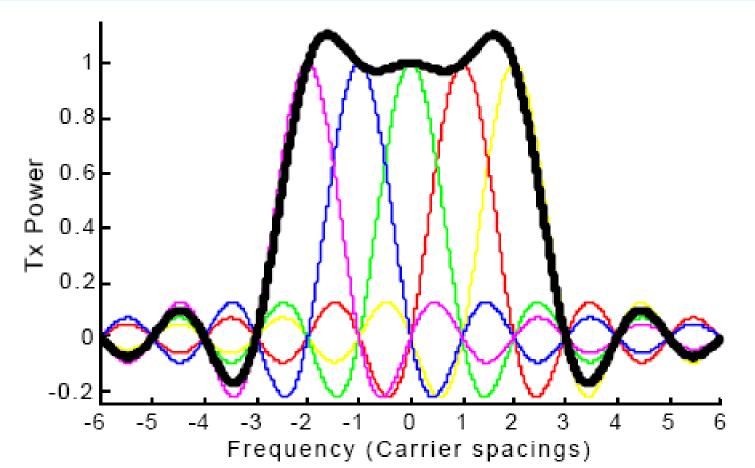
- Their spectral trends

OFDM signal

$$s(t) = \rho \sum_k \mathrm{Re}\{\sum_{n=0}^{N-1} D_{n,k} \exp[j(\omega_0 + n\omega_d)(t-kT)]\} \cdot p(t-kT)$$

N frequencies in OFDM signal spaced by $\omega_d = \frac{2\pi}{T_d}$

$p(t)$ is a unit rectangular pulse on $-T_g - T_d/2 \le t \le T_d/2$



Initial basis for rectangularly pulsed OFDM signal:

$$\Psi = \left\{ C_n^{(0)}(t), S_n^{(0)}(t) \right\}_{n=0}^{N-1} \qquad (1)$$

$$C_n^{(m)}(t) = \sqrt{\frac{2}{T_d}} \cos((\omega_0 + \frac{1}{2} m\omega_d + n\omega_d)t) \qquad S_n^{(m)}(t) = \sqrt{\frac{2}{T_d}} \sin((\omega_0 + \frac{1}{2} m\omega_d + n\omega_d)t)$$

$$\omega_d = \frac{2\pi}{T_d} \qquad |t| \le \frac{T_d}{2} \qquad n = 0,1,\ldots(N-1)$$

$$m = 0,1$$

N frequencies in OFDM signal spaced by $\omega_d = \frac{2\pi}{T_d}$

Bilinear transform $\left\{h_k(t)\right\}_{k=0}^{2K-1} \Rightarrow \left\{\frac{1}{\sqrt{2}}\left(h_{2k}(t) \pm h_{2k+1}(t)\right)\right\}_{k=0}^{K-1}$ (2)

Basis W

$$\Phi_1^{(c)} = \left\{P_1^{(c)}(t)C_{2n}^{(1)}(t),\ P_1^{(c)}(t)S_{2n}^{(1)}(t)\right\}_{n=0}^{N/2-1}$$

$$\Phi_u^{(c)} = \{P_1^{(s)}(t)\left(\prod_{k=1}^{u-2}P_{2k}^{(c)}(t)\right)P_{2^{u-1}}^{(s)}(t)C_{n2^u+\zeta(u)}^{(1)}(t),$$
$$P_1^{(s)}(t)\left(\prod_{k=1}^{u-2}P_{2k}^{(c)}(t)\right)P_{2^{u-1}}^{(s)}(t)S_{n2^u+\zeta(u)}^{(1)}(t)\}_{n=0}^{N/2^u-1}, u = 2,3,\ldots,\log_2 N \quad (3)$$

$$P_n^{(c)}(t) = \sqrt{2}\cos((1/2)n\omega_d t) \qquad \zeta(u) = \sum_{k=0}^{u-2}2^k = 2^{u-1}-1$$

$$P_n^{(s)}(t) = \sqrt{2}\sin((1/2)n\omega_d t)$$

$\dot{W}_L = \bigcup_{u=1}^{L}\Phi_u^{(c)}$ is a 2M-D basis set containing zero-edged continuous basis signals on $|t| \le \frac{T_d}{2}$

