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Signature design for oversaturated synchronous CDMA

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Direct Spreading CDMA

- Data stream of the k–th user $b_k(i)$, $i = \ldots, -1, 0, 1, \ldots$ modulates user–specific *signature* $\mathbf{s}_k = [s_k(0), s_k(1), \ldots s_k(N-1)]$
- All data streams and signatures are strictly time–aligned
- K – number of users
- $N = WT$ – spreading factor

Transmitted group signal

- $\mathbf{S} = [\mathbf{s}_1^T, \mathbf{s}_2^T, ..., \mathbf{s}_K^T]$ – signature matrix
- A_k – amplitude of the k–th user
- $\mathbf{A} = \mathrm{diag}[A_1, A_2, ..., A_K]$ – diagonal amplitude matrix
- $\mathbf{b} = [b_1, b_2, ..., b_K]$ – vector of current data bits
- Transmitted group signal:

$$\mathbf{y} = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k = \mathbf{S}^T \mathbf{A}\mathbf{b}$$

Oversaturation

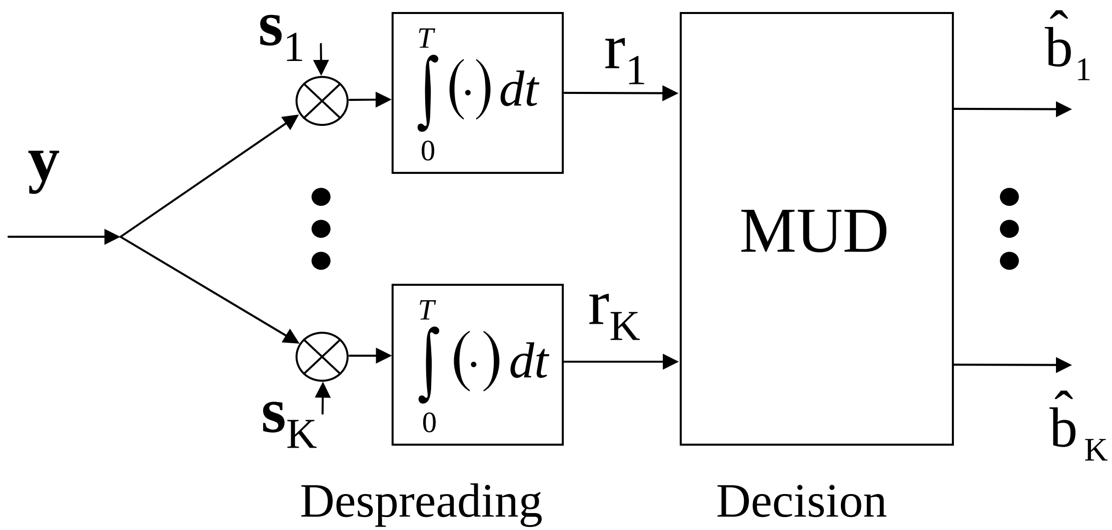
- Maximal number of orthogonal signatures is limited by $N = WT$
- Oversaturation (overloading) occurs when $K > N$
- Oversaturation efficiency is defined as $e_{ov} = K / N$

Oversaturation (cont.)

- With oversaturation, orthogonal signaling is not possible => unavoidable MAI occurs
- Conventional receiver is not applicable => multiuser receiver must be used
- The problem of optimal multiuser is its exponential complexity in the number of users (if no special care is taken in signature design)

Optimal multiuser receiver (cont.)

- Bank of matched filters followed by multiuser detector (MUD). Matched filter outputs are $\mathbf{r} = \mathbf{S}\left(\mathbf{S}^T \mathbf{A}\mathbf{b} + \mathbf{n}\right) = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}'$



Optimal multiuser receiver (cont.)