$L \in \left\{1, 2, \ldots, \log_2 N\right\}$

$M = \sum_{i=1}^{L}2^{-i}N = N(1-2^{-L})$

Basis V

$$\Theta_u^{(c)} = \{P_{N/2^u}^{(s)}(t)\prod_{k=1}^{u-1} P_{N/2^k}^{(c)}(t)C_{n+\frac{1}{2}\chi(u+1)}^{(0)}(t), P_{N/2^u}^{(s)}(t)\prod_{k=1}^{u-1} P_{N/2^k}^{(c)}(t)S_{n+\frac{1}{2}\chi(u+1)}^{(0)}(t)\}_{n=0}^{N/2^u-1}, u = 1, 2, ..., \log_2 N - 1$$

$$\Theta_{\log_2 N}^{(c)} = \{\prod_{k=1}^{\log_2 N} P_{N/2^k}^{(c)}(t)C_{n+\frac{1}{2}\chi(\log_2 N)}^{(1)}(t), \prod_{k=1}^{\log_2 N} P_{N/2^k}^{(c)}(t)S_{n+\frac{1}{2}\chi(\log_2 N)}^{(1)}(t)\} \quad (4)$$

$$\chi(u) = \sum_{k=1}^{u-1} N/2^k = N(1-2^{1-u})$$

$V_L = \bigcup_{u=1}^{L} \Theta_u^{(c)}$ is a 2M-D basis set containing zero-edged continuous basis signals on $|t| \le \frac{T_d}{2}$

$$L \in \{1, 2, ..., \log_2 N\}$$

$$M = \sum_{i=1}^{L} 2^{-i} N = N(1-2^{-L})$$

Alternative Formats for W_L and V_L

$$\Phi_u^{(c)} = \{2^{-u/2} \sum_{\upsilon=0}^{2^u-1} (-1)^{1+\psi_{u,\upsilon}} C_{n2^u+\upsilon}^{(0)}(t),\ 2^{-u/2} \sum_{\upsilon=0}^{2^u-1} (-1)^{1+\psi_{u,\upsilon}} S_{n2^u+\upsilon}^{(0)}(t)\}_{n=0}^{N/2^u-1} \quad (5)$$

$$\Theta_u^{(c)} = \{2^{-u/2} \sum_{\upsilon=0}^{2^u-1} \phi_{u,\upsilon} C_{n+\upsilon N/2^u}^{(0)}(t),\ 2^{-u/2} \sum_{\upsilon=0}^{2^u-1} \phi_{u,\upsilon} S_{n+\upsilon N/2^u}^{(0)}(t)\}_{n=0}^{N/2^u-1} \quad (6)$$

$u = 1, 2, \ldots, \log_2 N$

M complex data symbols $\{D_{m,k}\}_{m=0}^{M-1}$

New OFDM signal $$s(t) = \rho \sum_k \mathrm{Re}\{\sum_{n=0}^{N-1} B_{n,k} \exp[\,j(\omega_0 + n\omega_d)(t-kT)]\} \cdot p(t-kT) \qquad (7)$$

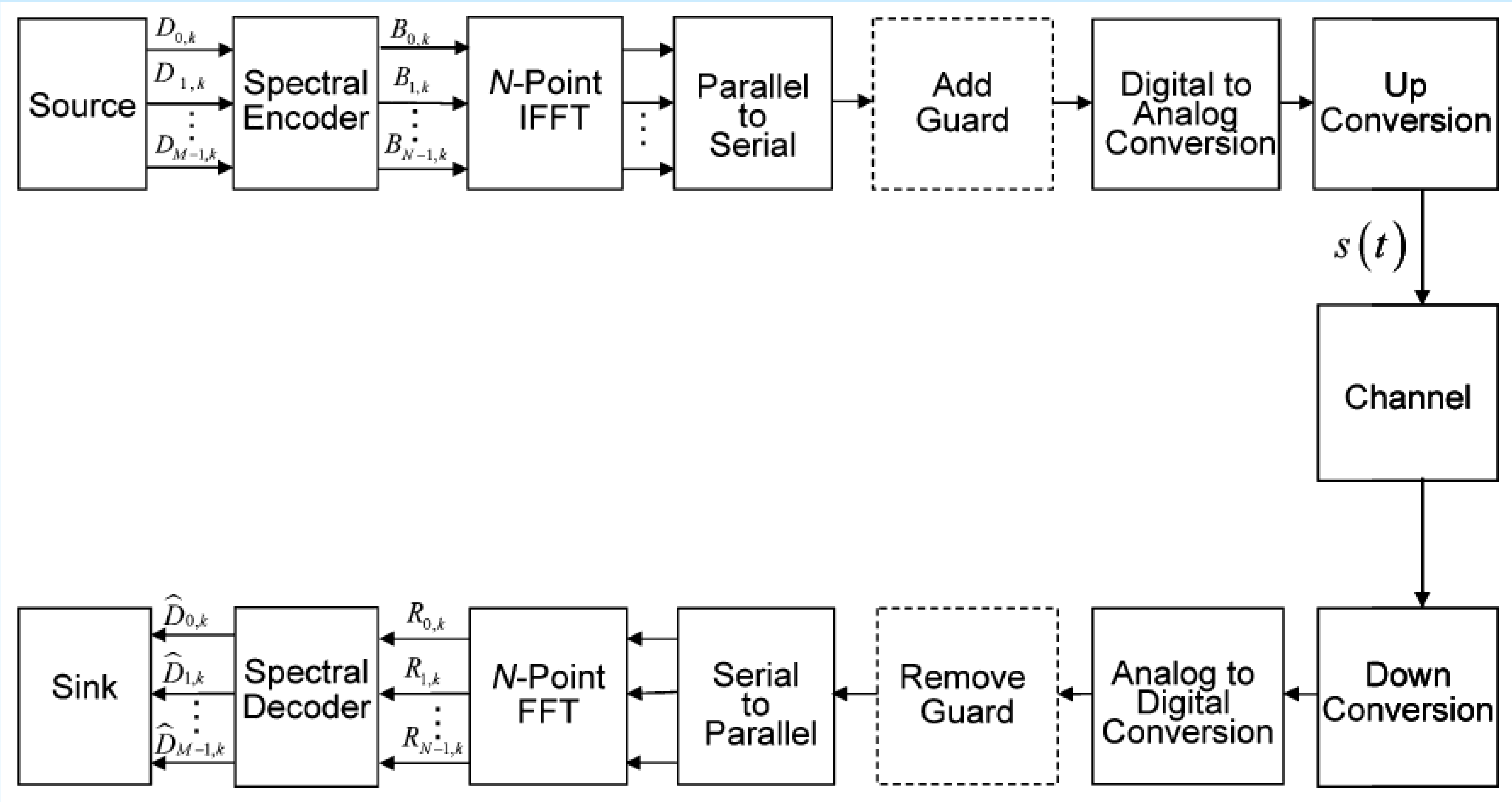
$$B_{n,k} = \sum_{m=0}^{M-1} G_{n,m}^{(\chi)} D_{m,k},\, n = 0,1,\ldots, N-1 \qquad \chi = W \text{ or } V$$

$$G^{(W)}_{n2^u+\upsilon,\chi(u)+n} = 2^{-u/2}(-1)^{1+\psi_{u,v}},\, n = 0,1,\ldots,\frac{N}{2^u}-1,\, \upsilon = 0,1,\ldots,2^u-1,\, u = 1,2,\ldots,L$$

$$G^{(V)}_{n+\frac{N}{2^u}\upsilon,\chi(u)+n} = 2^{-u/2}\phi_{u,v}, \qquad n = 0,1,\ldots,\frac{N}{2^u}-1, \qquad \upsilon = 0,1,\ldots,2^u-1, \qquad u = 1,2,\ldots,L$$