- Optimal multiuser detector calculates decision

$$\widehat{\mathbf{b}} = \arg\max\left(2\mathbf{r}^T\mathbf{A}\mathbf{b} - \mathbf{b}^T\mathbf{A}\mathbf{R}\mathbf{A}\mathbf{b}\right)$$

- Simplifications are possible, if e.g. signatures have hierarchical cross-correlation matrix structure, Learned's algrorithm having polynomial complexity can be applied.
- Optimal signature set provides maximal separation of all possible group signals

Group Orthogonal CDMA

- Another approach to simplify receiver: divide users to groups (subspaces) that don't interfere and oversaturate groups => MAI only inside groups
- Formally, signal space dimension N is divided to subspaces having dimension L. Each subspace is oversaturated by s users. The number of users served $K=N(L+s)/L=N+Ns/L$, $e_{ov}=1+s/L$
- Abbreviation: GO-CDMA (group orthogonal code division multiple access)
- It is possible to use optimal multiuser detector for each group due to small number of users inside group

GO-CDMA signatures

- Signature design performed for L=2,3,4 and s=1
  - L>4 cannot improve maximum minimum Euclidean distance further
  - s>1 did not produce good results
  - Symmetrical group signal set
  - L=4, s=1 results in R-T signatures

GO-CDMA signatures

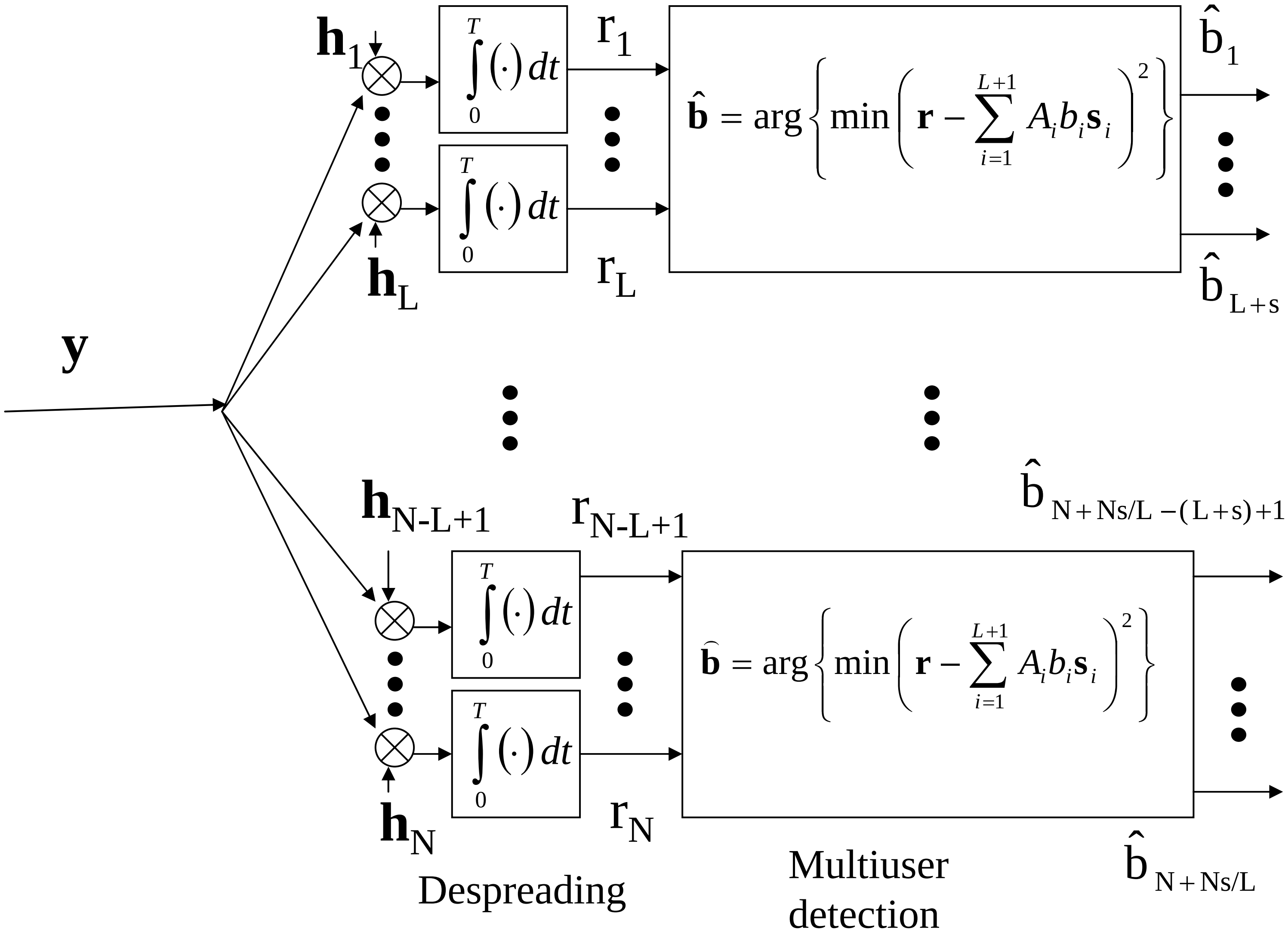
- H matrix with orthogonal rows $\mathbf{H}_N = \begin{pmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_N \end{pmatrix}$
- L=2, i-th group:

$$\mathbf{S}_i = \begin{pmatrix} s_1^i \\ s_2^i \\ s_3^i \end{pmatrix} = \frac{1}{\sqrt{2(1+\alpha^2)}} \begin{pmatrix} \sqrt{2} & \alpha\sqrt{2} \\ \alpha\sqrt{2} & \sqrt{2} \\ \sqrt{1+\alpha^2} & \sqrt{1+\alpha^2} \end{pmatrix} \begin{pmatrix} \mathbf{h}_{2i-1} \\ \mathbf{h}_{2i} \end{pmatrix}, \alpha = \frac{4-\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$$

- L=3, i-th group:

$$\mathbf{S}_i = \begin{pmatrix} \mathbf{s}_1^i \\ \mathbf{s}_2^i \\ \mathbf{s}_3^i \\ \mathbf{s}_4^i \end{pmatrix} = \frac{1}{\sqrt{3(1+2\beta^2)}} \begin{pmatrix} \sqrt{3} & \beta\sqrt{3} & \beta\sqrt{3} \\ \beta\sqrt{3} & \sqrt{3} & \beta\sqrt{3} \\ \beta\sqrt{3} & \beta\sqrt{3} & \sqrt{3} \\ \sqrt{1+2\beta^2} & \sqrt{1+2\beta^2} & \sqrt{1+2\beta^2} \end{pmatrix} \begin{pmatrix} \mathbf{h}_{3i-2} \\ \mathbf{h}_{3i-1} \\ \mathbf{h}_{3i} \end{pmatrix}, \beta = \frac{-12+\left(1+\sqrt{7}\right)\sqrt{14-\sqrt{7}}}{16-2\sqrt{7}}$$

GO-CDMA receiver(s)