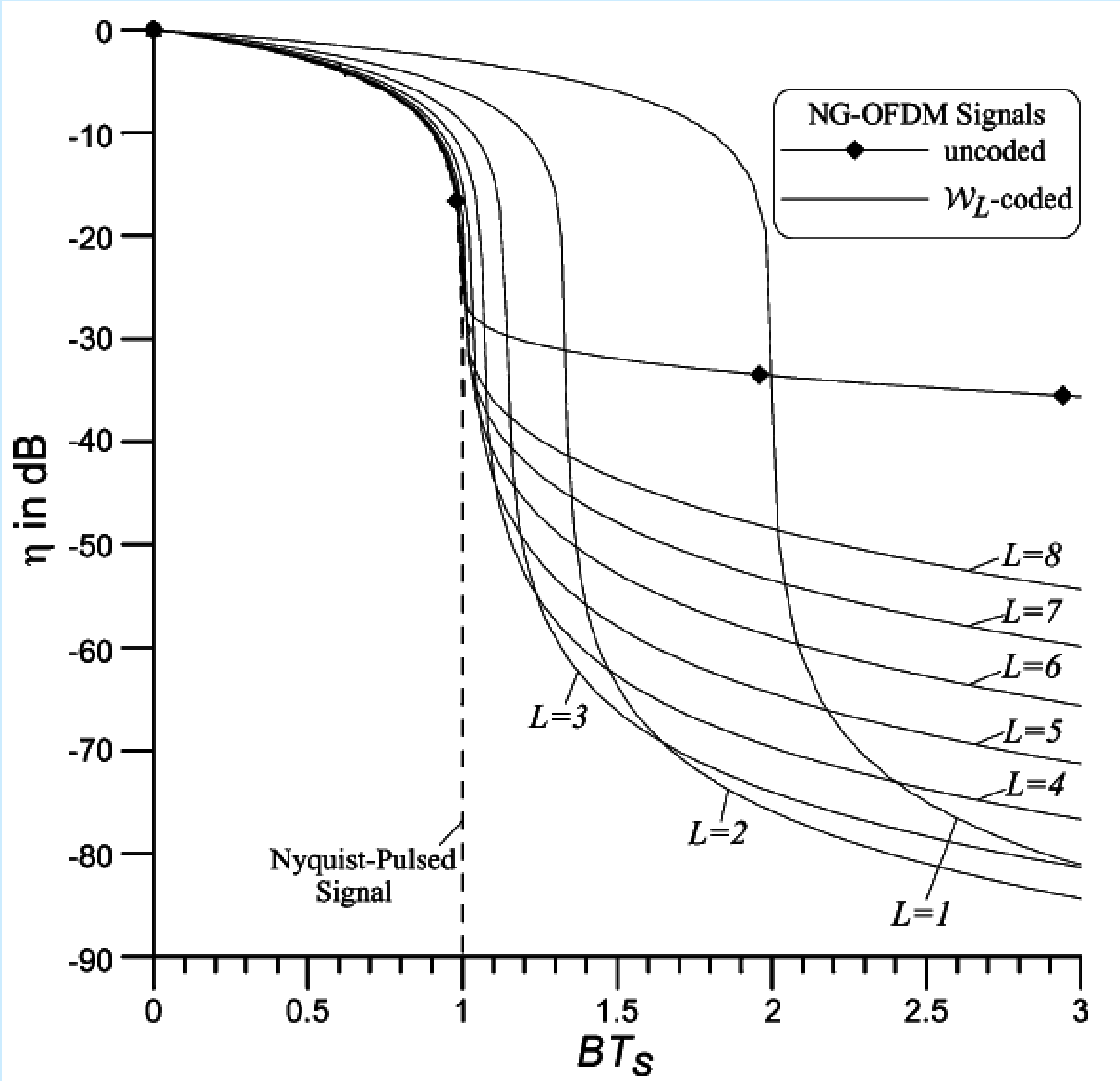
$p(t)$ is a unit rectangular pulse on $-T_g - T_d/2 \le t \le T_d/2$

System model



Spectral analysis

Fractional out-of-band power characteristics of W_L-coded OFDM signals
N=256



Spectral analysis

Fractional out-of-band power characteristics of V_L-coded OFDM signals N=256