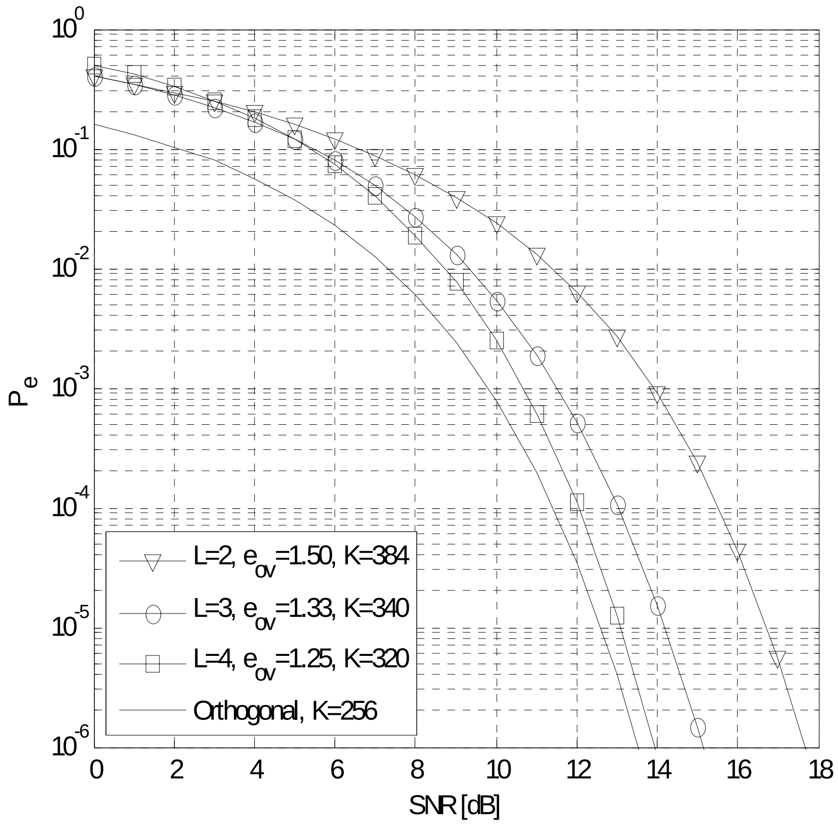


Performance of GO-CDMA

Euclidean distance

L	d^2_{min}	Loss [dB]	e_{ov}
1	4.00	0	1.00
2	1.53	4.18	1.50
3	2.71	1.69	1.33
4	4.00	0	1.25

Bit error rate (BER)



Collaboratively coded GO-CDMA (CCGO-CDMA)

- S-CDMA can be used in downlink of cellular system or in uplink in application of channelization (separation of channels of one user)
  - Transmitter controls all transmissions
  - Requiring signature per user seems unnecessary restriction
  - It is possible to collaboratively encode the data of users inside one group
  - Subspace acts as a common signature for users inside group

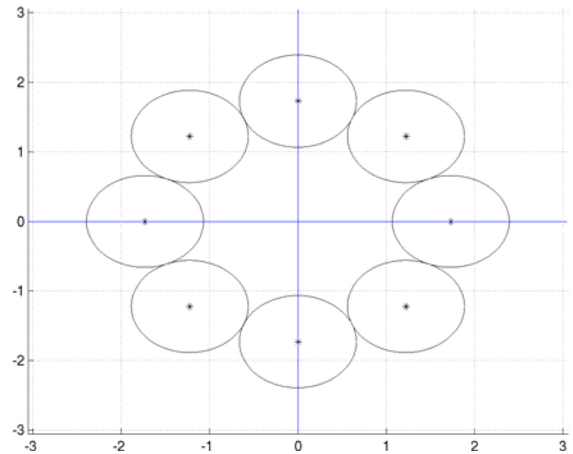
Signatures for CCGO-CDMA

- Data bits of L+s users are jointly mapped into 2^(L+s) L-dimensional signal vectors
- Signature design performed for L=2,3,4,5 and s=1,2. Now, it is possible to improve performance with L>4 and s>1.
- Sphere packing theory assists in finding good signal constellations. Signals treated as L-dimensional spheres having radius $d_{\min}/2$.
- Few different options investigated:
  - maximal minimum distance constellation
  - symmetrical constellations with or without zero-vector
  - equal energy constellation
- Modulation mapping not a trivial problem

CCGO-CDMA Signatures (L=2,s=1)

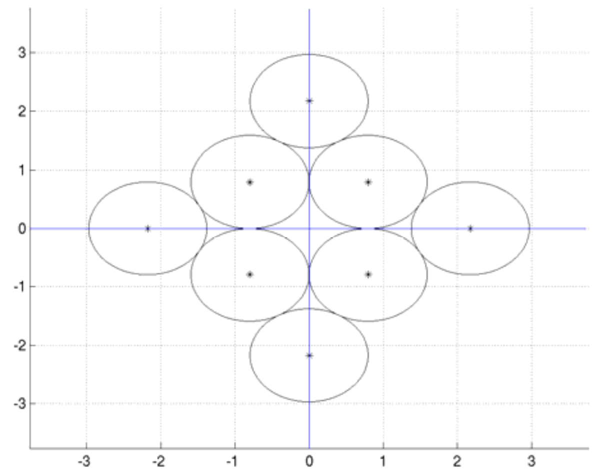
$d^2_{\min} = 1.76 \Rightarrow loss = 3.76\, dB$

A.



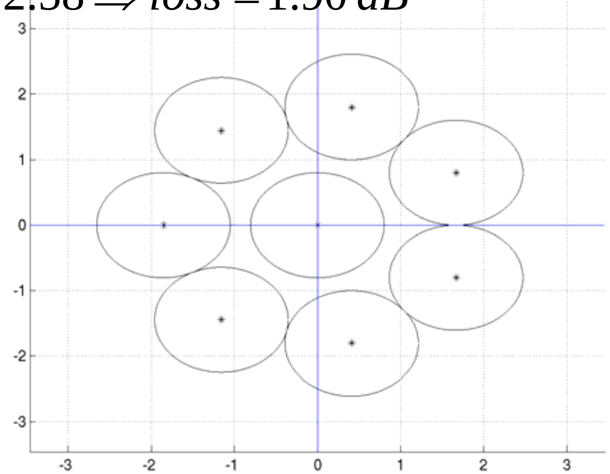
$d^2_{\min} = 2.54 \Rightarrow loss = 1.98\, dB$

B.



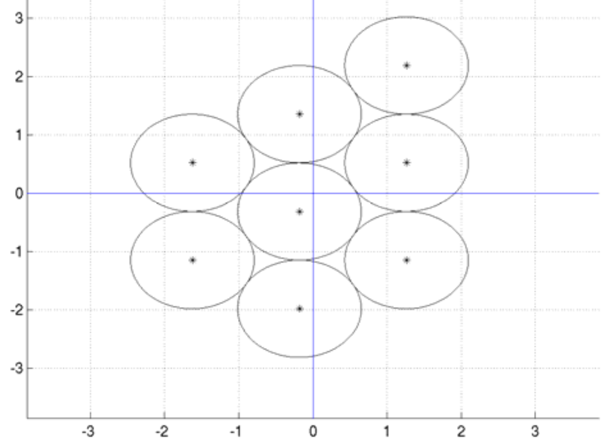
$d^2_{\min} = 2.58 \Rightarrow loss = 1.90\, dB$

C.

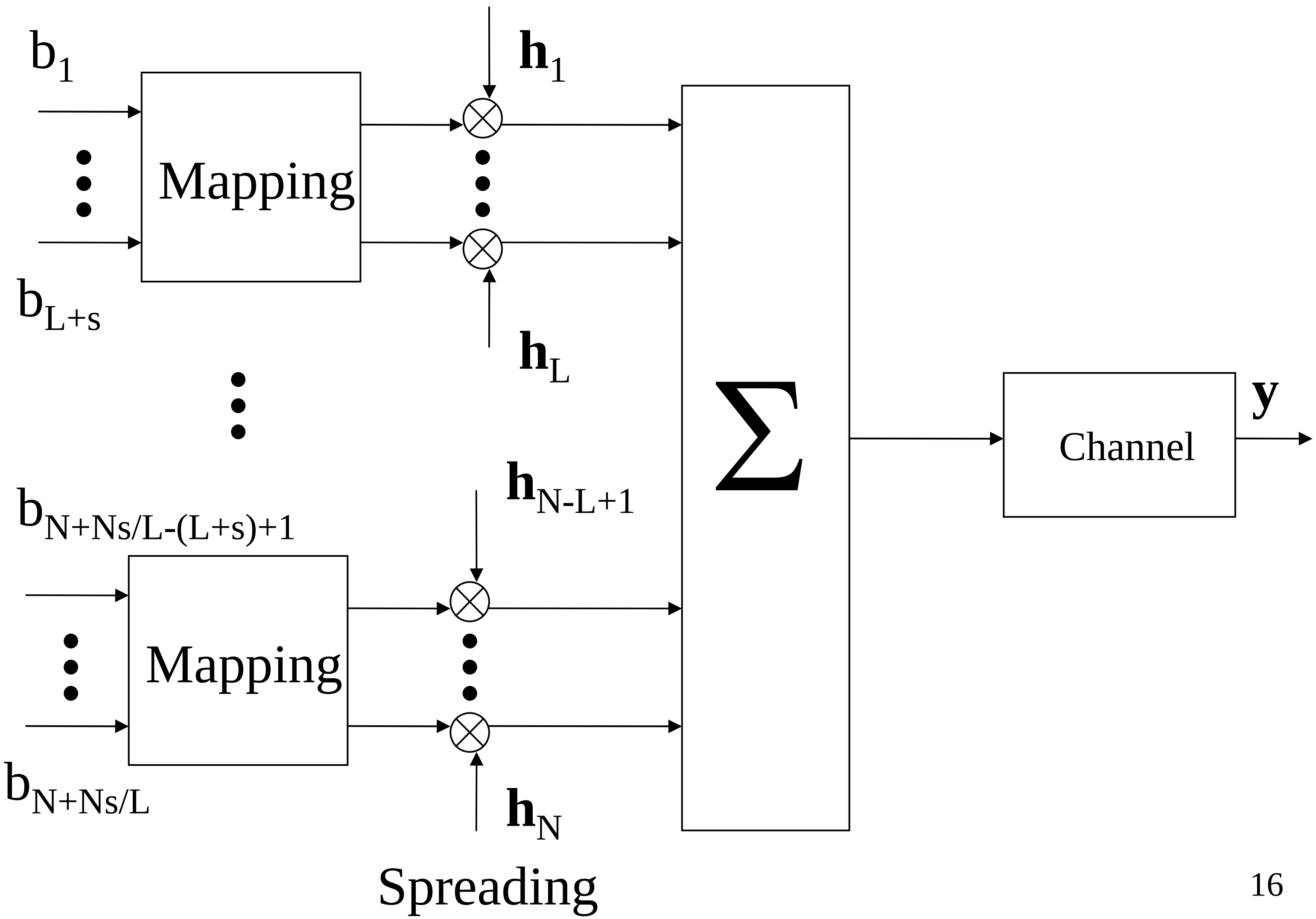


$d^2_{\min} = 2.78 \Rightarrow loss = 1.58\, dB$

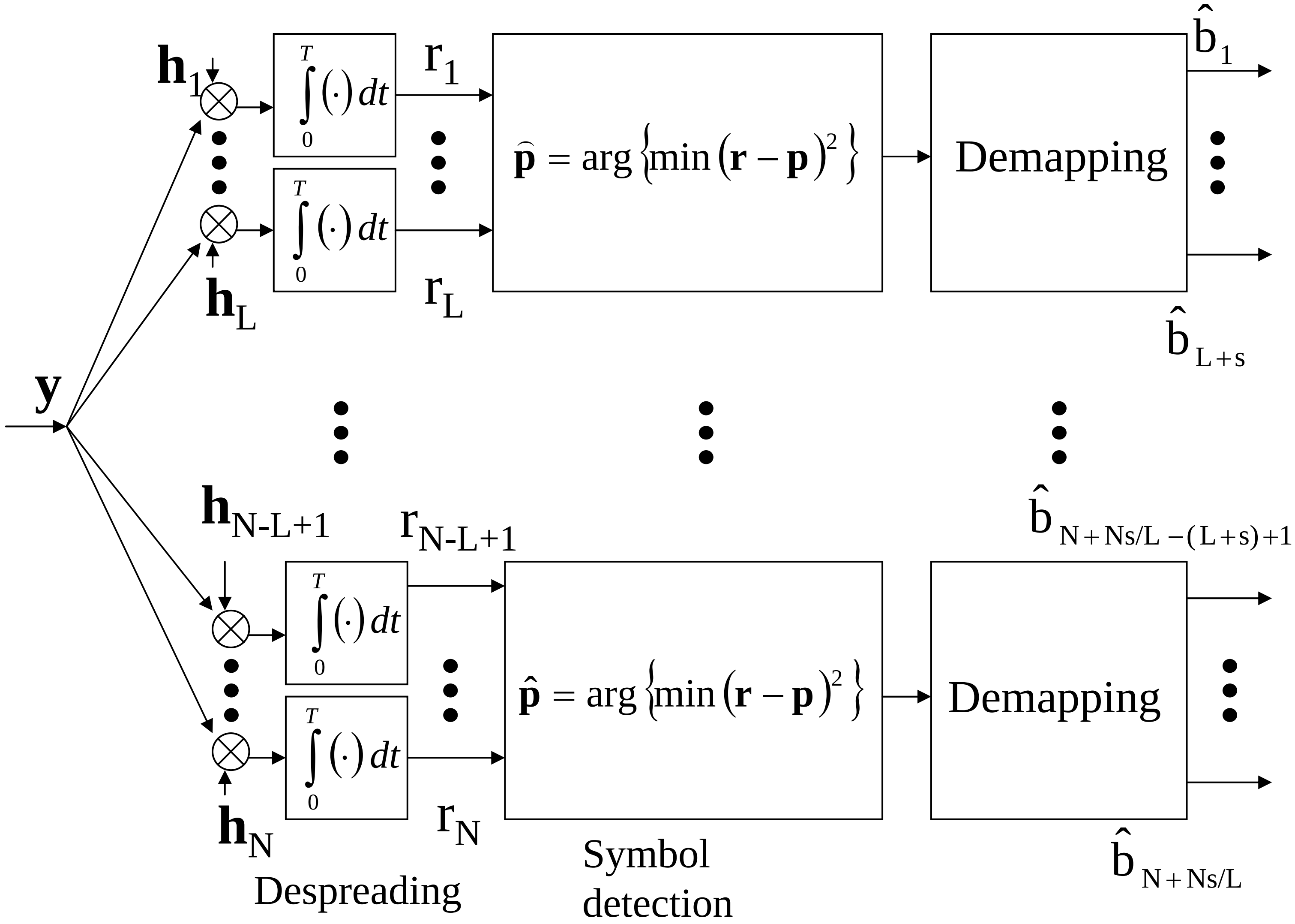
D.



CCGO-CDMA transmitter



CCGO-CDMA Receiver(s)



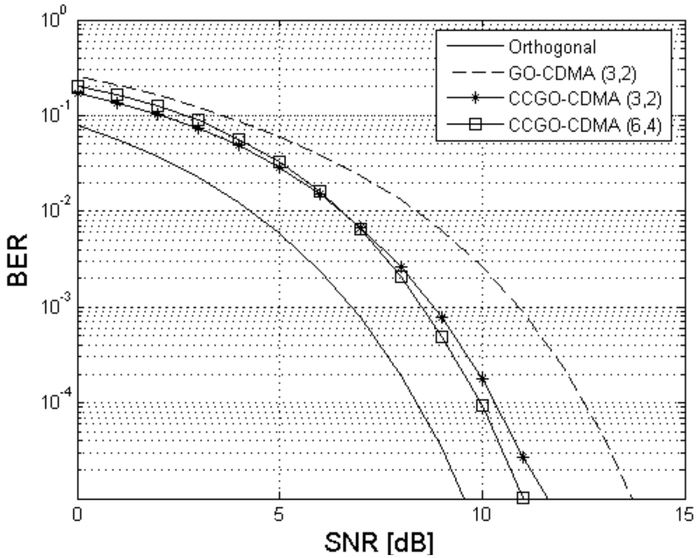
Performance of CCGO-CDMA (Euclidean distance)

Const.	L=2, s=1		L=3, s=1		L=4, s=1		L=5, s=1	
	d^2_{min}	loss	d^2_{min}	loss	d^2_{min}	loss	d^2_{min}	loss
A	1.76	3.57	3.10	1.10	4.04	-0.05	4.73	-0.73
B	2.54	1.98	3.25	0.90	4	0	4.36	-0.38
C	2.58	1.90	3.45	0.61	4.22	-0.24	4.83	-0.82
D	2.78	1.58	3.63	0.42	4.22	-0.23	4.47	-0.48

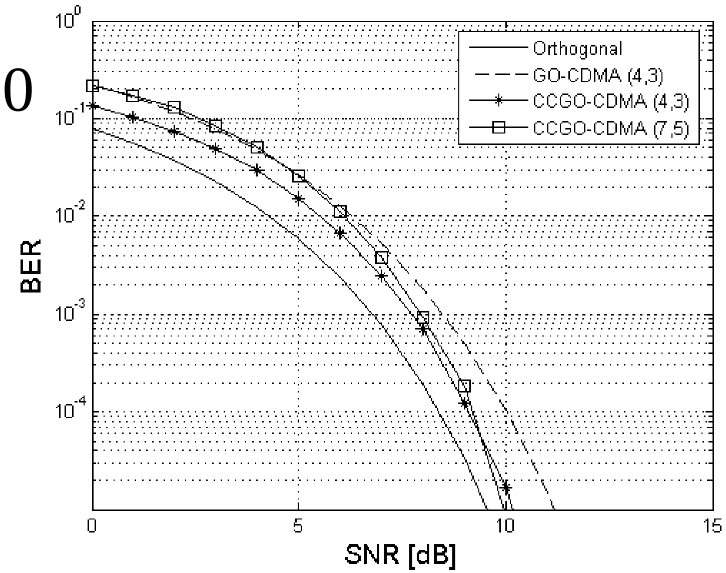
Const.	L=4, s=2		L=5, s=2	
	d^2_{min}	loss	d^2_{min}	loss
A	3.13	1.07	3.92	0.08
B	3.20	0.97	4.15	-0.16
C	3.19	0.99	3.97	0.03
D	3.28	0.86	4.19	-0.20

- Negative loss implies gain
- The order of superioirity changes when e_{ov} decreases

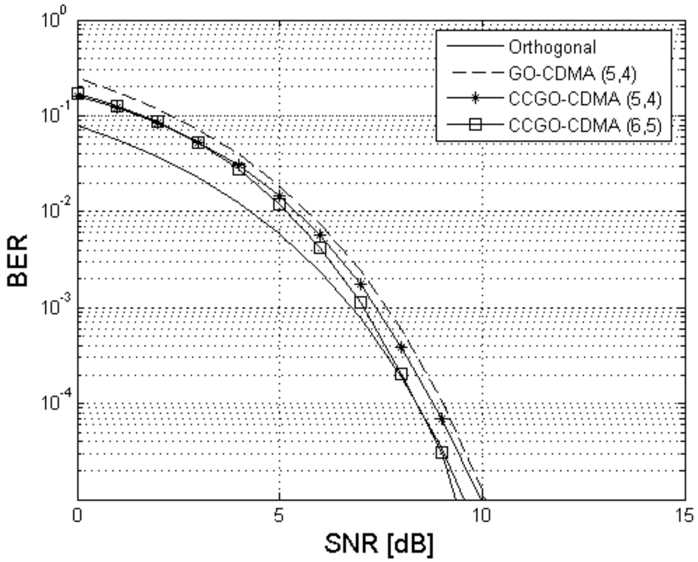
BER of constellation type C



e_{ov}=1.50



e_{ov}=1.33 and 1.40



e_{ov}=1.25 and 1.20